COURSE PROGRAMME

1. Information about the programme

1.1 University	**University "Alexandru Ioan Cuza" of Iasi**
1.2 Faculty	**Faculty of Mathematics**
1.3 Department	**Department of Mathematics**
1.4 Domain	**Mathematics**
1.5 Cycle	**Masters**
1.6 Programme / Qualification	**Applied Mathematics**

2. Information about the course

2.1 Course Name			Variational calculus and optimal control				
2.2 Course taught by			Prof. PhD. CATALIN GEORGE LEFTER				
2.3 Seminary / laboratory taught by			Prof. PhD. CATALIN GEORGE LEFTER				
2.4 Year	I	**2.5** Semester	I	**2.6** Type of evaluation*	E	**2.7** Course type**	Op

E – Exam / C – Colloquium / V – Verification
***OB – Obligatory / OP – Optionally / F – Facultative*

3. Total hours (estimated per semester and activities)

3.1 Number of hours per week	4	**3.2** course	2	**3.3** seminary/ laboratory	2
3.4 Total number of hours	56	**3.5** course	28	**3.6** seminary/ laboratory	28

Distribution	hours
Individual study using textbooks, course notes, bibliography items, etc.	40
Supplimentary study (library, on-line platforms, etc.)	35
Individual study for seminary/laboratory, homeworks, projects, etc.	40
Tutoring	0
Examination	4
Other activities	0
3.7 Total hours of individual activity*	119
3.8 Total hours per semester	175
3.9 Credit points	7

4. Pre-requisites - Curriculum (if necessary)

Differential equations. Differential geometry.

Operates with notions, methods, fundamental techniques in Mathematical Analysis, Licence level

5. Conditions (if necessary)

5.1 Course	Lecture hall
5.2 Seminary / Laboratory	Seminar hall

6. Objectives

Professional competencies
CP1.Knowing, understanding and using the fundamental concepts of mathematics and informatics, as well as the principles and methods used in the studied field
CP2.Creative application of research techniques and methods of using various software in engineering practice
Transversal competencies
CT1.The ability to quickly acquire new concepts and technologies that appear in the field of mathematics and computer science
CT3.Basic knowledge to be employed as a mathematician, statistician or actuarial expert
CT4.Acquisition of basic techniques in modeling and solving problems

7. Specific competencies/Learning outcomes

• applies scientific methods
• teach mathematics
• thinks abstractly
• performs analytical mathematical calculations
• think critically

8. Contents

8.1 Course	**Teaching methods**	**Remarks** (number oh hours, references)
Introductory notions. Examples of problems in Calculus of Variations. First variation of a functional . Euler-Lagrange equations.	Exposition/Interactive presentation	2 hours
Second variation of a functional. Sufficient conditions.	Exposition/Interactive presentation	2 hours
Hamiltonian systems. Hamiltonian formalism.	Exposition/Interactive presentation	2 hours
Total variation of a functional. Problems with end points on manifolds, transversality conditions. Non-smooth extremals, Weierstrass-Erdmann conditions.	Exposition/Interactive presentation	2 hours
Hamilton-Jacobi equation. Jacobi method for integrating hamiltonian systems.	Exposition/Interactive presentation	2 hours
Control of differential systems – an introduction. Linear systems – controllability.	Exposition/Interactive presentation	2 hours
Linear systems with convex cost functional. Optimality conditions. Pontriaghin maximum principle.	Exposition/Interactive presentation	2 hours
Dynamic programming equation. Linear equatyions with quadratic cost functional. Riccati equations.	Exposition/Interactive presentation	2 hours
Stabilization of linear differential equations. Algebraic Riccati equations.	Exposition/Interactive presentation	2 hours
Representation of differential equations on differentiable manifolds.	Exposition/Interactive presentation	2 hours
Controllability of differential systems. Orbit theorem and consequences.	Exposition/Interactive presentation	2 hours
Analytic systems. Integrability, Frobenius theorem.	Exposition/Interactive presentation	2 hours
Attainable sets and optimal control problems. Geometric form of Pontriaghin maximum principle.	Exposition/Interactive presentation	2 hours
Optimal control problems with free final time. Maximum principle for free time problems	Exposition/Interactive presentation	2 hours

Bibliography

Main references:
• V.Barbu, Mathematical methods in optimization of differential systems, Kluwer Academic Publishers, Dordrecht, 1994
• Cătălin Lefter, Calculus of Variations and Control Theory – available on the personal web page.
•
Supplementary references:
• I.M. Gelfand, S.V. Fomin, Calculus of variations, 2000
• J. Zabczyk, Mathematical control theory; an introduction, Birkhauser, Boston, 2008.

8.2 Seminary / Laboratory	Teaching methods	Remarks (number oh hours, references)
Introductory notions. Examples of problems in Calculus of Variations. First variation of a functional . Euler-Lagrange equations.	Interactive presentation/Exercises/Dialogue	2 hours
Second variation of a functional. Sufficient conditions.	Interactive presentation/Exercises/Dialogue	2 hours
Hamiltonian systems. Hamiltonian formalism.	Interactive presentation/Exercises/Dialogue	2 hours
Total variation of a functional. Problems with end points on manifolds, transversality conditions. Non-smooth extremals, Weierstrass-Erdmann conditions.	Interactive presentation/Exercises/Dialogue	2 hours
Hamilton-Jacobi equation. Jacobi method for integrating hamiltonian systems.	Interactive presentation/Exercises/Dialogue	2 hours
Control of differential systems – an introduction. Linear systems – controllability.	Interactive presentation/Exercises/Dialogue	2 hours
Linear systems with convex cost functional. Optimality conditions. Pontriaghin maximum principle.	Interactive presentation/Exercises/Dialogue	2 hours
Dynamic programming equation. Linear equatyions with quadratic cost functional. Riccati equations.	Interactive presentation/Exercises/Dialogue	2 hours
Stabilization of linear differential equations. Algebraic Riccati equations.	Interactive presentation/Exercises/Dialogue	2 hours
Representation of differential equations on differentiable manifolds.	Interactive presentation/Exercises/Dialogue	2 hours
Controllability of differential systems. Orbit theorem and consequences.	Interactive presentation/Exercises/Dialogue	2 hours
Analytic systems. Integrability, Frobenius theorem.	Interactive presentation/Exercises/Dialogue	2 hours
Attainable sets and optimal control problems. Geometric form of Pontriaghin maximum principle.	Interactive presentation/Exercises/Dialogue	2 hours
Optimal control problems with free final time. Maximum principle for free time problems	Interactive presentation/Exercises/Dialogue	2 hours

Bibliography

Bibliography

• V.Barbu, Mathematical methods in optimization of differential systems, Kluwer Academic Publishers, Dordrecht, 1994
• I.M. Gelfand, S.V. Fomin, Calculus of variations, 2000

9. Coordination of the contents with the expectations of the community representatives, professional associations and relevant employers in the corresponding domain

The material in this course is chosen in order to offer the students a unified understanding of the fundamental ideas studied in the courses of Theoretical mechanics, Differential geometry, Differential equations. It offers an introduction to the modern field of Control theory which is directly connected to the practical applications of Mathematics.

10. Assessment and examination

10.1 Continuous assessment		Percentage (min. 30%)		30
Course	Assessment type			Written assessment
	Percentage			30
	Failure to pass the continuous assessment results in failure to pass the final assessment			Yes
	Assessment methods	Details	Percentage	with reexamination
		Continuous written assessment	50	Yes
		Test	50	Yes

<table>
<tr><td rowspan="4">Seminary / Laboratory</td><td colspan="3">Assessment type</td><td>Written assessment</td></tr>
<tr><td colspan="3">Percentage</td><td>70</td></tr>
<tr><td colspan="3">Failure to pass the continuous assessment results in failure to pass the final assessment</td><td>Yes</td></tr>
<tr><td>Assessment methods</td><td>Details</td><td>Percentage</td><td>with reexamination</td></tr>
<tr><td></td><td>Continuous written assessment</td><td>50</td><td>Yes</td></tr>
<tr><td></td><td>Test</td><td>50</td><td>Yes</td></tr>
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10.2 Final assessment	Percentage (max. 70%)	70
	Assessment type	Final mixed assessment

10.3 Special notes (special situations is assessment)	

10.4 Minimum performance standard	

Date, **Course coordinator,**
Prof. PhD. CATALIN GEORGE LEFTER

Seminary coordinator,
Prof. PhD. CATALIN GEORGE LEFTER

Aproval date in the department,

Head of the departament,
Prof. PhD. IONEL DUMITREL GHIBA