Mircea BÎRSAN

Mathematical study of the generalized Cosserat models for thermoelastic rods and shells

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ABSTRACT

The theory of shells and the theory of rods are branches of the theory of elasticity, which have received considerable attention in the last decades. There is a growing interest in this field due to the intensive use of thin structures in mechanical and civil engineering. Nevertheless, the development of various rod and shell theories is far from being finalized. The emergence of new technologies and advanced materials in connection with rods and shells manufacturing lead to the necessity of elaborating adequate models and to extend the existing theories. The theories of rods and shells also offer very interesting and difficult problems from the mathematical point of view.

The classical theories of rods and shells are commonly used by engineers dealing with practical problems, but this approach has some disadvantages, since the simplifying assumptions restrict the range of applicability of such models. Also, the resulting equations can be too complicated, especially for thin structures with a more complex internal structure. An alternative approach to the description of the mechanical behavior of rods and shells is the direct approach, which will be employed in our work. The main idea of this approach is to use a two-dimensional continuum (deformable surface) with some physical properties as a shell model. The balances of mechanics and thermodynamics are applied directly to this two-dimensional continuum and no approximation is needed. Similarly, in the direct approach to rods, we start with a one-dimensional continuum (deformable curve) endowed with a certain microstructure, as a rod model. The Cosserat brothers were the first to elaborate a rigorous study on directed media (Cosserat and Cosserat, 1909). They have considered elastic continua in which every material point can undergo both displacements and rotations, so that it behaves like a infinitesimal rigid body. Later on, Ericksen and Truesdell (1958) presented a more general theory of oriented media and drew attention on this subject.

According to the original idea of Cosserat, the direct approach for shells

and rods investigates deformable continua (surfaces or curves) endowed with a triad of rigidly rotating orthonormal vectors connected to each point. The triad of orthonormal vectors (also called directors) is assigned to every point in order to characterize its orientation and independent rotations. Using this direct approach, the theories of shells and rods have been elaborated in details by Zhilin (1976, 2006a,b, 2007), who has supplemented the kinematical model suggested by Cosserat with appropriate constitutive equations, thus making the model applicable to solve practical problems. In our work, we employ this Cosserat-type approach to investigate the mechanical behavior of thermoelastic rods and shells with complex internal structure. The main difficulty of any direct approach is the formulation of constitutive equations for such models. Then, a crucial aspect in this model is the identification of the effective stiffness properties of the thin structure. In our work, we extend the theory of directed curves to the case of porous rods and determine the effective stiffness coefficients for composite rods and shells made of orthotropic thermoelastic materials.

The first chapter is devoted to the theory of curved rods and beams modeled by the direct approach. We extend the model to the case of porous thermoelastic materials and we present a mathematical study of the governing equations. To describe the porosity, we employ the Nunziato-Cowin theory for elastic materials with voids (Nunziato and Cowin, 1979; Cowin and Nunziato, 1983), which is intended for the study of porous solids and granular materials. We prove the uniqueness of classical solutions and the existence of weak solutions for both the equilibrium and the dynamical equations of curvilinear thermoelastic porous rods. We investigate in details the case of initially straight rods, when the extension-torsion and bending-shear problems become decoupled. For orthotropic porous thermoelastic rods we identify the effective stiffness coefficients for our model by comparison of solutions to certain problems of bending, extension, torsion, and shear vibrations, obtained in these two approaches: deformable curves and three-dimensional rods.

We also presents an extended thermodynamic theory for rods, which employs two independent temperature fields to describe the thermal effects in rods. The first temperature field is a mean value of the absolute temperature in the cross-section of the rod, while the second measures the spatial variation of the temperature in the cross-section. Both thermal fields are needed for the better description of the thermo-mechanical behavior of rods. Next, we show that the model of directed curves is an efficient approach for analyzing elastic beams and rods with a complex internal structure, such as functionally graded, composite, or non-homogeneous rods. We identify the effective stiffness properties for general non-homogeneous rods, such as the effective bending stiffness, extensional stiffness, torsional rigidity and other coupling coefficients for non-homogeneous beams with arbitrary cross-section shape. These formulas are expressed in terms of the solutions to some auxiliary plane strain boundary-value problems defined on the cross-section domain. Then, we employ our analytical modeling to analyze the deformation of functionally graded beams made of metal foams. Finally, we verify our analytical modeling by comparison with the numerical solutions to various bending problems, computed by the finite element method.

Chapter 2 is devoted to the investigation of thermoelastic shells made of orthotropic non-homogeneous materials. We employ a Cosserat model which takes into account the thermal effects in shells by introducing two temperature fields assigned to the points of the directed surface. The two thermal fields describe the temperature on the two major surfaces (top and bottom) of the shell, respectively. We present a mathematical study of the field equations, which shows that the generalized shell model is mathematically well-formulated. Finally, we investigate the deformation of cylindrical shells with arbitrary shape of cross-section made of orthotropic materials, under the action of resultant forces and moments applied to the end edges, and the action of given body loads. We consider a quite general set of constitutive equations which can describe various types of composite shells (such as multi-layered, non-homogeneous, or reinforced shells) and present a general analytical solution procedure for this problem. In the case of three-layered circular cylindrical shells we apply this general method to determine the closed-form solution for the displacement and rotation fields, and we compare the theoretical results with corresponding numerical solutions.

The investigated models are quite general, since they can handle composite porous rods with natural twisting, or orthotropic thermoelastic multilayered shells. We have established the governing equations of these generalized Cosserat models and we have proved some important mathematical properties of the solutions to these equations, pertaining to existence, uniqueness, continuous dependence on data, reciprocity, variational characterization, and stability. Moreover, we have determined the effective stiffness properties of various types of composite thin structures. The effective stiffness concept makes these models useful and efficient tools for the treatment of complex practical problems concerning rods or shells.