Review by H.H. Bauschke in Mathematical Reviews MR:

According to the author, the aims of this book are "to present the conjugate and subdifferential calculus using the method of perturbation functions in order to obtain the most general results in this field" and "to provide important applications of this calculus and of the properties of convex functions". Zălinescu assumes that the reader has a solid background in the basics of functional analysis. Except for some results from that area, all results presented in his book come with complete and detailed proofs.

The book is organized as follows.

A short (37-page) first chapter starts with a review of the necessary results from functional analysis as well as various closure and interiority notions of convex sets. These notions are needed to obtain general open mapping theorems. This chapter features also the variational principles by Ekeland and by Borwein and Preiss; it concludes with Baire's theorem, which Zălinescu presents via a useful identity due to Ursescu.

The second chapter (120 pages) is devoted to convex analysis in locally convex spaces. The author introduces and studies convex functions, directional derivatives, and continuity properties; the conjugate function and the subdifferential; the general convex optimization problem and its associated value or perturbed function. Given two separated locally convex spaces X and Y and a proper convex function Φ from $X \times Y$ to $]-\infty, +\infty]$, Zălinescu provides nine conditions sufficient for the fundamental duality formula

$$\inf_{x \in X} \Phi(x, 0) = - \min_{y^* \in Y^*} \Phi^*(0, y^*),$$

which serves as a link between the primal and the dual problem. While these sufficient conditions are too technical to state here, the fundamental duality formula is broadly applicable, very flexible, and one of the main tools of this book. Armed with this result, Zălinescu discusses its many remarkable applications: conjugate and subdifferential calculus (formulae involving sums, infimal convolutions, etc.); Fenchel duality; necessary and sufficient optimality conditions for convex optimization problems with constraints, including Lagrangians and a useful normal cone formula. The chapter concludes with a minimax theorem.

The final Chapter 3 (137 pages) discusses convex analysis and its applications in normed spaces. Zălinescu starts with a result on ϵ-subdifferentials (due to Borwein) that immediately leads to the Brøndstedt-Rockafellar theorem. Additional formulae for the subdifferentials of (linear) compositions and sums follow, as well as the Bishop-Phelps theorem. Rockafellar's maximal monotonicity theorem (on the subdifferential of a proper lower semicontinuous convex function) is proved twice, first by an approach originally due to Simons. The author then studies the relationship between convex functions and types of subdifferentials (abstract, convex, and in the sense of Clarke). Zagrodny's approximate mean value theorem is proven and applications are provided (including a second proof of Rockafellar's maximal monotonicity theorem). Zălinescu then carefully analyzes convex functions with additional "good" properties (such as well-conditioned, uniformly convex, or uniformly smooth). Several notions from the geometry of normed spaces—for instance, uniform smoothness, uniform convexity, and the Kadets-Klee property—are characterized from within convex analysis by utilizing weight functions. The chapter is completed by detailed sections on applications of the developed body of results to the best approximation problem, to Tikhonov well-posedness, to weak sharp minima, to error bounds, and to maximal monotone multifunctions.

Zălinescu concludes every chapter with a large, well-chosen set of exercises, and detailed bibliographical notes. I was delighted to find that the solutions to all exercises are provided as well; the corresponding section in the book spans 52 pages. This feature makes the book very valuable to researchers—results presented in the exercises can be cited with confidence—and to anyone working through the material on his or her own.

Two minor quibbles: first, the index could be more detailed. For instance, I recalled reading about the useful Toland-Singer duality but could not find it in the index (it turns out to be Exercise 2.27). Secondly, given a convex function f defined on a Banach space X, I would have liked to see the biconjugate f^{**} be defined and analyzed in the bidual X^{**} rather than X. While considering f^{**} only on X is the classical and more symmetric approach followed in this book, understanding and analyzing f^{**} as a function on X^{**} is ultimately more useful: see, for instance, Chapter VII in the recent monograph by S. Simons [Minimax and monotonicity, Lecture Notes in Math., 1693, Springer, Berlin, 1998; MR 2001h:49002], where this viewpoint is utilized in the study of stronger maximal monotonicity properties of subdifferentials in nonreflexive Banach spaces.

Returning to the author's aims stated at the beginning of this review, I believe that Zălinescu has succeeded quite admirably with this book. A novice may find it somewhat challenging to discriminate major from minor results; however, researchers will appreciate Convex analysis in general vector spaces as a useful and comprehensive reference. In summary, this book is a very welcome addition to the bookshelf of every convex analyst working in infinite-dimensional spaces!