Doctoral School of Mathematics

ALGEBRA

- topics for admission to the PhD programme -

A. ALGEBRAIC STRUCTURES

1. Groups

1.1. Groups, subgroups. Homomorphisms. Examples. Lagrange's Theorem. Normal subgroups, quotient groups. Products of groups.
1.2. *G*-sets. *p*-groups and Sylow's Theorems.
1.3. Characteristic and fully invariant subgroups. Center of a grup. Derived subgroup. Nilpotent groups. Solvable groups.

2. Rings. Divisibility in integral domains

2.1. Rings, subrings, ideals. Examples. Polynomial rings. Ideals, quotient rings. Prime fields.
2.2. Rings of fractions. The field of fractions of an integral domain. Constructions of the ring of integers **Z** and of the field of rationals **Q**.
2.3. Divisibility in semigroups and rings. Unique factorization domains, principal ideal domains, Euclidean domains. Examples. Arithmetic in polynomial rings. Prime ideals and maximal ideals in commutative rings.

B. COMMUTATIVE ALGEBRA AND GALOIS THEORY

1. Field extensions

1.1. Algebraic extensions, algebraically closed fields, the fundamental theorem of algebra.
1.2. Finite fields. Wedderburn's Theorem. Subfields of a finite field.
1.3. Separable extensions, normal extensions, ruler and compass numbers.
1.4. Transcendental extensions, transcendence degree.

2. The Galois group of a Galois extension

2.1. Galois group, fundamental theorem of Galois theory.
2.2. The correspondence between normal extensions and normal subgroups.

REFERENCES

1. I.D. Ion, N. Radu, *Algebră*, Ed. Didactică şi Pedagogică, Bucureşti, 1981.
2. I. Purdea, Gh. Pic, *Tratat de algebră modernă I+II*, Ed. Academiei, Bucureşti, 1977, 1982
3. V. Leoreanu, *Fundamente de algebra*, Matrix Rom, 2001
4. M. Tarnauceanu, *Probleme de algebra*, vol.I+II , Ed. Univ. Cuza, 2003, 2004