UNIQUENESS OF ENTIRE FUNCTIONS

BY

YI ZHANG and WEI-LING XIONG

Abstract. In this paper, we study the uniqueness problems on meromorphic functions sharing a finite set. The results extend and improve some theorems obtained earlier by Fang (2002) and Zhang-Lin (2008).

Mathematics Subject Classification 2000: 30D35.

Key words: uniqueness, meromorphic function, entire function, share value.

1. Introduction and results

In this paper, we will use the standard notations of Nevanlinna’s value distribution theory (cf. [2], [5]).

Let $f$ be a nonconstant meromorphic function in the whole complex plane $\mathbb{C}$, we set $E(a, f) = \{z | f(z) - a = 0, \text{counting multiplicities}\}$. In general, put $E(S, f) = \bigcup_{a \in S} E(a, f)$, where $S$ denotes a set of complex numbers. Let $k$ be a positive integer. Set

$$E_k(S, f) = \bigcup_{a \in S} \{z | f(z) - a = 0, \exists i, 0 < i \leq k, \text{ s.t. } f^{(i)}(z) \neq 0\},$$

where each zero of $f(z) - a$ with multiplicity $m$ counted $m$ times when $m \leq k$ in $E(S, f)$.

Let $f$ and $g$ be two nonconstant entire functions, $n, m, l, t$ and $p$ be positive integers, we set

$$F = [f^n(f^l - 1)^t]^{(p)}, G = [g^n(g^l - 1)^t]^{(p)},$$

*Supported by NSF of Guangxi China (0728041).
Theorem A ([1]). Let $f$ and $g$ be two nonconstant entire functions, and let $n, k$ be tow positive integers with $n > 2k + 8$. If $[f^n(z)(f(z) - 1)]^{(k)}$ and $[g^n(z)(g(z) - 1)]^{(k)}$ share $1$ CM, then $f(z) \equiv g(z)$.

In a latter paper, Zhang and Lin improved Theorem A and obtained the following result.

Theorem B ([4]). Let $f$ and $g$ be two nonconstant entire functions, and let $n, m$ and $k$ be three positive integer with $n > 2k + m + 4$. If $[f^n(z)(f(z) - 1)^m]^{(k)}$ and $[g^n(z)(g(z) - 1)^m]^{(k)}$ share $1$ CM, then $f(z) \equiv g(z)$ or $f$ and $g$ satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^m(\omega_1 - 1)^m - \omega_2^m(\omega_2 - 1)^m$.

In this article, we prove

Theorem 1. Let $f$ and $g$ be two transcendental entire functions, $n, m, t, l, p$ be positive integers. If $E_1(S_m, [f^n(f^l - 1)^t]^{(p)}) = E_1(S_m, [g^n(g^l - 1)^t]^{(p)})$ and $n > \frac{6}{m} + 3l + 4p$, then $f(z) \equiv bg(z)$, where $b^l = 1$.

2. Lemmas

To prove the theorem, we need the following lemmas.

Lemma 1 ([3]). Let $f(z)$ be a nonconstant meromorphic function, $k$ be a positive integer, if $f^{(k)} \not\equiv 0$, then $N(r, \frac{1}{f^{(k)}}) \leq N(r, \frac{1}{f}) + kN(r, f) + S(r, f)$.

Lemma 2. Let $F, G$ be defined as (1.1) and (1.2). If $E_1(S_m, F) = E_1(S_m, G)$, and $n > \frac{6}{m} + 3l + 4p$, then $H_m \equiv 0$.

Proof. If $H_m \neq 0$, then $E_1(1, F^m) = E_1(1, G^m)$, since $E_1(S_m, F) = E_1(S_m, G)$. Suppose that $z_0$ is a common simple zero point of $F^m - 1$ and $G^m - 1$, then it follows from (1.2) that $z_0$ is a zero point of $H_m$, and zero point of $F^m$ or $G^m$ with multiplicity 1 also are not poles of $H_m$. Thus, we
UNIQUENESS OF ENTIRE FUNCTIONS

331

have
\[ N_1 \left( r, \frac{1}{F_{m} - 1} \right) = N_1 \left( r, \frac{1}{G_{m} - 1} \right) \leq N \left( r, \frac{1}{H_{m}} \right) \leq T(r, H_m) + O(1) \]
\[ \leq N(r, H_m) + S(r). \]

By the definition of $H_m$, we have poles of $H_m$ with multiplicity 1. Thus
\[ N_1 \left( r, \frac{1}{F_{m} - 1} \right) = N_1 \left( r, \frac{1}{G_{m} - 1} \right) \leq N \left( r, \frac{1}{F_{m}} \right) \]
\[ + N \left( r, \frac{1}{(F_m)^{\gamma}} \right) + N \left( r, \frac{1}{(G_m)^{\gamma}} \right) + N \left( r, \frac{1}{G_{m} - 1} \right) \]
\[ + N \left( r, \frac{1}{(G_m)^{\gamma}} \right) + S(r). \]

Where $S(r) = \max\{S(r, f), S(r, g)\}$.

By the second fundamental theorem, we have
\[ T(r, F_{m}) + T(r, G_{m}) \leq N \left( r, \frac{1}{F_{m}} \right) + N \left( r, \frac{1}{F_{m} - 1} \right) + N \left( r, \frac{1}{G_{m}} \right) \]
\[ + N \left( r, \frac{1}{(F_m)^{\gamma}} \right) + N \left( r, \frac{1}{(G_m)^{\gamma}} \right) + N \left( r, \frac{1}{G_{m} - 1} \right) - N \left( r, \frac{1}{(G_m)^{\gamma}} \right) \]
\[ + S(r). \]

By Lemma 1, we get $N \left( r, \frac{1}{(G_m)^{\gamma}} \right) \leq N(r, \frac{1}{G_{m}}) + S(r)$. Thus
\[ N_0 \left( r, \frac{1}{(G_m)^{\gamma}} \right) + N \left( r, \frac{1}{G_{m} - 1} \right) \leq N \left( r, \frac{1}{G_{m}} \right) + S(r). \]
\[ \leq N \left( r, \frac{1}{G_{m}} \right) \leq N \left( r, \frac{1}{G_m} \right) + S(r). \]

It follows that
\[ N_0 \left( r, \frac{1}{(G_m)^{\gamma}} \right) + N \left( r, \frac{1}{G_{m} - 1} \right) \leq N \left( r, \frac{1}{G_m} \right) + S(r). \]

Similarly, we have
\[ N_0 \left( r, \frac{1}{F_{m}^{\gamma}} \right) + N \left( r, \frac{1}{F_{m} - 1} \right) \leq N \left( r, \frac{1}{F_m} \right) + S(r). \]
From (2.1)-(2.4) we have

\[
m(T(r, F) + T(r, G)) \leq N\left(r, \frac{1}{F^m}\right) + N_{1}\left(r, \frac{1}{F^{m-1}}\right) + N_{2}\left(r, \frac{1}{F^{m-1}}\right) + N_{3}\left(r, \frac{1}{G^m}\right) + N_{4}\left(r, \frac{1}{G^{m-1}}\right)
\]

(2.5)

\[
+ S(r) \leq 4N\left(r, \frac{1}{F^m}\right) + 4N\left(r, \frac{1}{G^m}\right) + 2N_{2}\left(r, \frac{1}{F^m}\right) + 2N_{2}\left(r, \frac{1}{G^m}\right) + S(r).
\]

Since

\[
N\left(r, \frac{1}{F^m}\right) + N_{2}\left(r, \frac{1}{F^m}\right) \leq N\left(r, \frac{1}{F^m}\right) - \left[N_{3}\left(r, \frac{1}{F^m}\right) + N_{3}\left(r, \frac{1}{F^m}\right)\right],
\]

and

\[
N_{3}\left(r, \frac{1}{F^m}\right) - 2N_{3}\left(r, \frac{1}{F^m}\right) \geq [m(n - p) - 2]N\left(r, \frac{1}{f}\right),
\]

we have

(2.6)

\[
N\left(r, \frac{1}{F^m}\right) + N_{2}\left(r, \frac{1}{F^m}\right) \leq N\left(r, \frac{1}{F^m}\right) - [m(n - p) - 2]N\left(r, \frac{1}{f}\right).
\]

Similarly,

(2.7)

\[
N\left(r, \frac{1}{G^m}\right) + N_{2}\left(r, \frac{1}{G^m}\right) \leq N\left(r, \frac{1}{G^m}\right) - [m(n - p) - 2]N\left(r, \frac{1}{g}\right).
\]
Combining (2.5)-(2.7), we have
\[ m \left[ T(r, F) + T(r, G) \right] \leq 2N \left( r, \frac{1}{F^m} \right) + 2N \left( r, \frac{1}{G^m} \right) \]
\[ + 2 \left[ N \left( r, \frac{1}{F^m} \right) - (m(n - p) - 2)N \left( r, \frac{1}{f} \right) \right] \]
\[ + 2 \left[ N \left( r, \frac{1}{G^m} \right) - (m(n - p) - 2)N \left( r, \frac{1}{g} \right) \right] + S(r) \]
\[ \leq 2N \left( r, \frac{1}{F^m} \right) + 2N \left( r, \frac{1}{G^m} \right) + 2N \left( r, \frac{1}{F^m} \right) + 2N \left( r, \frac{1}{G^m} \right) \]
\[ - 2(m(n - p) - 2) \left[ N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{g} \right) \right] + S(r) = 4N \left( r, \frac{1}{F^m} \right) \]
\[ + 4N \left( r, \frac{1}{G^m} \right) - [4m(n - p) - 6] \left[ N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{g} \right) \right] + S(r). \]

By Lemma 1, we have
\[ m \left( m \left( r, \frac{1}{F_1} \right) + m \left( r, \frac{1}{G_1} \right) \right) \leq m \left( m \left( r, \frac{1}{F} \right) + m \left( r, \frac{1}{G} \right) \right) + S(r) \]
\[ \leq 3m \left[ N \left( r, \frac{1}{F_1} \right) + N \left( r, \frac{1}{G_1} \right) \right] - [4m(n - p) - 6] \left[ N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{g} \right) \right] \]
\[ + S(r) \leq 3m \left[ N \left( r, \frac{1}{F_1} \right) + N \left( r, \frac{1}{G_1} \right) \right] \]
\[ - [4m(n - p) - 6] \left[ N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{g} \right) \right] + S(r), \]
where \( F_1 = f^n(f^l - 1)^l, G_1 = g^a(g^l - 1)^l. \) It follows that
\[ m \left[ T(r, F_1) + T(r, G_1) \right] \leq 4m \left[ N \left( r, \frac{1}{F_1} \right) + N \left( r, \frac{1}{G_1} \right) \right] \]
\[ - [4m(n - p) - 6] \left[ N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{g} \right) \right] + S(r). \]

We get \([m(n + tl) - 4ml - 4mp - 6]T(r, f) + T(r, g) \leq S(r), \) which contradicts the assumption that \( n > \frac{6}{m} + 3lt + 4p \). Therefore \( H_m \equiv 0, \) which completes the proof of Lemma 2.

**Lemma 3.** Let \( f \) be a transcendental meromorphic functions, \( a_1 \) and \( a_2 \) be two meromorphic functions such that \( T(r, a_j) = S(r, f)(j = 1, 2) \) and
Then
\[ T(r, f) \leq N(r, f) + N \left( r, \frac{1}{f - a_1} \right) + N \left( r, \frac{1}{f - a_2} \right) + S(r, f). \]

**Lemma 4.** Let \( f \) be a transcendental entire function, \( k \) be a positive integer, and \( c \) be a nonzero finite complex number. Then
\[ T(r, f) \leq N \left( r, \frac{1}{f} \right) + N \left( r, \frac{1}{f^{(k)} - c} \right) - N \left( r, \frac{1}{f^{(k+1)}} - c \right) + S(r, f), \]

where \( N_0(r, 1/f^{(k+1)}) \) is the counting function which only counts those points such that \( f^{(k+1)} = 0 \) but \( f^{(k)} - c \neq 0 \).

### 3. Proof of Theorem 1

Let \( F, G \) and defined as (1.1) and (1.2).

By Lemma 2, we have \( H_m \equiv 0 \), that is
\[
\frac{(F^m)''}{(F^m)'} - 2 \frac{(F^m)'}{F^m} - 2 = \frac{(G^m)''}{(G^m)'} - 2 \frac{(G^m)'}{G^m}.
\]
Thus
\[
\frac{1}{G^m - 1} \equiv \frac{A}{F^m - 1} + B;
\]
where \( A \neq 0 \) and \( B \) are two constants. Hence \( E(1, F^m) = E(1, G^m) \), \( T(r, F) = T(r, G) \).

(1) Now we claim that
\[
f^n(f^l - 1)^l \equiv a g^n(g^l - 1)^l.
\]

Next we consider the following two cases.

**Case 1.** When \( B = 0 \), by (3.1), we have
\[
F^m = AG^m + (1 - A).
\]
(a) If \( A = 1 \), then by (3.3), we have \( F^m = G^m \), and hence \( f^n(f^l - 1)^l \equiv a g^n(g^l - 1)^l \).
(b) If $A \neq 1$, then by (3.3), we have

$$F^{m-1}F' = AG^{m-1}G'. \tag{3.4}$$

From (3.3) and (3.4) we get: when $F = 0$, $G^m \neq 0, 1$ and $G' = 0$, when $G = 0$, $F^m \neq 0, 1$ and $F' = 0$. Thus

$$N\left(r, \frac{1}{F}\right) - N_0\left(r, \frac{1}{(G^m)'}\right) = S(r, F), \tag{3.5}$$

$$N\left(r, \frac{1}{G}\right) - N_0\left(r, \frac{1}{(F^m)'}\right) = S(r, F).$$

By the second fundamental theorem, we have

$$T(r, F^m) \leq N\left(r, \frac{1}{F^m}\right) + N\left(r, \frac{1}{(F^m)'(1-A)}\right) - N_0\left(r, \frac{1}{(F^m)'}\right) + S(r, F) \tag{3.6}$$

Similarly, we have

$$T(r, G^m) \leq N\left(r, \frac{1}{G^m}\right) + N\left(r, \frac{1}{G}\right) - N_0\left(r, \frac{1}{(G^m)'}\right) + S(r, F). \tag{3.7}$$

From (3.5)-(3.7), we have

$$2mT(r, F) \leq \left[N\left(r, \frac{1}{G}\right) - N_0\left(r, \frac{1}{(G^m)'}\right)\right] + S(r, F) \leq 2T(r, F) + S(r, F).$$

Hence $m = 1$. By (3.3) we get

$$f^n(f^l - 1)^l \equiv ag^n(g^l - 1)^l + P(z), \tag{3.8}$$

where $P(z)$ is a polynomial of degree at most $p - 1$.

If $P(z) \neq 0$, by (3.8) and Lemma 3, we have

$$T(r, f^n(f^l - 1)^l) \leq N\left(r, f^n(f^l - 1)^l\right) + N\left(r, \frac{1}{f^n(f^l - 1)^l - P}\right) + S(r, f) \leq N\left(r, \frac{1}{f^l}\right) + N\left(r, \frac{1}{f^l - 1}\right) + S(r, f) \leq 2(1 + l)T(r, f) + S(r, f).$$
Thus, \( n + tl \leq 2(1 + l) \), which contradicts the assumption that \( n > \frac{6}{m} + 3tl + 4p \).

**Case 2.** When \( B \neq 0 \), by (3.1), we have

\[
\frac{1}{G^{m} - 1} = B \frac{F^{m} + \left( \frac{A}{B} - 1 \right)}{F^{m} - 1}, \quad \frac{A}{F^{m} - 1} = -B \frac{G^{m} - \left( \frac{1}{B} + 1 \right)}{G^{m} - 1},
\]

and

\[
\frac{G^{m-1}G'}{(G^{m} - 1)^2} = A \frac{F^{m-1}F'}{(F^{m} - 1)^2}.
\]

Thus

\[
(3.9) \quad F^{m} + \left( \frac{A}{B} - 1 \right) \neq 0, \quad G^{m} - \left( \frac{1}{B} + 1 \right) \neq 0.
\]

(a) If \( A = B \).

By (3.9), we have \( F \neq 0 \). Since \( F = (f^{n}(f^{l} - 1)^{t}(p) \) and \( n > p \), thus \( f \neq 0 \). Let \( f = e^{\alpha} \), where \( \alpha \) is a nonconstant entire function. Thus

\[
f^{n}(f^{l} - 1)^{t} = e^{n\alpha} \sum_{j=0}^{t} (-1)^{t-j} C_{t}^{j} e^{lj\alpha} = \sum_{j=0}^{t} (-1)^{t-j} C_{t}^{j} e^{(n+j)\alpha}.
\]

Let

\[
((-1)^{t-j} C_{t}^{j} e^{(n+j)\alpha})^{(p)} = P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)} e^{(n+j)\alpha},
\]

where \( P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)}(j = 0, 1, 2, \cdots, t) \) are differential polynomials. Thus

\[
F = \sum_{j=0}^{t} P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)}) e^{(n+j)\alpha} = e^{n\alpha} \sum_{j=0}^{t} P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)}) e^{lj\alpha}
\]

= \( e^{n\alpha} F_{0} \),

where \( F_{0} = \sum_{j=0}^{t} P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)}) e^{lj\alpha} \).

Obviously, there exists \( j(0 \leq j \leq t) \), such that \( P_{j}(\alpha', \alpha'', \cdots, \alpha^{(p)}) \neq 0 \). Suppose \( P_{0}(\alpha', \alpha'', \cdots, \alpha^{(p)}) \neq 0 \). Since \( F \neq 0 \), thus \( F_{0} \neq 0 \). Since \( f \) is a
nonconstant entire function we use Lemma 3 to obtain

\[ lT(r, e^\alpha) = T(r, F_0) \leq N \left( r, \frac{1}{F_0} \right) \]

\[ + N \left( r, \frac{1}{F_0 - P_0(\alpha', \alpha'', \ldots, \alpha^{(p)})} \right) + N(r, F_0) + S(r, e^\alpha) \]

\[ = N \left( r, \frac{1}{\sum_{j=1}^{t} P_j(\alpha', \alpha'', \ldots, \alpha^{(p)}) e^{j\alpha}} \right) + N(r, F_0) + S(r, e^\alpha) \]

\[ \leq l(t - 1)T(r, e^\alpha) + S(r, e^\alpha), \]

which is a contradiction.

(b) If \( A \neq B \) and \( B = -1 \).

By (3.9), we have \( G \neq 0 \). Since \( G = (g^n(g^l - 1)^l)^{p+1} \) and \( n > p \), thus \( g \neq 0 \). Let \( g = e^\beta \), where \( \beta \) is a nonconstant entire function. Similarly, we have \( lT(r, e^\beta) \leq l(t - 1)T(r, e^\beta) + S(r, e^\beta) \), which is a contradiction.

(c) If \( A \neq B \) and \( B \neq -1 \).

When \( m > 1 \), by (3.10) and the second fundamental theorem, we have

\[ T(r, G^m) \leq N(r, \frac{1}{G^m}) + N(r, \frac{1}{G^m - (\frac{A}{B} + 1)}) + N(r, G^m) + S(r, G) \]

\[ \leq N(r, \frac{1}{G}) + S(r, G) \leq T(r, G) + S(r), \]

thus \( G \) is constant, hence \( g \) is constant, which is a contradiction.

When \( m = 1 \), by (3.10), we have \( F + (\frac{A}{B} - 1) \neq 0 \), thus \( (f^n(f^l - 1)^l)^{p+1} + (\frac{A}{B} - 1) \neq 0 \), by Lemma 4, we have

\[ (n + lt)T(r, f) = T(r, f^n(f^l - 1)^l) + S(r, f) \leq N_{p+1} \left( r, \frac{1}{f^n(f^l - 1)^l} \right) \]

\[ + N \left( r, \frac{1}{f^n(f^l - 1)^l} + (\frac{A}{B} - 1) \right) - N_0(r, \frac{1}{(f^n(f^l - 1)^l)^{p+1}}) + S(r, f) \]

\[ \leq N_{p+1} \left( r, \frac{1}{f^n(f^l - 1)^l} \right) + S(r, f) \leq (p + 1)N \left( r, \frac{1}{f} \right) \]

\[ + N_{p+1} \left( r, \frac{1}{(f^l - 1)^l} \right) + S(r, f) \leq (p + 1 + lt)T(r, f) + S(r, f), \]
thus \( n \leq p + 1 \), which contradicts the assumption that \( n > \frac{6}{m} + 3t + 4p \).

By case 1 and case 2, we get (3.2).

By (3.2), we have

\[
f^{n-1}(f^l - 1)^{t-1} \left( f^l - \frac{n}{n + tl} \right) f'
\]

\[
= ag^{n-1}(g^l - 1)^{t-1} \left( g^l - \frac{n}{n + tl} \right) g'.
\]

From (3.2) and (3.11), we get

(i) When \( f = 0 \), have \( g = 0 \) or \( g^l = 1 \).

(ii) When \( f^l = 1 \), have \( g^l = 1 \) or \( g = 0 \).

(iii) When \( f^l = \frac{n}{n + tl} \), have \( g^l = \frac{n}{n + tl} \) or \( g' = 0 \) (such that \( g^l \neq \frac{n}{n + tl} \), \( g \neq 0 \), \( g^l \neq 1 \)).

By (3.2), (i) and (ii), we have

\[
N \left( r; \frac{1}{f^l - 1} \right) - N \left( r; \frac{1}{f^l - 1}, \frac{1}{g^l - 1} \right) \leq \frac{t}{n} N \left( r; \frac{1}{f^l - 1} \right),
\]

and

\[
N \left( r; \frac{1}{f} \right) - N \left( r; \frac{1}{f}, \frac{1}{g} \right) \leq \frac{t}{n} N \left( r; \frac{1}{g^l - 1} \right).
\]

Using the fact that \( f \) and \( g \) are nonconstant entire functions and the second fundamental theorem, we have

\[
2lT(r, f) \leq N \left( r; \frac{1}{f} \right) + N \left( r; \frac{1}{f^l - 1} \right) + N \left( r; \frac{1}{f^l - \frac{n}{n + tl}} \right) - N_0 \left( r; \frac{1}{f^l} \right) + S(r, f)
\]

\[
(3.14)
\]

and

\[
2lT(r, g) \leq N \left( r; \frac{1}{g} \right) + N \left( r; \frac{1}{g^l - 1} \right) + N \left( r; \frac{1}{g^l - \frac{n}{n + tl}} \right) - N_0 \left( r; \frac{1}{g^l} \right) + S(r, g).
\]

\[
(3.15)
\]

If \( f^l = g^l \), then there exists constant \( b \), such that \( f \equiv bg \), where \( b^l = 1 \).
If \( f^l \neq g^l \), by (3.12)-(3.15),(i)-(iii), we have
\[
4lT(r, f) = 2l[T(r, f) + T(r, g)] \leq 2N\left( r, \frac{1}{f^l} g \right) + 2N\left( r, \frac{1}{f^l - 1}, \frac{1}{g^l - 1} \right) + 2N\left( r, \frac{1}{f^l - \frac{n}{n+1}}, \frac{1}{g^l - \frac{n}{n+1}} \right)
\]
\[
+ \frac{2t}{n} N\left( r, \frac{1}{f^l - 1} \right) + \frac{2t}{n} N\left( r, \frac{1}{g^l - 1} \right) + S(r, f)
\]
\[
\leq 2N\left( r, \frac{1}{f^l - 1} \right) + \frac{2t}{n} N\left( r, \frac{1}{f^l - 1} \right) + \frac{2t}{n} N\left( r, \frac{1}{g^l - 1} \right) + S(r, g) \leq \left( 2l + \frac{4tl}{n} \right) T(r, f) + S(r, f).
\]

Thus \( 2l \leq \frac{4t}{n} \), which contradicts the assumption that \( n > \frac{6}{m} + 3tl + 4p \).

Summarizing the above discussion we obtain Theorem 1.

**REFERENCES**


Received: 16.III.2010 Liuzhou City Vocational College, Liuzhou Guangxi, 545001, P. R. CHINA
Revised: 2.VI.2010

Department of Information and Computing Science of Guangxi University of Technology, Liuzhou Guangxi, 545006, P. R. CHINA

.cnzy2006@163.com