```
(* PARABOLA  *)

(* polinomul de gradul al doilea care genereaza parabola *)

P[x_, y_] := 9 x^2 - 6 x y + y^2 + 20 x;

(* coeficientii formei patratice *)

a11 = Coefficient[P[x, y], x^2];
a12 = Coefficient[P[x, y], x y] / 2;
a22 = Coefficient[P[x, y], y^2];

A = {{a11, a12}, {a12, a22}};

MatrixForm[A]
```

$$\begin{pmatrix} 9 & -3 \\ -3 & 1 \end{pmatrix}$$

```
(* avem gen parabolic *)

δ = Det[A]

0

(* I1 este urma matricii A; simbolul I este protejat *)

I1 = Tr[A]

10

a10 = Coefficient[Coefficient[P[x, y], x], y, 0] / 2;
a20 = Coefficient[Coefficient[P[x, y], y], x, 0] / 2;

a00 = Coefficient[Coefficient[P[x, y], x, 0], y, 0];

(*
   Simbolul D este protejat, foloses DP
*)

DP = {{a11, a12, a10}, {a12, a22, a20}, {a10, a20, a00}}

{{9, -3, 10}, {-3, 1, 0}, {10, 0, 0}}

MatrixForm[DP]
```

$$\begin{pmatrix} 9 & -3 & 10 \\ -3 & 1 & 0 \\ 10 & 0 & 0 \end{pmatrix}$$

```
(* avem nedegenerare *)

Δ = Det[DP]

-100

(* Invariantul urmator intra in joc doar daca Δ=0 !!! *)

Δ1 = δ + Det[{{a11, a10}, {a10, a00}}] + Det[{{a22, a20}, {a20, a00}}]

-100

(* autovalorile si vectorii proprii matricii A ; una este 0 *)

λ = Eigenvalues[A];
```

```
λ1 = λ[[1]]
λ2 = λ[[2]]
```

10

0

```
vect = Eigenvectors[A];

nr1 = Sqrt[vect[[1]].vect[[1]]];
nr2 = Sqrt[vect[[2]].vect[[2]]];

v1 = vect[[1]] / nr1
v2 = vect[[2]] / nr2
```

$$\left\{-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right\}$$

$$\left\{\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right\}$$

```
(* Vreau ca valoarea proprie 0 sa fie λ1 *)

{λ1, λ2, v1, v2} = If[λ1 == 0, {λ1, λ2, v1, v2}, {λ2, λ1, v2, v1}]
```

$$\left\{0, 10, \left\{\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right\}, \left\{-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right\}\right\}$$

```
(* orientarea *)

or = Det[{v1, v2}]
```

1

```
(*
    Daca orientarea eset negativa schimb semnul la v2
*)

w2 = If[or > 0, v2, -v2];
v2 = w2
```

$$\left\{-\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right\}$$

```
(* varful parabolei *)

eq1 = D[P[x, y], x]
eq2 = D[P[x, y], y]
```

$20 + 18 x - 6 y$

$-6 x + 2 y$

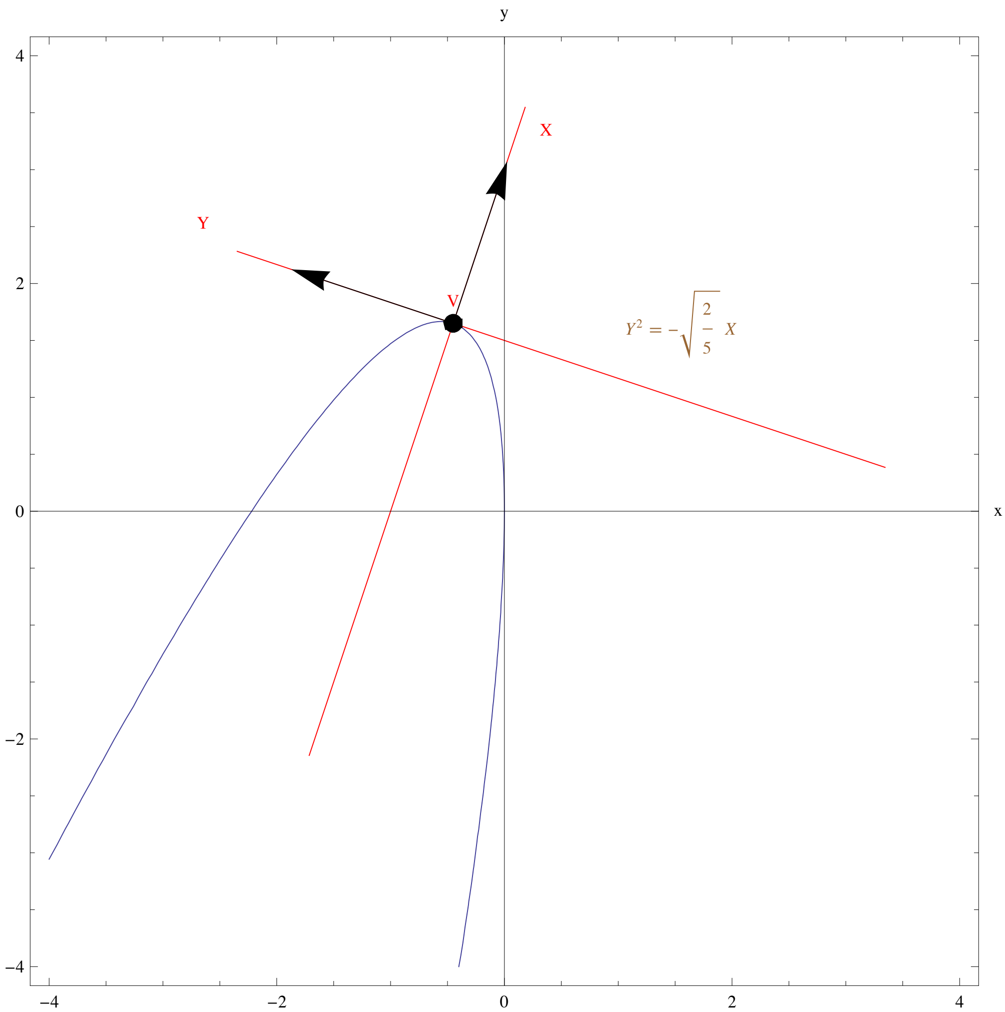
```
sol = Solve[{eq1 == a v1[[1]], eq2 == a v1[[2]], P[x, y] == 0}, {x, y, a}]
```

$$\left\{\left\{x \to -\frac{9}{20}, y \to \frac{33}{20}, a \to 2\sqrt{10}\right\}\right\}$$

```
x0 = x /. sol[[1]];
y0 = y /. sol[[1]];

r = 0.04;
```

```
varf = ParametricPlot[{x0, y0} + r {Cos[t], Sin[t]},
   {t, 0, 2 Pi}, PlotStyle → {Black, Thickness[0.01]}];

(* focarul : se foloseste axa focala definita mai inainte *)

F = ParametricPlot[{x0, y0} + c v + r {Cos[t], Sin[t]},
   {t, 0, 2 Pi}, PlotStyle → {Purple, Thickness[0.005]}];

axa1 = ParametricPlot[{x0, y0} + t v1,
   {t, -4, 2}, AspectRatio → Automatic, PlotStyle → Red];
axa2 = ParametricPlot[{x0, y0} + t v2, {t, -4, 2},
   AspectRatio → Automatic, PlotStyle → Red];

VX = Graphics[Arrow[{{x0, y0}, {x0, y0} + 1.5 v1}]];
VY = Graphics[Arrow[{{x0, y0}, {x0, y0} + 1.5 v2}]];

axVX = Graphics[{Red, Text["X", {x0, y0} +  v1 + {0.5, 0.75}]}];
axVY = Graphics[{Red, Text["Y", {x0, y0} +  2 v2 + {-0.3, 0.25}]}];

varfV = Graphics[{Red, Text["V", {x0, y0} + {0, 0.2}]}];

foc = Graphics[{Purple, Text["F", {x0, y0} + c v + {0, 0.2}]}];

(* reprezentare grafica *)

Parabola = ContourPlot[P[x, y] == 0, {x, -4, 4}, {y, -4, 4},
   Axes → True, AxesLabel → {"x", "y"}, AspectRatio → Automatic];

(* p se calculeaza mai jos *)

eqred = Graphics[
   {Brown, Text["Hiperbola:", {x0 - 5, y0 + 5}], Text[ Y^2 == 2 p X, {x0 + 2, y0}]}];
```

```
Show[Parabola, axa1, axa2, varf, axVX, axVY, eqred, VX, VY, varfV]
```




```
(* translatia in varful parabolei; scrierea in noile coordonate *)

(*
   SCHIMBAREA COORDONATELOR, PENTRU A ADUCE CONICA LA FORMA REDUSA
*)

(* ROTATIA : {x, y} --> {xp, yp} *)

{x, y} = Transpose[{v1, v2}].{xp, yp}
```

$$\left\{\frac{xp}{\sqrt{10}} - \frac{3\ yp}{\sqrt{10}},\ \frac{3\ xp}{\sqrt{10}} + \frac{yp}{\sqrt{10}}\right\}$$

```
(* ecuatia elipsei in noile coordonate xp, yp *)

P1[xp_, yp_] := Simplify[P[x, y]] // Expand

P1[xp, yp]
```

$$2\sqrt{10}\ xp - 6\sqrt{10}\ yp + 10\ yp^2$$

```
(* A DOUA SCHIMBARE DE COORDONATE *)

(* refacem coordonatele varfului in noile coordonate xp, yp *)
```

```
{xp0, yp0} = {v1, v2}.{x0, y0}
```

$$\left\{\frac{9}{2\sqrt{10}}, \frac{3}{\sqrt{10}}\right\}$$

```
(* FACEM TRANSLATIA IN VARF, centrul noului reper *)
```

```
xp = X + xp0
yp = Y + yp0
```

$$\frac{9}{2\sqrt{10}} + X$$

$$\frac{3}{\sqrt{10}} + Y$$

```
P2[X_, Y_] := Simplify[P1[xp, yp]]
```

$$p = \frac{-1}{2}\,\frac{\text{Coefficient}[P2[X, Y], X]}{\text{Coefficient}[P2[X, Y], Y, 2]}$$

$$-\frac{1}{\sqrt{10}}$$

```
(* PARABOLA  *)

(* polinomul de gradul al doilea care genereaza hparabola *)

P[x_, y_] := 16 x^2 - 24 x y + 9 y^2 - 94 x - 42 y + 205;

(* coeficientii formei patratice *)

a11 = Coefficient[P[x, y], x^2];
a12 = Coefficient[P[x, y], x y] / 2;
a22 = Coefficient[P[x, y], y^2];

A = {{a11, a12}, {a12, a22}};

MatrixForm[A]
```

$$\begin{pmatrix} 16 & -12 \\ -12 & 9 \end{pmatrix}$$

```
(* avem gen hiperbolic *)

δ = Det[A]

0

(* I1 este urma matricii A; simbolul I este protejat *)

I1 = Tr[A]

25

a10 = Coefficient[Coefficient[P[x, y], x], y, 0] / 2;
a20 = Coefficient[Coefficient[P[x, y], y], x, 0] / 2;

a00 = Coefficient[Coefficient[P[x, y], x, 0], y, 0];

(*
   Simbolul D este protejat, foloses DP
*)

DP = {{a11, a12, a10}, {a12, a22, a20}, {a10, a20, a00}}

{{16, -12, -47}, {-12, 9, -21}, {-47, -21, 205}}

MatrixForm[DP]
```

$$\begin{pmatrix} 16 & -12 & -47 \\ -12 & 9 & -21 \\ -47 & -21 & 205 \end{pmatrix}$$

```
(* avem nedegenerare *)

Δ = Det[DP]

-50 625

(* Invariantul urmator intra in joc doar daca Δ=0 !!! *)

Δ1 = δ + Det[{{a11, a10}, {a10, a00}}] + Det[{{a22, a20}, {a20, a00}}]

2475

(* autovalorile si vectorii proprii matricii A ; una este 0 *)

λ = Eigenvalues[A];
```

```
λ1 = λ[[1]]
λ2 = λ[[2]]
```

25

0

```
vect = Eigenvectors[A];

nr1 = Sqrt[vect[[1]].vect[[1]]];
nr2 = Sqrt[vect[[2]].vect[[2]]];

v1 = vect[[1]] / nr1
v2 = vect[[2]] / nr2
```

$\left\{-\frac{4}{5}, \frac{3}{5}\right\}$

$\left\{\frac{3}{5}, \frac{4}{5}\right\}$

```
(* Vreau ca valoarea proprie 0 sa fie λ1 *)

{λ1, λ2, v1, v2} = If[λ1 == 0, {λ1, λ2, v1, v2}, {λ2, λ1, v2, v1}]
```

$\left\{0, 25, \left\{\frac{3}{5}, \frac{4}{5}\right\}, \left\{-\frac{4}{5}, \frac{3}{5}\right\}\right\}$

```
(* orientarea *)

or = Det[{v1, v2}]
```

1

```
(*
    Daca orientarea eset negativa schimb semnul la v2
*)

w2 = If[or > 0, v2, -v2];
v2 = w2
```

$\left\{-\frac{4}{5}, \frac{3}{5}\right\}$

```
(* varful parabolei *)

eq1 = D[P[x, y], x]
eq2 = D[P[x, y], y]
```

$-94 + 32\,x - 24\,y$

$-42 - 24\,x + 18\,y$

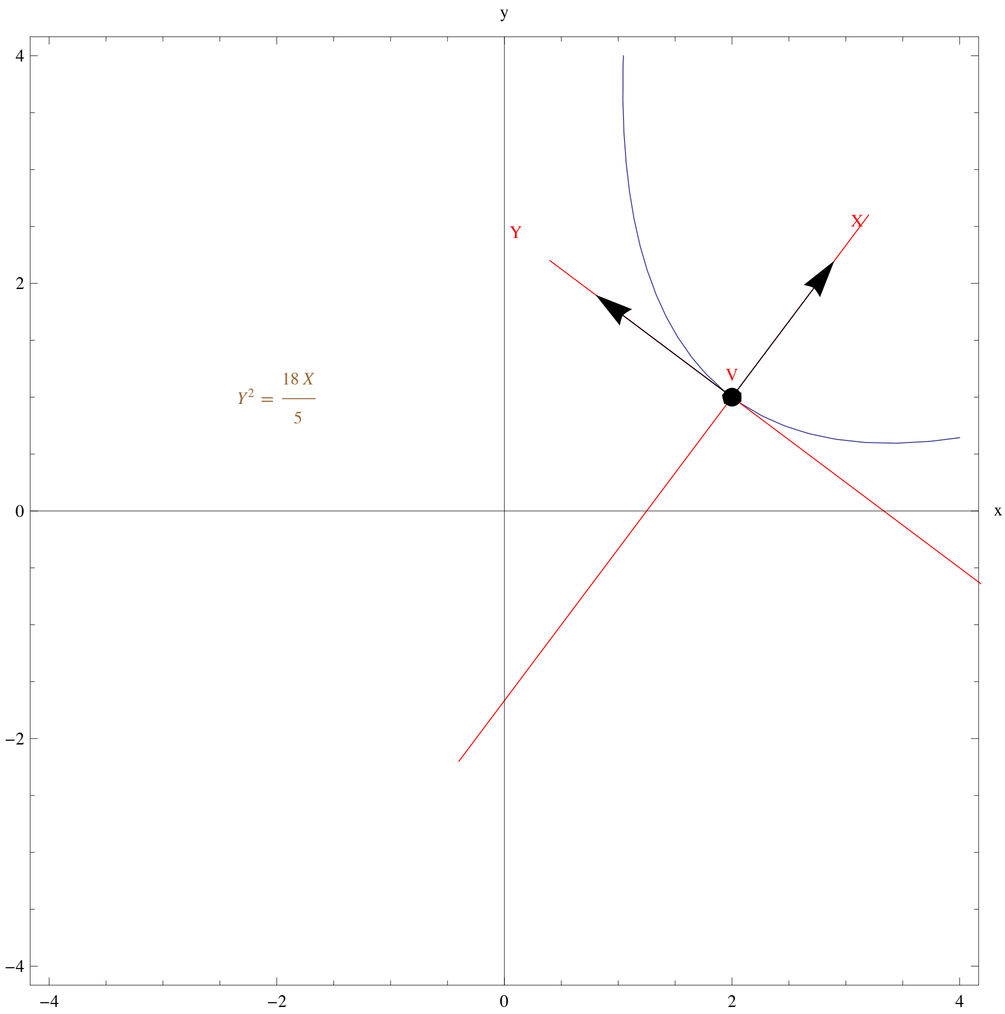
```
sol = Solve[{eq1 == a v1[[1]], eq2 == a v1[[2]], P[x, y] == 0}, {x, y, a}]
```

{{x → 2, y → 1, a → -90}}

```
x0 = x /. sol[[1]];
y0 = y /. sol[[1]];

r = 0.04;

varf = ParametricPlot[{x0, y0} + r {Cos[t], Sin[t]},
   {t, 0, 2 Pi}, PlotStyle → {Black, Thickness[0.01]}];
```

```
(* focarul : se foloseste axa focala definita mai inainte *)

F = ParametricPlot[{x0, y0} + c v + r {Cos[t], Sin[t]},
   {t, 0, 2 Pi}, PlotStyle → {Purple, Thickness[0.005]}];

axa1 = ParametricPlot[{x0, y0} + t v1,
   {t, -4, 2}, AspectRatio → Automatic, PlotStyle → Red];
axa2 = ParametricPlot[{x0, y0} + t v2, {t, -4, 2},
   AspectRatio → Automatic, PlotStyle → Red];

VX = Graphics[Arrow[{{x0, y0}, {x0, y0} + 1.5 v1}]];
VY = Graphics[Arrow[{{x0, y0}, {x0, y0} + 1.5 v2}]];

axVX = Graphics[{Red, Text["X", {x0, y0} +  v1 + {0.5, 0.75}]}];
axVY = Graphics[{Red, Text["Y", {x0, y0} +  2 v2 + {-0.3, 0.25}]}];

varfV = Graphics[{Red, Text["V", {x0, y0} + {0, 0.2}]}];

foc = Graphics[{Purple, Text["F", {x0, y0} + c v + {0, 0.2}]}];

(* reprezentare grafica *)

Parabola = ContourPlot[P[x, y] == 0, {x, -4, 4}, {y, -4, 4},
   Axes → True, AxesLabel → {"x", "y"}, AspectRatio → Automatic];

(* p se calculeaza mai jos *)

eqred = Graphics[
   {Brown, Text["Hiperbola:", {x0 - 5, y0 + 5}], Text[ Y^2 == 2 p X, {x0 - 4, y0}]}];
```

```
Show[Parabola, axa1, axa2, varf, axVX, axVY, eqred, VX, VY, varfV]
```




```
(* translatia in varful parabolei; scrierea in noile coordonate *)

(*
   SCHIMBAREA COORDONATELOR, PENTRU A ADUCE CONICA LA FORMA REDUSA
*)

(* MAI INTAI TRANSLATIA IN VERF, centrul noului reper
             {x,y} --> {xp,yp}
*)

x = xp + x0
y = yp + y0
```

2 + xp

1 + yp

```
P1[xp_, yp_] := Simplify[P[x, y]]

P1[xp, yp] // Expand
```

$-54\, xp + 16\, xp^2 - 72\, yp - 24\, xp\, yp + 9\, yp^2$

```
(* ROTATIA : {xp, yp} --> {X, Y} *)
```

```
{xp, yp} = Transpose[{v1, v2}].{X, Y}
```

$$\left\{\frac{3\,X}{5}-\frac{4\,Y}{5},\ \frac{4\,X}{5}+\frac{3\,Y}{5}\right\}$$

```
(* ecuatia elipsei in noile coordonate xp, yp *)

P2[X_, Y_] := Simplify[P1[xp, yp]] // Expand

P2[X, Y]
```

$$-90\,X+25\,Y^2$$

$$p=\frac{-1}{2}\,\frac{\text{Coefficient}[P2[X,Y],X]}{\text{Coefficient}[P2[X,Y],Y,2]}$$

$$\frac{9}{5}$$