



Doctoral School of Mathematics

# GEOMETRY

- *topics for admission to the PhD programme* -

## A. DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES

### 1. The Geometry of Space and Plane Curves

- 1.1. The arclength parameter; Frenet frame and Frenet formulas for space curves
- 1.2. The fundamental theorem of space curves
- 1.3. The case of plane curves; the isoperimetric inequality for closed curves

### 2. Surfaces in Space

- 2.1. Definitions and examples
- 2.2. Characterization theorem of surfaces
- 2.3. Change of parameters of a surface
- 2.4. The tangent space and the tangent plane to a surface
- 2.5. Smooth maps between surfaces and their differential
- 2.6. The first fundamental form; applications. Isometries between surfaces
- 2.7. The Gauss map and the second fundamental form of oriented surfaces
- 2.8. The differential of the Gauss map in a coordinate system
- 2.9. The mean curvature and the Gaussian curvature; computing formulas
- 2.10. Lines of curvature
- 2.11. Compatibility equations for a surface; the Gauss theorem and the statement of the fundamental theorem of surfaces in space (Bonnet theorem)
- 2.12. Parallel transport; geodesic curves and parametrized geodesics

## B. DIFFERENTIABLE MANIFOLDS

### 1. Differentiable Manifolds and Maps Between Manifolds

- 1.1. The definitions of topological manifolds and differentiable manifolds
- 1.2. Examples of manifolds: the sphere, the projective space, the product manifold etc.
- 1.3. Smooth functions on manifolds
- 1.4. Smooth maps between manifolds; the rank theorem; immersions, submersions, embeddings
- 1.5. Submanifolds

### 2. Tensor Bundles

- 2.1. The tangent and cotangent spaces to a manifold at a point; the tangent and cotangent maps
- 2.2. Tensor spaces to a manifold at a point
- 2.3. The tangent and cotangent bundles; vector fields and 1-forms



### 3. Calculus on Manifolds

- 3.1. The bracket of two vector fields
- 3.2. The flow of a vector field
- 3.3. Lie derivative
- 3.4. Differential forms; exterior derivative
- 3.5. Affine connections

### 4. Riemannian Manifolds

- 4.1. Riemannian metrics on a manifold
- 4.2. Levi-Civita connection; geodesics on riemannian manifolds, exponential map
- 4.3. Sectional curvature; the Schur theorem

### BIBLIOGRAPHY

1. M. Berger, B. Gostiaux, *Differential geometry: manifolds, curves, and surfaces*, Graduate Texts in Mathematics, 115, Springer-Verlag, New York, 1988.
2. M.P. do Carmo, *Riemannian Geometry*, Birkhauser, Boston Inc., 1992.
3. M.P. do Carmo, *Differential Geometry of Curves & Surfaces*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976
4. B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, *Modern Geometry – Methods and Applications: Part I, II, III*, Springer, 1985.
5. C. Oniciuc, *Lecții de geometrie diferențială a curbilor și suprafețelor*, Casa Editorială DEMIURG, Iași, 2018.
6. C. Oniciuc, *O introducere în teoria aplicațiilor armonice*, Casa Editorială DEMIURG, Iași, 2007.
7. V. Oproiu, *Geometrie diferențială*, Ed.Universitatii „Al. I. Cuza” Iasi, 2002.