Preface to the Second Edition

Since vector variational methods have been a growing field of research over the past few years, our main goal when preparing the second edition of *Variational Methods in Partially Ordered Spaces* was to expand on it without changing the structure of the initial book. This new edition, as suggested by the publisher, has benefited from the comments of the authors and many individuals, which have resulted in the addition of some new sections, and the reorganization of all others. Moreover, in the references, currently published articles and books related to vector variational methods are added.

The main changes are:

- In Chapter 1, an additional example is added in Section 1.2 in order to solve a vector convex minimization problem by the asymptotic behavior of solutions of a continuous dynamical system. Section 1.6 on uncertainty is new and briefly explains in two examples how the solution of a vector problem reacts to disturbances in the data.
- Chapter 2 contains several new notions and results on Functional Analysis over Cones. In particular, it improves the presentation of Sections 2.1.2 and 2.2. Sections 2.3, 2.5, 2.9, and 2.12 are entirely new, and add notions necessary for reading the new sections in the chapters that follow.
- Chapter 3 contains an additional Section 3.2 on Solution Concepts in Set-Valued Optimization.
- Chapter 4 is new. It offers a brief overview on Generalized Differentiation and Optimality Conditions, and presents in Section 4.1 recent concepts of Mordukhovich's generalized differentiation in order to express optimality conditions for optimization problems with set-valued objectives.
- Chapter 5 contains an additional Section 5.5 on asymptotic behavior of multiobjective Pareto-equilibrium problems. Especially applications are suitable for asymptotic behavior of multiobjective optimization and saddle-point problems.

- Chapter 6 is new. Dealing with scalar problems under uncertainty, this chapter introduces three general approaches (vector approach, set approach, and nonlinear scalarization) which permit a unified treatment of a large variety of models from robust optimization and stochastic programming.

The book was written by four authors, we wrote it together, and it was at every time stimulating and profitable to consider the problems from different sides. A. Göpfert, Chr. Tammer, and C. Zălinescu contributed to Section 1.1, 2.1, 2.2, 2.4, 3.1, and 3.11; C. Zălinescu wrote Sections 2.6, 2.7, 2.8, 2.10, 2.11 and 3.3–3.7; H. Riahi wrote Sections 2.9, 2.12, 3.9, 3.10, 5.2.3, 5.2.4, 5.5; and A. Göpfert and Chr. Tammer wrote Sections 1.2–1.6, 2.3, 2.5, 3.2, 3.8, 3.12, 4.1–4.4, 5.1, 5.2.1, 5.2.2, 5.3, 5.4, and 6.1–6.5.

Here, in this second edition, we adapted and updated Chapters 1, 2, 3 and 5. Chapters 4 and 6 are new. Especially, C. Zălinescu revised and extended Sections 2.1, 2.2, 2.4, 2.6, 2.7, 2.8, 2.10, 2.11, 3.1, 3.3–3.7, 3.11; H. Riahi added an example in Section 1.2, revised and extended Sections 3.9, 3.10, and added the new Sections 5.2.4, 5.5; and A. Göpfert and Chr. Tammer added the new Sections 1.6, 2.3, 2.5, 3.2, 4.1–4.4 and 6.1–6.5.

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With great dismay, we had to learn that our co-author and very good friend, Alfred Göpfert, passed away on January 22, 2023. We were privileged to complete the substantive work on the second edition of our book together with him. We have lost an excellent mathematician and a warm-hearted friend. With his charisma, Alfred Göpfert inspired generations of mathematicians for his science. We have learned a lot from him. It is a need for us to express our sincere gratitude to Alfred and our condolences to his family and loved ones.

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