Convex Analysis in General Vector Spaces, World Scientific, 2002

Errata:

Page xii, last line of Example 4: A **instead of** T.

Page 1, line -12: $\Lambda \cdot A = \{\lambda a \mid \ldots\}$ **instead of** $\Lambda \cdot A = \{\gamma a \mid \ldots\}$.

Page 3, relation (1.1): A **instead of** C.

Page 4, line -11: "V is a convex" **instead of** "V is a".

Page 17, line -13: $0 = \langle z, x^* \rangle \ldots$ **instead of** $0 \le \langle z, x^* \rangle \ldots$

Page 31, line 15: "*there exist the sequence* $(u_n)_{n\ge 0} \subset B(x_0, \lambda)$ *converging to some* $u \in X$ *and* $(\mu_n)_{n\ge0} \subset \mathbb{R}_+$ *with* $\sum_{n\ge0} \mu_n = 1$ *such that*" **instead of** "*there exists a sequence* $(u_n)_{n\ge0} \subset B(x_0, \lambda)$ *converging to some* $u \in X$ *such that*".

Page 32, line 12: (1.14) **instead of** (1.15).

Page 33, line 8: $\Theta_p(x_0) = \cdots \le \sum_{n\ge1} \mu_n \gamma \delta^{-1} =$ **instead of** $\Theta_p(x_0) \le \cdots \le \sum_{n\ge1} \mu_k \gamma \delta^{-1} =$.

Page 34, line 11: $B(x, r) \cap \bigcap_{n\in\mathbb{N}} D_n \ne \emptyset$ **instead of** $B(x, r) \cap \bigcap_{n\in\mathbb{N}} D_n$.

Page 35, line 17: $P(\alpha) := \{0\} \cup$ **instead of** $P(\alpha) :=$.

Page 35, line 18: "Arccos $\frac{(x|y)}{\|x\|\cdot\|y\|}$ for $x, y \in X \setminus \{0\}$" **instead of** "Arccos $\frac{(x|y)}{\|x\|\cdot\|y\|}$".

Page 60, -8: $f(x) \cdot f(y) > 0$ **instead of** $f(x) \cdot f(y) \ge 0$.

Page 72, line 16: U'_x **instead of** U_x;
line 17: r'_x **instead of** r_x.

Page 74, line 2: $(f_n(x_k))_{n\in\mathbb{N}}$ **instead of** $(f_n(x_k) - n \in \mathbb{N})$.

Page 74, line -10: "$\dim X < \infty$ and one of its nonempty level sets is bounded" **instead of** "$\dim X < \infty$".

Page 78, line 6: "$f^{**}(\overline{x})$" **instead of** "$f^{**}(x)$".

Page 81, line 12: "$= f(\overline{x})$ and $x^* \in \partial(\overline{\mathrm{co}} f)(\overline{x})$. This" **instead of** "$= f(\overline{x})$. This".

Page 81, line 14: "$\partial(\overline{\mathrm{co}} f)(\overline{x}) \subset \partial\overline{f}(\overline{x}) \subset \partial f(\overline{x})$. On the other hand, replacing $\overline{x}^*$ with an arbitrary $x^* \in \partial f(\overline{x})$, we obtain that $\partial f(\overline{x}) \subset \partial(\overline{\mathrm{co}} f)(\overline{x})$." **instead of** "$\partial(\overline{\mathrm{co}} f)(\overline{x}) \supset \partial\overline{f}(\overline{x}) \supset \partial f(\overline{x})$. If $x^* \in \partial(\overline{\mathrm{co}} f)(\overline{x})$, then $\forall x \in X \ : \ \langle x - \overline{x}, x^* \rangle \le (\overline{\mathrm{co}} f)(x) - (\overline{\mathrm{co}} f)(\overline{x}) \le f(x) - f(\overline{x})$, whence $\partial(\overline{\mathrm{co}} f)(\overline{x}) \subset \partial f(\overline{x})$.".

Page 83, lines 4, 6: x^* **instead of** $\overline{x}^*$.

Page 85, lines 4, 8: $i = 1$ **instead of** $i = 0$.

Page 95, line 5: T **instead of** A (two times).

Page 96, lines 13–16: T **instead of** A (7 times) (once $T^* y^*$ **instead of** Ay^*).

Page 97, line -7: "semicontinuous for every $x \in X$, and" **instead of** "semicontinuous and";
line -5: $\overline{\mathrm{co}}^{w^*}$ **instead of** $\overline{\mathrm{co}}$.

Page 100, condition (iii) of Theorem 2.5.2 replaced by *"there exist* $x_0, x_1 \in \operatorname{dom} f$ *and* $r > 0$ *such that* $f(x_0) < f(x_1)$, $D := [f \le f(x_1)] + rU_X \subset \operatorname{dom} f$*, and* f *is Lipschitz on* D*"*. In the proof, L is the Lipschitz constant on D. The rest of the proof is the same.

Page 101, line -8: $f - \gamma \le g \le f + \gamma$ **instead of** $f - \gamma \le f \le f + \gamma$;
line -5: $|g_1(x) - g_2(x)|$ **instead of** $|f(x) - g(x)|$.

Page 104, line 12: "K. Take $y := nr^{-1}z \in K$; then $h(y) \le h(0) + h_\infty(y) \le \varepsilon$ and $(y + K) \cap nU_X = \emptyset$." **instead of** "$K$.".

Page 108, line -9: $\Phi^*(0, y^*)$ **instead of** $\Phi(0, y^*)$;
line -6: $h^{**}(0)$ **instead of** $x^{**}(0)$.

Page 109, line 7: (v) **instead of** (iv).

Page 111, line 5: $\inf_{z\in V} h(z)$ **instead of** $\inf_{y\in V} h(y)$;
line 11: $\Phi(x, 0)$ **instead of** $F(x, 0)$.

Page 114, line -7: "follows by Theorem" **instead of** "follows Theorem".

Page 124, line 10: $A^*(\partial_{\varepsilon_2} g(Ax))$ **instead of** $\partial_{\varepsilon_2} g(Ax)$.

Page 125, line -5: $\mathcal{C}$ **instead of** $\mathcal{A}$.

Page 126, line 13: $\mathcal{C}$ **instead of** $\mathcal{A}$.

Page 132, line 3: ε **instead of** ε_0.

Page 151, line -4: v **instead of** y.

Page 154, Exercise 2.46: lim inf **instead of** lim.

Page 171, line -10: $\sup\limits_{\substack{0<t\leq\delta,\, \|x-\overline{x}\|\leq\delta \\ \max\{f(x),f(\overline{x})-\delta\}\leq\alpha\leq f(\overline{x})+\delta}}$ **instead of** $\sup\limits_{0<t\leq\delta,\, \|x-\overline{x}\|\leq\delta,\, f(x)\leq\alpha\leq f(\overline{x})+\delta}$;

line -8: $\inf\limits_{\|v-u\|\leq\varepsilon} \dfrac{f(x+tv)-\max\{f(x),f(\overline{x})-\delta\}}{t}$ **instead of** $\inf\limits_{\|v-u\|\leq\varepsilon} \dfrac{f(x+tv)-f(x)}{t}$.

Page 172, line -14: $\sup\limits_{\substack{0<t\leq\delta_k,\, \|x-\overline{x}\|\leq\delta_k \\ \max\{f(x),f(\overline{x})-\delta_k\}\leq\alpha\leq f(\overline{x})+\delta_k}}$ **instead of** $\sup\limits_{0<t\leq\delta_k,\, \|x-\overline{x}\|\leq\delta_k,\, f(x)\leq\alpha\leq f(\overline{x})+\delta_k}$.

Page 173, line 9: The statement "$\limsup_{x\to_f\overline{x}} f^{\uparrow}(x,u)\leq f^{\uparrow}(\overline{x},u)$" is false for f not lsc at $\overline{x}$ as the example on lines $4, 5$ shows; it is even false for f continuous, as the example on page 171, lines -3, -2 shows (for $u=0$). Of course, its proof is wrong.

Page 173, lines 10, 11: $\sup\limits_{\|x-\overline{x}\|\leq\delta,\, f(x)\leq f(\overline{x})+\delta,\, 0<t\leq\delta}$ **instead of** $\sup\limits_{\|x-\overline{x}\|\leq\delta,\, 0<t\leq\delta}$.

Page 173, line 13: $f(x+tv)$ **instead of** $f(y+tv)$.

Page 175, line -5: "proper functions, lsc at $\overline{x}$" **instead of** "proper functions and $\overline{x}$".

Page 176, line 1: I don't know if the assertion (iv) is true (the proof being based on Proposition 3.2.3 (i)).

Page 177, lines 12, 13: (v) **instead of** (iv).

Page 179, line 2: "lsc and proper, $a,b\in X$" **instead of** "lsc, $a,b\in X$".

Page 179, line 4: "$\overline{\partial}$" **instead of** "∂".

Page 183, lines 13–19 to be replaced by

$$\text{“}\liminf\,\langle c-x_n,x_n^*\rangle\geq 0,\quad \liminf\,\langle b'-a,x_n^*\rangle\geq r-f(a).$$

Observe that $b-c=\eta(b'-a)$ with $\eta:=\|b-c\|\,/\,\|b'-a\|\geq\|b-b'\|\,/\,\|b'-a\|=\lambda/(1-\lambda)$. Since $\overline{\partial}f$ is monotone we get the contradiction

$$\begin{aligned}\|b-a\|\cdot\|y^*\|\geq\|b-c\|\cdot\|y^*\| &\geq\langle b-c,y^*\rangle\\ &=\liminf\,\langle b-x_n,y^*\rangle\geq\liminf\,\langle b-x_n,x_n^*\rangle\\ &=\liminf\,(\langle\eta(b'-a),x_n^*\rangle+\langle c-x_n,x_n^*\rangle)\\ &\geq\eta\liminf\,\langle b'-a,x_n^*\rangle+\liminf\,\langle c-x_n,x_n^*\rangle\\ &\geq\eta(r-f(a))\geq\lambda(1-\lambda)^{-1}(r-f(a)).\text{”}\end{aligned}$$

Page 202, line -3: 3.3 **instead of** 2.24.

Page 220, line -13: $[f\leq f(x_1)]+\frac{r}{4}U_X\subset D(x_0,\frac{3r}{4})$ **instead of** $[f\leq f(x_1)]$.

Page 234, line -2: "give" **instead of** "give dual";
line -1: "spaces using weight functions" **instead of** "spaces".

Page 250, line -1: $\alpha\,\|x^*\|^{-1}\,x^*$ **instead of** $\alpha\,\|x-\overline{x}\|^{-1}\,(x-\overline{x})$.

Page 291, Exercise 3.16: w **instead of** w^*.

Page 294, line 9: "(x) of Theorem 3.10.1 is" **instead of** "(x) is".

Page 297, line -7: ", where $\frac{1}{0}:=\infty$; for" **instead of** "; for".

Page 299, line -6: "(b) Let" **instead of** "(c) Let".

Page 300, line 3: (a) **instead of** (b).

Page 300, line 7: $\mathbb{R}_+$ **instead of** $\mathbb{R}$.

Page 322, line 11: "Consider the topology generated by the family of all semi-norms on X; so" **instead of** "Consider the topology $\sigma(X,X')$ on X; so".

Page 326, line -16: "holds and $0 \notin C$" **instead of** "holds".

Acknowledgements. The author thanks Prof. S. Cobzaş from University Babeş–Bolyai, Cluj-Napoca, and especially Dr. M. Voisei from University of Ohio for pointing out most of the above corrections.