Review by R. Pini in Zentralblatt für Mathematik, pre01807400

The primary aim of this book is to present the conjugate and subdifferential calculus using the method of perturbation functions in order to obtain the most general results in this field. The secondary aim is to give important applications of this calculus and of the properties of convex functions. A solid background in functional analysis is required, together with a good knowledge of topology and topological vector spaces, while no prior knowledge of convex analysis is assumed. The book is divided into three chapters. Every chapters ends with exercises (overall, more than 80) and bibliographical notes; the complete solutions of all exercises are given.

In the first chapter, preliminary results on functional analysis together with some various closure and interiority notions for closed sets are presented. Besides, Ursescu's theorem and an improvement of Simons's open mapping theorem are proved, as well as the Ekeland's variational principle, and the smooth variational principle of Borwein and Preiss. The last result is an extension of Baire's theorem obtained via two Uresescu's results.

In the second chapter convex analysis in locally convex spaces is presented. This chapter is dedicated, mainly, to conjugate and ε-subdifferential calculus. The author introduces convex functions and their characterizations via the epigraph, or, in case of differentiability, via the gradient. Classical results about conjugate functions are presented; properties of the subdifferential are introduced and studied. To a convex function $f : X \to \overline{\mathbb{R}}$, a perturbation function $\Phi : X \times Y \to \overline{\mathbb{R}}$ is associated (X, Y are separated locally convex spaces): Φ is a proper convex function, with the property that $f(x) = \Phi(x, 0)$. In this chapter is presented one of the main tools of the book, the fundamental duality formula, that relates the primal and the dual problem:

$$\inf_{x\in X} \Phi(x, 0) = \max_{y^*\in Y^*} (-\Phi^*(0, y^*)).$$

Its proof is stated under very general conditions by using open mapping theorems; the author proves nine sufficient conditions which ensure its validity. The formula is applied to many situations: calculus for conjugates and subdifferentials, duality relations, necessary and sufficient conditions in convex optimization problems with constraints.

The framework of the third chapter is that of convex analysis in normed spaces. From some classical results in convex analysis, like the theorems of Borwein and of Brøndsted-Rockafellar, the author obtains formulas for the subdifferential of compositions and sums of suitable functions, as well as the Bishops-Phelps theorem. A simple proof of Rockafellar's theorem on the maximal monotonicity of the subdifferential of a convex function is given, using a recent theorem of Simons. Convex functions are characterized via an abstract subdifferential, whose particular cases are the subdifferentials of Clarke and Fenchel cases; in the last case, a characterization is provided using Zagrodny's approximate mean value theorem. Classes of more regular convex functions, like well-conditioned, uniformly convex and uniformly smooth convex functions are introduced, analysed and characterized. The geometry of the normed space X, like its strict convexity, smoothness and reflexivity, is investigated via suitable weight functions. Other applications of the results presented in the book are considered; among them, best approximation problems, weak sharp minima, well-behaved functions, global error bounds, monotone multifunctions. [Rita Pini (Milano)]

MSC 2000: *46-01 Textbooks (functional analysis) 46A55 Convexity in topological linear spaces 52A07 Convex sets in topological vector spaces (convex geometry)

Keywords: convex functions; conjugate functions; subdifferentials; perturbed problems; duality; ε-subdifferentials; convex optimization; well conditioned functions.