

```

(* interpolare Lagrange - forma esplicita *)

x0 = 0;
x1 = -1;
x2 = 3;

y0 = 2;
y1 = 1;
y2 = 0;

L0[x_] := 
$$\frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)};$$

L1[x_] := 
$$\frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)};$$

L2[x_] := 
$$\frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)};$$


Simplify[L[x]]


$$\frac{1}{12} (24 + 7x - 5x^2)$$

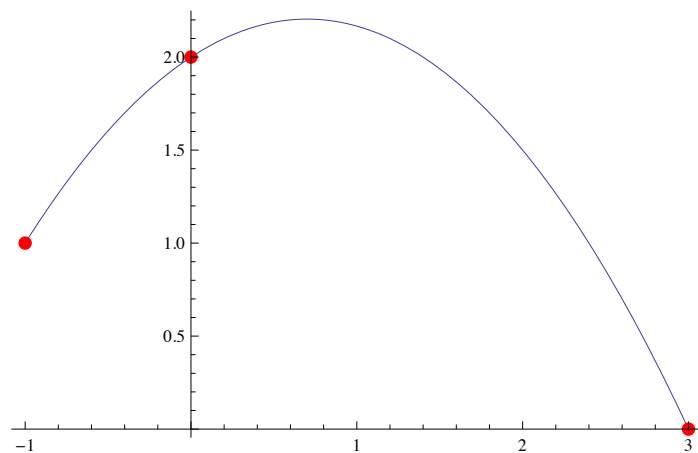

L[x_] := y0 L0[x] + y1 L1[x] + y2 L2[x]

PA = Graphics[{Red, PointSize[Large], Point[{0, 2}]}];
PB = Graphics[{Red, PointSize[Large], Point[{-1, 1}]}];
PC = Graphics[{Red, PointSize[Large], Point[{3, 0}]}];

lagr1 = Plot[L[x], {x, -1, 3}];

Show[lagr1, PA, PB, PC]

```



```

(* interpolare Lagrange - forma parametrica *)

```

```

t0 = 0;
t1 = 1 / 2;
t2 = 1;

Lt0[t_] := 
$$\frac{(t - t_1)(t - t_2)}{(t_0 - t_1)(t_0 - t_2)};$$

Lt1[t_] := 
$$\frac{(t - t_0)(t - t_2)}{(t_1 - t_0)(t_1 - t_2)};$$

Lt2[t_] := 
$$\frac{(t - t_0)(t - t_1)}{(t_2 - t_0)(t_2 - t_1)};$$


```

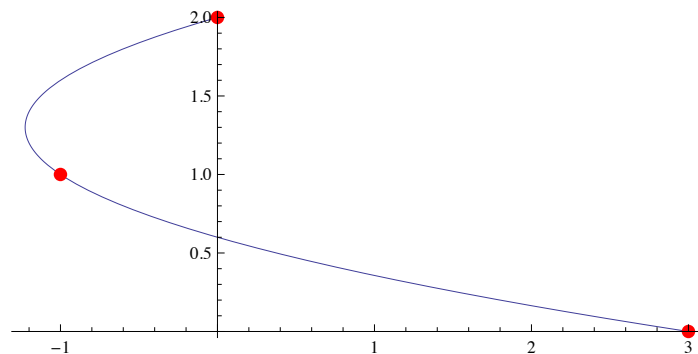
```
Lt[t_] := Lt0[t] {x0, y0} + Lt1[t] {x1, y1} + Lt2[t] {x2, y2}
```

```
Lt[t] // Expand
```

```
{-7 t + 10 t^2, 2 - 2 t}
```

```
c1 = ParametricPlot[Lt[t], {t, t0, t2}];
```

```
Show[c1, PA, PB, PC]
```



```
t0 = -1;
```

```
t1 = 1;
```

```
t2 = 2;
```

```
Lt0[t_] := (t - t1) (t - t2) /
```

```
(t0 - t1) (t0 - t2);
```

```
Lt1[t_] := (t - t0) (t - t2) /
```

```
(t1 - t0) (t1 - t2);
```

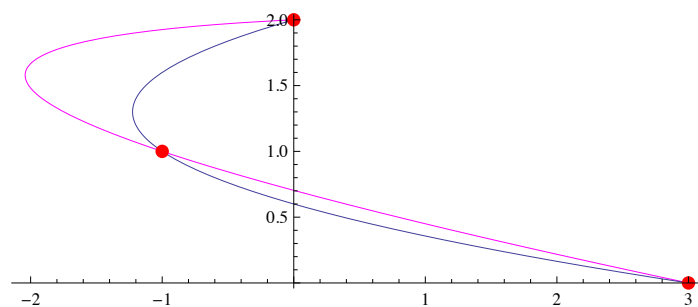
```
Lt2[t_] := (t - t0) (t - t1) /
```

```
(t2 - t0) (t2 - t1);
```

```
Lt[t_] := Lt0[t] {0, y0} + Lt1[t] {x1, y1} + Lt2[t] {x2, y2}
```

```
c2 = ParametricPlot[Lt[t], {t, t0, t2}, PlotStyle -> Magenta];
```

```
Show[c2, c1, PA, PB, PC]
```



```
t0 = -1;
```

```
t1 = 1;
```

```
t2 = 3;
```

```
(* interpolare Lagrange de grad 3 - forma explicita *)
```

```
x0 = 0;
x1 = 1;
x2 = 8;
x3 = 27;
```

```
y0 = 0;
y1 = 1;
y2 = 2;
y3 = 3;
```

```
L0[x_] := 
$$\frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)};$$

```

```
L1[x_] := 
$$\frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)};$$

```

```
L2[x_] := 
$$\frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)};$$

```

```
L3[x_] := 
$$\frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)};$$

```

```
L[x] // Simplify
```

```

$$\frac{x (70825 - 8820 x + 239 x^2)}{62244}$$

```

```
L[x_] := y0 L0[x] + y1 L1[x] + y2 L2[x] + y3 L3[x]
```

```
PA = Graphics[{Red, PointSize[Large], Point[{x0, y0}]}];
```

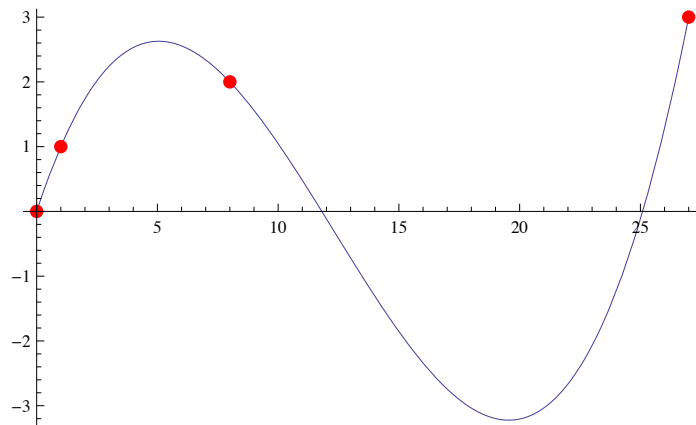
```
PB = Graphics[{Red, PointSize[Large], Point[{x1, y1}]}];
```

```
PC = Graphics[{Red, PointSize[Large], Point[{x2, y2}]}];
```

```
PD = Graphics[{Red, PointSize[Large], Point[{x3, y3}]}];
```

```
lagr1 = Plot[L[x], {x, x0, x3}];
```

```
Show[lagr1, PA, PB, PC, PD]
```

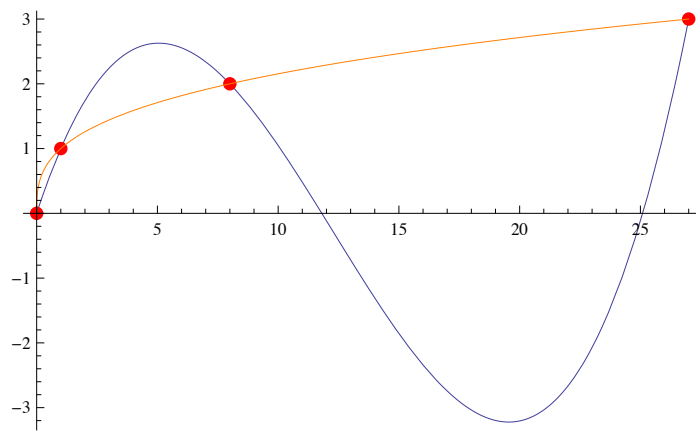


```
f[x_] := 
$$\sqrt[3]{x};$$

```

```
grafic = Plot[f[x], {x, x0, x3}, PlotStyle -> Orange];
```

```
Show[lagr1, PA, PB, PC, PD, grafic]
```



```

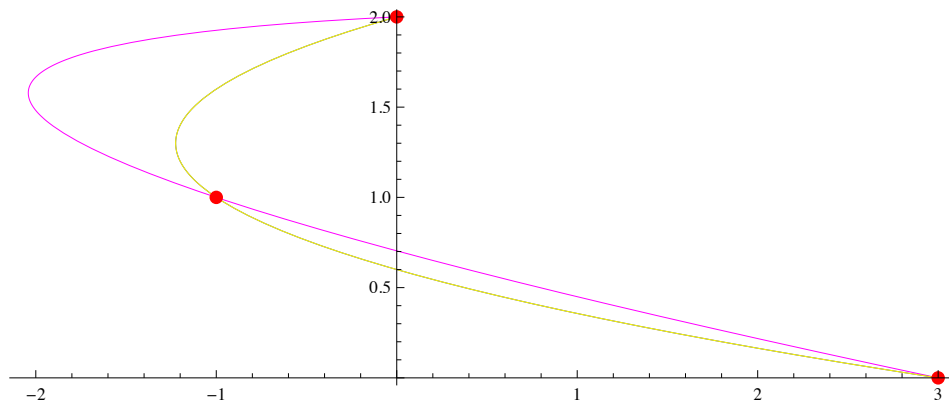
Lt0[t_] := (t - t1) (t - t2) / ((t0 - t1) (t0 - t2));
Lt1[t_] := (t - t0) (t - t2) / ((t1 - t0) (t1 - t2));
Lt2[t_] := (t - t0) (t - t1) / ((t2 - t0) (t2 - t1));

Lt[t_] := Lt0[t] {0, y0} + Lt1[t] {x1, y1} + Lt2[t] {x2, y2}

c3 = ParametricPlot[Lt[t], {t, t0, t2}, PlotStyle -> Yellow];

Show[c2, c1, c3, PA, PB, PC]

```



```

(*
    c3 si c1 se suprapun!!!
    Diviziunile:
        0 < 1/2 < 1
        -1 < 1 < 3
*)

t0 = -1;
t1 = 0;
t2 = 3;

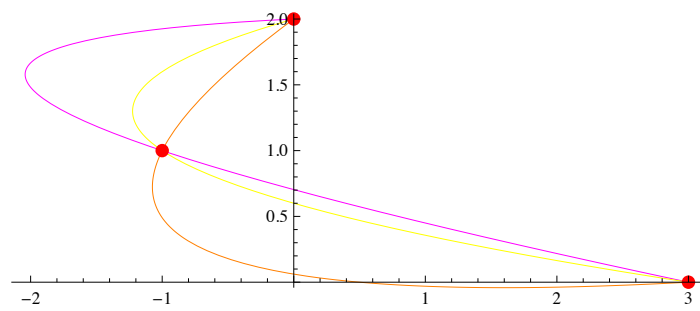
Lt0[t_] := (t - t1) (t - t2) / ((t0 - t1) (t0 - t2));
Lt1[t_] := (t - t0) (t - t2) / ((t1 - t0) (t1 - t2));
Lt2[t_] := (t - t0) (t - t1) / ((t2 - t0) (t2 - t1));

Lt[t_] := Lt0[t] {x0, y0} + Lt1[t] {x1, y1} + Lt2[t] {x2, y2}

c4 = ParametricPlot[Lt[t], {t, t0, t2}, PlotStyle -> Orange];

```

Show[c2, c4, c3, PA, PB, PC]



Lt[u] // Expand

$$\left\{ -1 - \frac{5u}{12} + \frac{7u^2}{12}, 1 - \frac{5u}{6} + \frac{u^2}{6} \right\}$$

```
(* interpolare Lagrange de grad 3 - forma explicita *)
```

```
x0 = 0;
x1 = 1 / 8;
x2 = 1 / 3;
x3 = 1;
```

```
y0 = 0;
y1 =  $\sqrt[3]{1/8}$ ;
y2 =  $\sqrt[3]{1/3}$ ;
y3 = 1;
```

```
L0[x_] :=  $\frac{(x - x1)(x - x2)(x - x3)}{(x0 - x1)(x0 - x2)(x0 - x3)}$ ;
```

```
L1[x_] :=  $\frac{(x - x0)(x - x2)(x - x3)}{(x1 - x0)(x1 - x2)(x1 - x3)}$ ;
```

```
L2[x_] :=  $\frac{(x - x0)(x - x1)(x - x3)}{(x2 - x0)(x2 - x1)(x2 - x3)}$ ;
```

```
L3[x_] :=  $\frac{(x - x0)(x - x1)(x - x2)}{(x3 - x0)(x3 - x1)(x3 - x2)}$ ;
```

```
L[x] // Simplify
```

```
 $\frac{12}{35} \left( 64 (-1 + x) \left( -\frac{1}{3} + x \right) - 21 \times 3^{2/3} (-1 + x) \left( -\frac{1}{8} + x \right) + 5 \left( -\frac{1}{3} + x \right) \left( -\frac{1}{8} + x \right) \right) x$ 
```

```
L[x_] := y0 L0[x] + y1 L1[x] + y2 L2[x] + y3 L3[x]
```

```
PA = Graphics[{Red, PointSize[Large], Point[{x0, y0}]}];
```

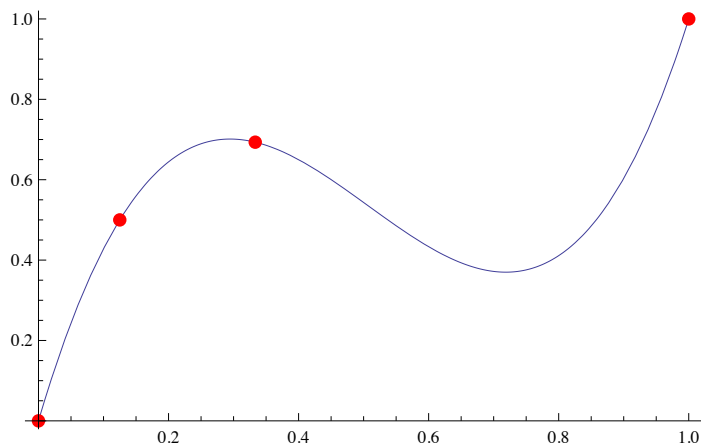
```
PB = Graphics[{Red, PointSize[Large], Point[{x1, y1}]}];
```

```
PC = Graphics[{Red, PointSize[Large], Point[{x2, y2}]}];
```

```
PD = Graphics[{Red, PointSize[Large], Point[{x3, y3}]}];
```

```
lagr1 = Plot[L[x], {x, x0, x3}];
```

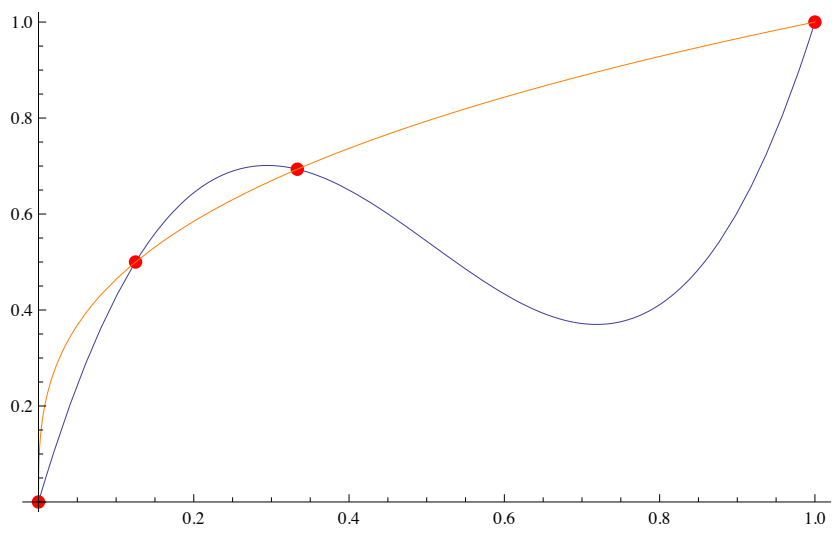
```
Show[lagr1, PA, PB, PC, PD]
```



```
f[x_] :=  $\sqrt[3]{x}$ ;
```

```
grafic = Plot[f[x], {x, x0, x3}, PlotStyle -> Orange];
```

```
Show[lagr1, PA, PB, PC, PD, grafic]
```



(* interpolare Lagrange de grad 4 - aproximarea functiei tangenta *)

```
x0 = -0.45 Pi;
x1 = -Pi / 4;
x2 = 0;
x3 = Pi / 4;
x4 = 0.45 Pi;
```

```
y0 = Tan[x0];
y1 = Tan[x1];
y2 = Tan[x2];
y3 = Tan[x3];
y4 = Tan[x4];
```

$$L0[x_] := \frac{(x - x_1)(x - x_2)(x - x_3)(x - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)};$$

$$L1[x_] := \frac{(x - x_0)(x - x_2)(x - x_3)(x - x_4)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)};$$

$$L2[x_] := \frac{(x - x_0)(x - x_1)(x - x_3)(x - x_4)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)};$$

$$L3[x_] := \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_4)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)};$$

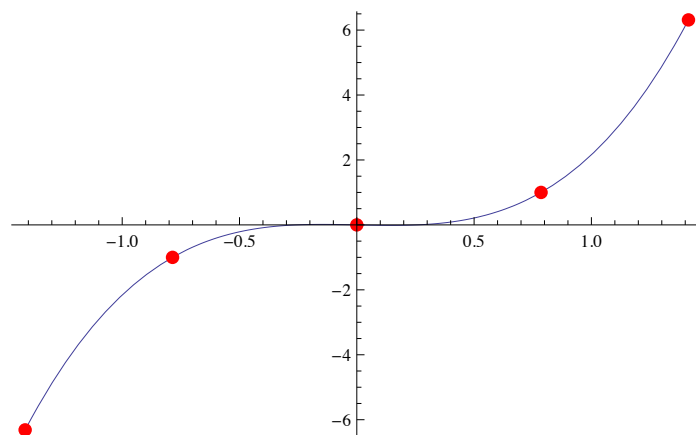
$$L4[x_] := \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_3)}{(x_4 - x_0)(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)};$$

```
L[x_] := y0 L0[x] + y1 L1[x] + y2 L2[x] + y3 L3[x] + y4 L4[x]
```

```
PA = Graphics[{Red, PointSize[Large], Point[{x0, y0}]}];
PB = Graphics[{Red, PointSize[Large], Point[{x1, y1}]}];
PC = Graphics[{Red, PointSize[Large], Point[{x2, y2}]}];
PD = Graphics[{Red, PointSize[Large], Point[{x3, y3}]}];
PE = Graphics[{Red, PointSize[Large], Point[{x4, y4}]}];
```

```
lagr1 = Plot[L[x], {x, x0, x4}];
```

```
Show[lagr1, PA, PB, PC, PD, PE]
```



```
f[x_] := Tan[x];
```

```
grafic = Plot[f[x], {x, x0, x4}, PlotStyle -> Orange];
```

```
Show[lagr1, PA, PB, PC, PD, PE, grafic]
```

