

```
(*
Scriti un program care sa ne furnizeze
polinoamele Lagrange de baza pentru un set de noduri date.
*)
```

```
X = {x0, x1, x2, x3};
```

```
n = Length[X];
```

```
For[i = 1, i ≤ n, i++,
```

```
  a = 1;
```

```
  For[j = 1, j ≤ n, j++,
```

```
    If[j ≠ i, a = a *  $\frac{x - X[[j]]}{X[[i]] - X[[j]]}$  ]];
```

```
  L[i, x_] = a;
```

```
  Print[L[i, x]]]
```

$$\frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$\frac{(x - x_0)(x - x_2)(x - x_3)}{(-x_0 + x_1)(x_1 - x_2)(x_1 - x_3)}$$

$$\frac{(x - x_0)(x - x_1)(x - x_3)}{(-x_0 + x_2)(-x_1 + x_2)(x_2 - x_3)}$$

$$\frac{(x - x_0)(x - x_1)(x - x_2)}{(-x_0 + x_3)(-x_1 + x_3)(-x_2 + x_3)}$$

```
L[1, x]
```

$$\frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

```
 $\sum_{i=1}^n X[[i]] L[i, x]$  // Simplify
```

```
x
```

```
 $\sum_{i=1}^n L[i, x]$  // Simplify
```

```
1
```

```

x = {-1, -1/√2, 0, 1/√2, 1};
y = {0, 1/√2, 1, 1/√2, 0};
n = Length[X];

For[i = 1, i ≤ n, i++,
  a = 1;
  For[j = 1, j ≤ n, j++,
    If[j ≠ i, a = a *  $\frac{x - X[[j]]}{X[[i]] - X[[j]]}$ ]];
  L[i, x_] = a]

c[s_] := {Cos[s], Sin[s]};

cerc = ParametricPlot[c[s], {s, 0, Pi}, PlotStyle → Blue];

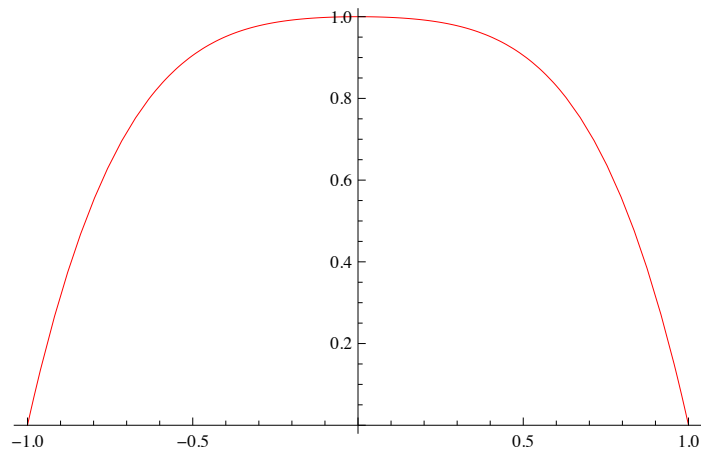
Table[A[i], {i, 1, n}];

For[i = 1, i ≤ n, i++,
  A[i] = c[X[[i]] Pi]];
PA = Graphics[{Red, PointSize[Large], Point[c[0]]}];
PB = Graphics[{Red, PointSize[Large], Point[c[Pi/4]]}];
PC = Graphics[{Red, PointSize[Large], Point[c[Pi/2]]}];
PD = Graphics[{Red, PointSize[Large], Point[c[3 Pi/4]]}];
PE = Graphics[{Red, PointSize[Large], Point[c[Pi]]}];

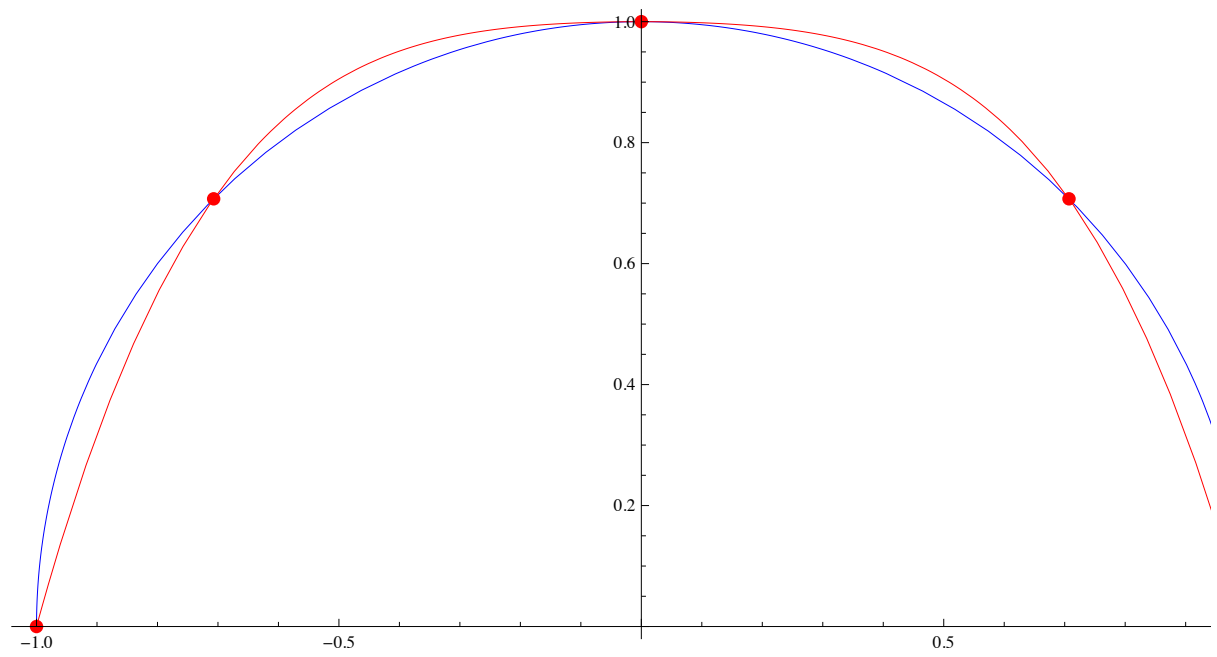
L[x_] := Simplify[ $\sum_{i=1}^n (L[i, x] Y[[i]])$ ]

Lg = Plot[L[x], {x, X[[1]], X[[n]]}, PlotStyle → Red]

```



```
Show[cerc, Lg, PA, PB, PC, PD, PE]
```



```
T = {0, 1 / 3, 2 / 3, 1};
```

```
n = Length[T];
```

```
For[i = 1, i ≤ n, i++,
```

```
  a = 1;
```

```
  For[j = 1, j ≤ n, j++,
```

```
    If[j ≠ i, a = a *  $\frac{t - T[[j]]}{T[[i]] - T[[j]]}$ ]]];
```

```
  L[i, t_] = a;
```

```
  Print[L[i, t]]]
```

$$\frac{9}{2} (1-t) \left(-\frac{2}{3}+t\right) \left(-\frac{1}{3}+t\right)$$

$$\frac{27}{2} (-1+t) \left(-\frac{2}{3}+t\right) t$$

$$-\frac{27}{2} (-1+t) \left(-\frac{1}{3}+t\right) t$$

$$\frac{9}{2} \left(-\frac{2}{3}+t\right) \left(-\frac{1}{3}+t\right) t$$

```
c[s_] := {Cos[2 s Pi], Sin[2 s Pi], s};
```

```
Table[A[i], {i, 1, n}];
```

```
For[i = 1, i ≤ n, i++, A[i] = c[T[[i]]];
```

```
  Print[A[i]]]
```

```
{1, 0, 0}
```

$$\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{3}\right\}$$

$$\left\{-\frac{1}{2}, -\frac{\sqrt{3}}{2}, \frac{2}{3}\right\}$$

```
{1, 0, 1}
```

```
(*
```

```
A0=c[t0];A1=c[t1];A2=c[t2];A3=c[t3]
```

```
*)
```

```
L[t_] := Simplify[ $\sum_{i=1}^n (L[i, t] A[i])$ ]
```

```
L[t]
```

$$\left\{\frac{1}{4} (4-27 t+27 t^2), \frac{27}{4} \sqrt{3} (-1+t) t (-1+2 t), t\right\}$$

```
PA = Graphics3D[{Red, PointSize[Large], Point[c[T[[1]]]]}];
```

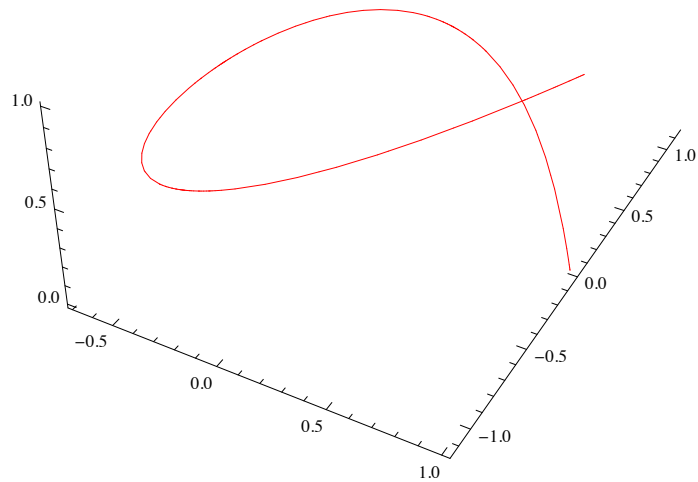
```
PB = Graphics3D[{Red, PointSize[Large], Point[c[T[[2]]]]}];
```

```
PC = Graphics3D[{Red, PointSize[Large], Point[c[T[[3]]]]}];
```

```
PD = Graphics3D[{Red, PointSize[Large], Point[c[T[[4]]]]}];
```

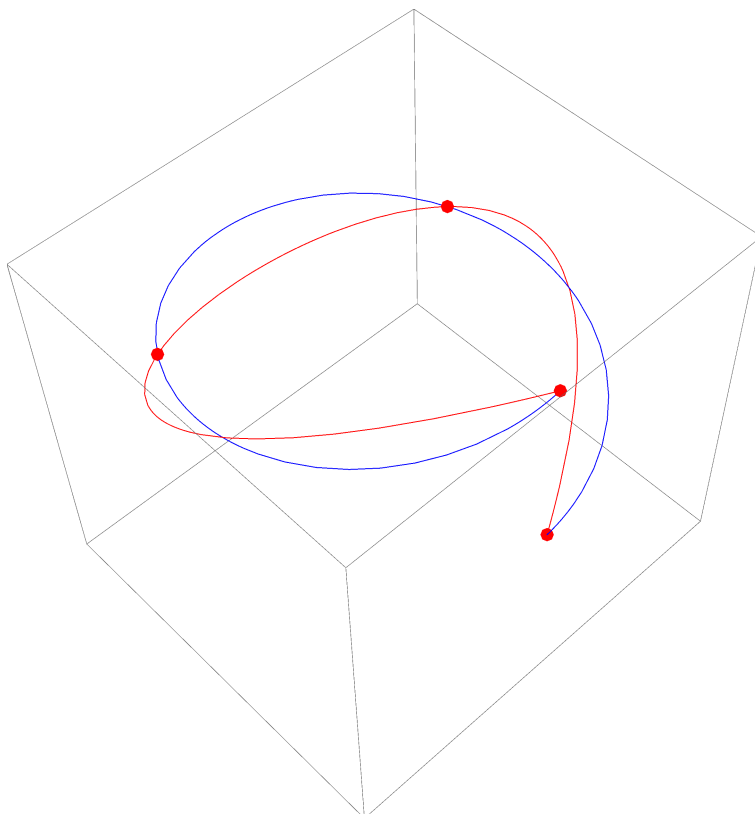
```
El = ParametricPlot3D[c[s], {s, 0, 1}, PlotStyle → Blue, Boxed → False];
```

```
Lg = ParametricPlot3D[L[t], {t, 0, 1}, PlotStyle -> Red, Boxed -> False]
```



```
S = Graphics3D[{Opacity[0], Sphere[{0, 0, 0}, 1]}];
```

```
Show[S, Lg, E1, PA, PB, PC, PD]
```



```

T = {0, 1 / 3, 2 / 3, 1};
n = Length[T];

For[i = 1, i ≤ n, i++,
  a = 1;
  For[j = 1, j ≤ n, j++,
    If[j ≠ i, a = a *  $\frac{t - T[[j]]}{T[[i]] - T[[j]]}$ ]];
  L[i, t_] = a]

c[s_] := {2 Cos[s], 2 Sin[s]};
c[s_] := {5 Cos[s], 3 Sin[s]};
c[s_] := {3 Cos[s] + 1, 2 Sin[s] + s};
c[s_] := {Cos[2 s], Sin[s] + Cos[s]};
c[s_] := {2 s + Cos[s], Sin[s] + Cos[2 s]};
c[s_] := {s + Cos[3 s], Sin[s] + Cos[2 s]};

cerc = ParametricPlot[c[s], {s, 0, 2 Pi}, PlotStyle → Blue];

Table[A[i], {i, 1, n}];

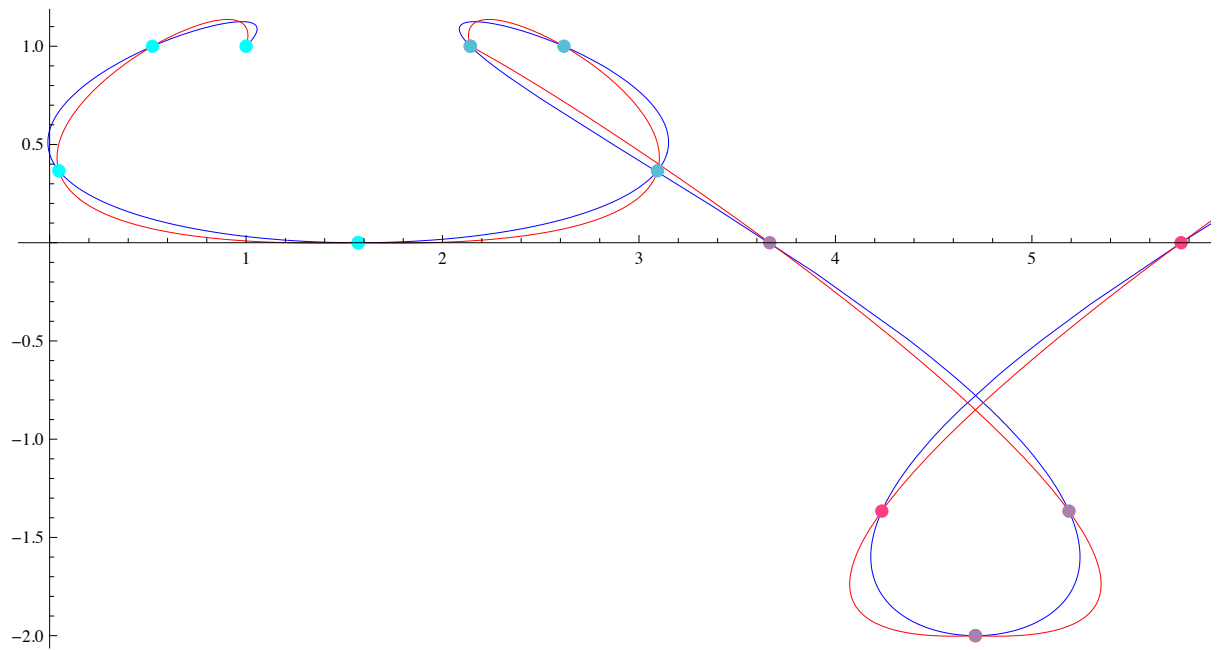
For[k = 0, k ≤ 3, k++,
  For[i = 1, i ≤ n, i++,
    A[i] = c[T[[i]] Pi / 2 + k / 2 Pi];
    PA[k] = Graphics[{RGBColor[k / 3, 1 - k / 4, 1 - k / 6],
      PointSize[Large], Point[c[T[[1]] Pi / 2 + k / 2 Pi]}];
    PB[k] = Graphics[{RGBColor[k / 3, 1 - k / 4, 1 - k / 6],
      PointSize[Large], Point[c[T[[2]] Pi / 2 + k / 2 Pi]}];
    PC[k] = Graphics[{RGBColor[k / 3, 1 - k / 4, 1 - k / 6],
      PointSize[Large], Point[c[T[[3]] Pi / 2 + k / 2 Pi]}];
    PD[k] = Graphics[{RGBColor[k / 3, 1 - k / 4, 1 - k / 6],
      PointSize[Large], Point[c[T[[4]] Pi / 2 + k / 2 Pi]}];

  L[t_] := Simplify[ $\sum_{i=1}^n (L[i, t] A[i])$ ];

  Lg[k] = ParametricPlot[L[t], {t, 0, 1}, PlotStyle → Red]

```

```
Show[cerc, Lg[1], Lg[2], Lg[3], Lg[0], PA[1], PA[2], PA[3], PA[0], PB[1],  
PB[2], PB[3], PB[0], PC[1], PC[2], PC[3], PC[0], PD[1], PD[2], PD[3], PD[0]]
```



```
X = {-3, -2, -1, 0, 0.5, 1.8, 3};
```

```
n = Length[X];
```

```
For[i = 1, i ≤ n, i++,
```

```
  a = 1;
```

```
  For[j = 1, j ≤ n, j++,
```

```
    If[j ≠ i, a = a *  $\frac{x - X[[j]]}{X[[i]] - X[[j]]}$ ]];
```

```
  L[i, x_] = a]
```

```
L[2, x] // Expand
```

```
0.0852632 x - 0.132632 x2 - 0.132632 x3 + 0.109474 x4 + 0.0136842 x5 - 0.0105263 x6
```

```
f[x_] :=  $\frac{2x + 1}{x^4 + 1}$ ;
```

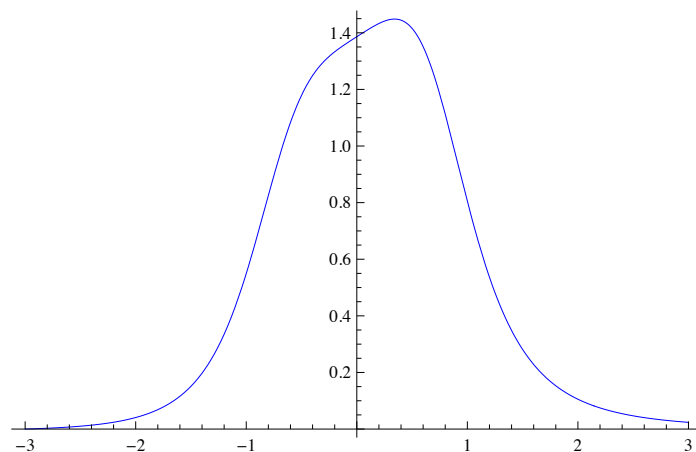
```
f[x_] :=  $\frac{\text{Log}[x + 4]}{x^4 + 1}$ ;
```

```
(*  
A0=c[t0];A1=c[t1];A2=c[t2];A3=c[t3]  
*)
```

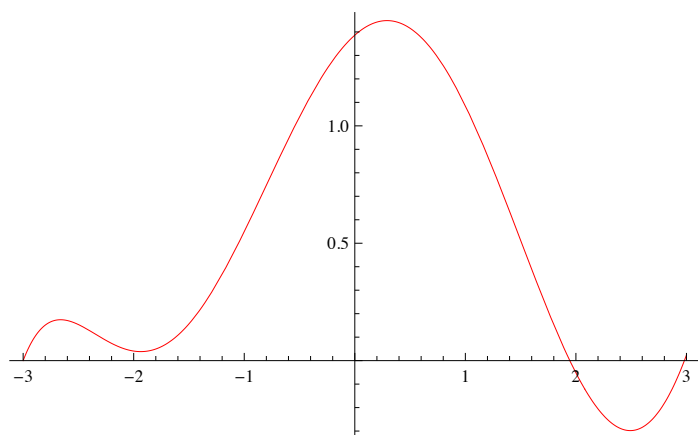
```
L[x_] := Simplify[ $\sum_{i=1}^n (L[i, x] f[X[[i]])$ ]
```

```
PA = Graphics[{Red, PointSize[Large], Point[{X[[1]], f[X[[1]]}]}];  
PB = Graphics[{Red, PointSize[Large], Point[{X[[2]], f[X[[2]]}]}];  
PC = Graphics[{Red, PointSize[Large], Point[{X[[3]], f[X[[3]]}]}];  
PD = Graphics[{Red, PointSize[Large], Point[{X[[4]], f[X[[4]]}]}];  
PE = Graphics[{Red, PointSize[Large], Point[{X[[5]], f[X[[5]]}]}];  
PF = Graphics[{Red, PointSize[Large], Point[{X[[6]], f[X[[6]]}]}];  
PG = Graphics[{Red, PointSize[Large], Point[{X[[7]], f[X[[7]]}]}];
```

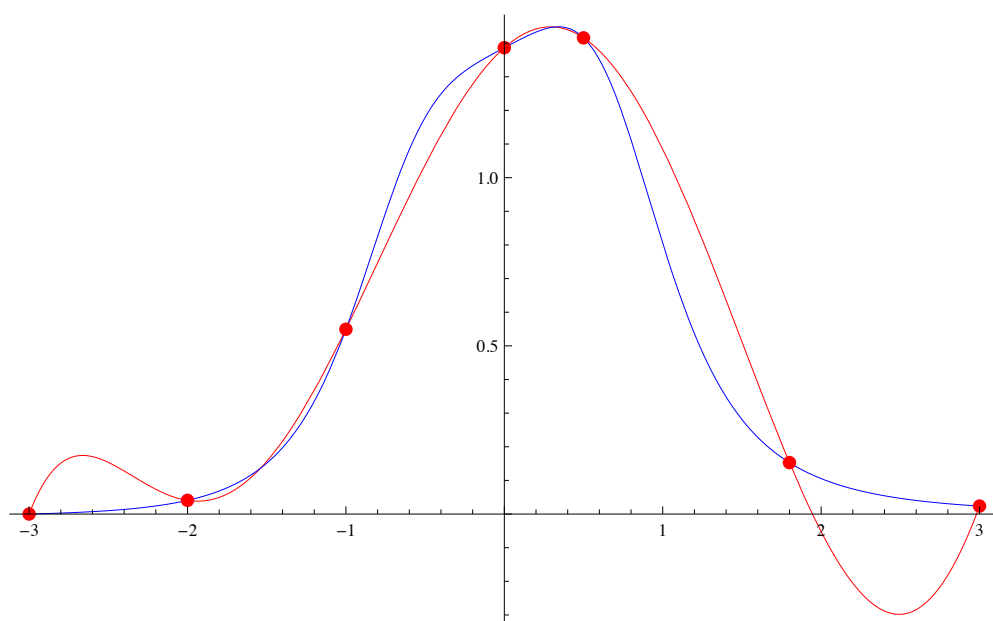
```
curba = Plot[f[x], {x, X[[1]], X[[n]]}, PlotStyle → Blue]
```




```
Lg = Plot[L[x], {x, X[[1]], X[[n]]}, PlotStyle -> Red]
```



```
Show[Lg, curba, PA, PB, PC, PD, PE, PF, PG]
```



```
X = {-3, -2, -1, 0};
```

```
n = Length[X];
```

```
For[i = 1, i ≤ n, i++,
```

```
  a = 1;
```

```
  For[j = 1, j ≤ n, j++,
```

```
    If[j ≠ i, a = a *  $\frac{x - X[[j]]}{X[[i]] - X[[j]]}$ ]];
```

```
  L[i, x_] = a]
```

```
L[2, x] // Expand
```

$$\frac{3x}{2} + 2x^2 + \frac{x^3}{2}$$

```
Lstg[x_] := Simplify[ $\sum_{i=1}^n (L[i, x] f[X[[i]])$ ]
```

```

Cstg = Plot[Lstg[x], {x, X[[1]], X[[n]]}, PlotStyle → Brown];

X = {0, 0.5, 1.8, 3};

n = Length[X];

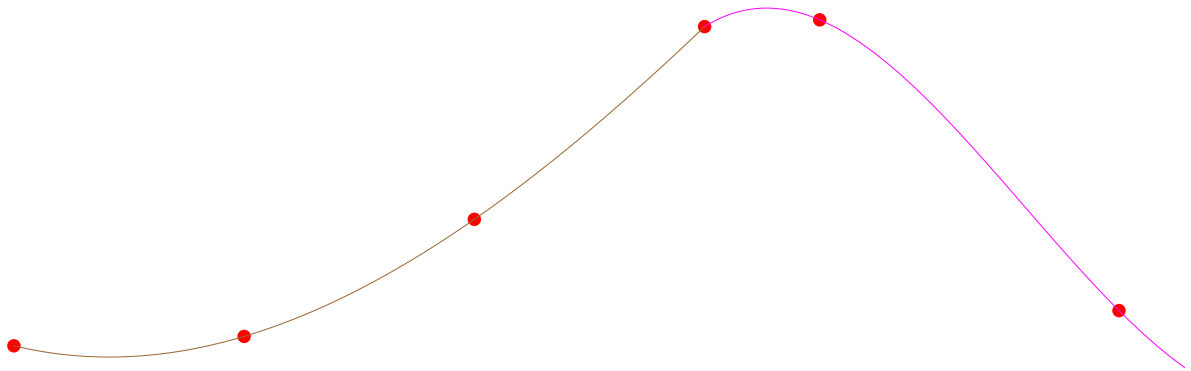
For[i = 1, i ≤ n, i++,
  a = 1;
  For[j = 1, j ≤ n, j++,
    If[j ≠ i, a = a *  $\frac{x - X[[j]]}{X[[i]] - X[[j]]}$ ]];
  L[i, x_] = a]

Ldr[x_] := Simplify[ $\sum_{i=1}^n (L[i, x] f[X[[i]])$ ]

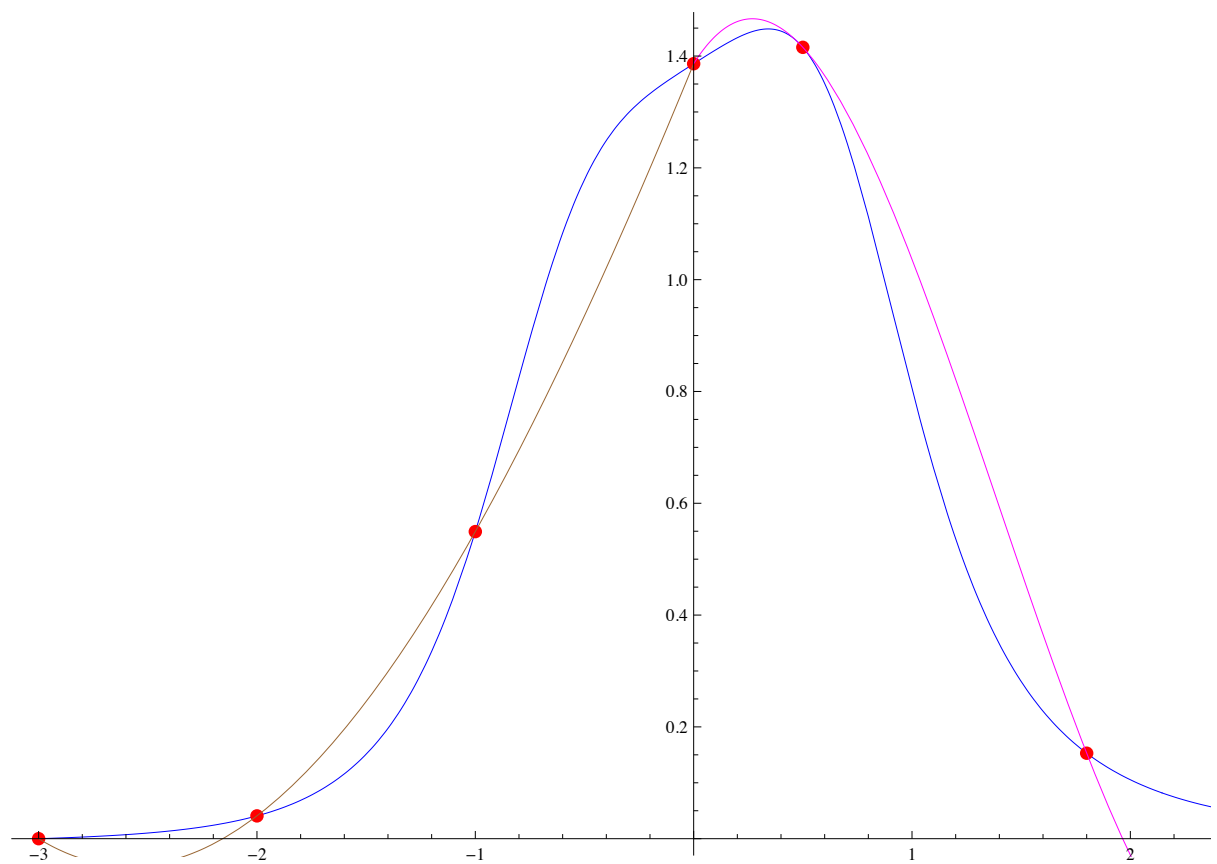
Cdr = Plot[Lstg[x], {x, X[[1]], X[[n]]}, PlotStyle → Magenta];

Show[PA, PB, PC, PD, PE, PF, PG, Cstg, Cdr]

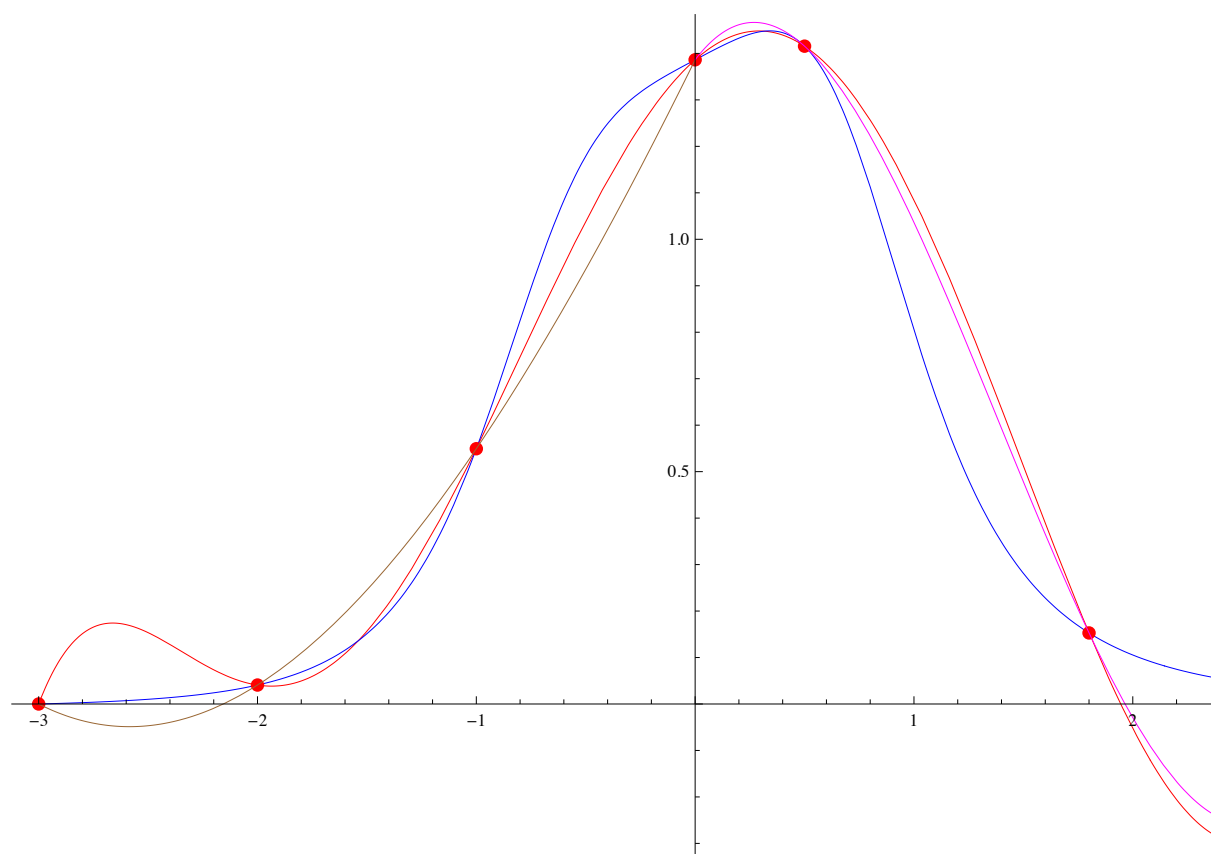
```



Show[curba, PA, PB, PC, PD, PE, PF, PG, Cstg, Cdr]



Show[Lg, curba, PA, PB, PC, PD, PE, PF, PG, Cstg, Cdr]



TEMA :

Folosind interpolarea Lagrange (pe $[0, 1]$), sa se interpoleze punctele :

1 :

$$(R, 0), (R \cos \pi / 6, R \sin \pi / 6), (R \cos 2 \pi / 6, R \sin 2 \pi / 6), \\ (R \cos 3 \pi / 6, R \sin 3 \pi / 6) = (0, R),$$

2 :

$$(0, R), (R \cos 4 \pi / 6, R \sin 4 \pi / 6), (R \cos 5 \pi / 6, R \sin 5 \pi / 6), \\ (R \cos 6 \pi / 6, R \sin 6 \pi / 6) = (-R, 0),$$

3 :

$$(-R, 0), (R \cos 7 \pi / 6, R \sin 7 \pi / 6), \\ (R \cos 8 \pi / 6, R \sin 8 \pi / 6), (R \cos 9 \pi / 6, R \sin 9 \pi / 6) = (0, -R),$$

4 :

$$(0, -R), (R \cos 10 \pi / 6, R \sin 10 \pi / 6), \\ (R \cos 11 \pi / 6, R \sin 11 \pi / 6), (R \cos 12 \pi / 6, R \sin 12 \pi / 6) = (R, 0),$$