Existence and stability of periodic solutions for impulsive fuzzy Cohen-Grossberg neural networks on time scales

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Abstract By applying the method of coincidence degree and constructing a suitable Lyapunov functional, some sufficient conditions are established for the existence and globally exponential stability of periodic solutions for a kind of impulsive fuzzy Cohen-Grossberg neural networks on time scales. Moreover an example is given to illustrate our results.

Keywords periodic solutions · fuzzy Cohen-Grossberg neural networks · impulses

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1 Introduction

In the recent years, Cohen-Grossberg [11] neural networks have been extensively studied and applied in many different fields such as associative memory, signal processing and some optimization problems. In such applications, it is of prime importance to ensure that the designed neural networks are stable (see [19]). In practice, due to the finite speeds of the switching and transmission of signals, time delays do exist in a working network and thus should be incorporated into the model equation (see [1], [7]-[9], [20], [25], [29]-[30]). In the recent years, the dynamical behaviors, including the existence and global stability of equilibrium and periodic solutions of Cohen-Grossberg

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neural networks with time delays, have been studied (see [1]-[2], [7]-[10], [20], [22]-[25], [29]-[30] and the references therein).

In this paper, we would like to integrate fuzzy operations into Cohen-Grossberg neural networks. Speaking of fuzzy operations, YANG and YANG [26] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have found that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs (see [12]-[13], [26]-[28], [31]-[32]).

Motivated by the above discussions, we consider the following impulsive fuzzy Cohen-Grossberg neural networks on time scales:

\[
\begin{align*}
\dot{x}_i(t) & = -a_i(x_i(t)) \left[ b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) \right] \\
& \quad - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) \\
& \quad - \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) - I_i(t) \right], t \in \mathbb{T}^+, t \neq t_k, \\
\Delta_i(x_i(t_k)) & = I_i(x_i(t_k)) = x_i(t_k^+) - x_i(t_k^-), k \in \mathbb{N}, i = 1, 2, \ldots, n,
\end{align*}
\]

(1.1)

where $\mathbb{T}$ is an $\omega$-periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. $n$ corresponds to the number of units in the neural networks. For $i = 1, 2, \ldots, n, x_i(t)$ corresponds to the state of the $i$th neuron. $f_j(\cdot)$ and $g_j(\cdot)$ are signal transmission functions. $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of the $j$-th unit from the $i$-th unit and satisfies $0 \leq \tau_{ij}(t) \leq \tau (\tau$ is a constant). $a_i(x_i(t)) > 0$ represents an amplification function at time $t$. $b_i(x_i(t))$ is an appropriately behaved function at time $t$; $c_{ij}(t)$ represents the elements of the feedback template. $I_i(t)$ is an external input to the $i$-th unit. $\alpha_{ij}(t)$ and $\beta_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $x_i(t_k^+)$ and $x_i(t_k^-)$ represent the right and left limit of $x_i(t_k)$ in the sense of time scales, $\{t_i\}$ is a sequence of real numbers such that $0 < t_1 < t_2 < \ldots < t_l \rightarrow \infty$, as $l \rightarrow \infty$, there exists a positive integer $q$ such that $t_{i+q} = t_i + \omega, J_{i(l+q)}(x_i(t_{k+q})) = J_{i(l)}(x_i(t_k))), l \in \mathbb{Z}, i = 1, 2, \ldots, n.$

For each interval $L$ of $\mathbb{R}$ we denote $L_{\mathbb{T}} = L \cap \mathbb{T}$. Without loss of generality, we also assume that $\{0, \omega\}_\mathbb{T} \cap \{t_i, l \in \mathbb{Z}\} = \{t_1, t_2, \ldots, t_q\}$. Let $\mathbb{R}^+ = (0, \infty), \mathbb{T}^+ = \mathbb{R}^+ \cap \mathbb{T}$. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in C(\mathbb{T}, \mathbb{R}^n), \|x\| = \sum_{i=1}^{n} \max_{t \in [0, \omega]} |x_i(t)|$. The
initial conditions associated with system (1.1) are of the form
\[ x_i(t) = \varphi_i(t), \quad t \in [-\tau, 0], \quad \tau = \max \sup_{1 \leq i,j \leq n} \{\tau_{ij}(t)\}, \quad (1.2) \]
where \( \varphi_i(t), i = 1, 2, \ldots, n \) are continuous functions on \([-\tau, 0]\).

For the sake of convenience, we introduce some notations
\[
\bar{f} = \max_{t \in [0, \omega]} |f(t)|, \quad \underline{f} = \min_{t \in [0, \omega]} |f(t)|, \quad \hat{f} = \frac{1}{\omega} \int_0^\omega f(t) \Delta t, \\
\|f\|_2 = \left( \int_0^\omega |f(t)|^2 \Delta t \right)^{1/2}, \quad \|g\|_2 = \left( \int_0^\omega |g(t)|^2 \Delta t \right)^{1/2},
\]
where \( f \) is an \( \omega \)-periodic function.

Throughout this paper, we make the following assumptions:

(A1) \( c_{ij}, \alpha_{ij}, \beta_{ij}, \tau_{ij}, I_i \in C(T, \mathbb{R}) \) are \( \omega \)-periodic functions, \( i, j = 1, 2, \ldots, n \).

(A2) \( a_i \in C(\mathbb{R}, \mathbb{R}^+) \) are bounded functions, namely there exist positive constants \( a_i, \underline{a}_i \) such that \( 0 < a_i \leq \alpha_i(\cdot) \leq \underline{a}_i, \quad i = 1, 2, \ldots, n \).

(A3) \( b_i \in C(\mathbb{R}, \mathbb{R}^+) \) are delta differentiable, and there exist positive constants \( b_i, \delta_i \) such that \( 0 < b_i \leq \frac{db_i(x_i)}{dx_i} \leq \delta_i, \quad b_i(0) = 0, \quad i = 1, 2, \ldots, n \).

(A4) \( f_j, g_j \in C(\mathbb{R}, \mathbb{R}) \), and there exist \( M_f, M_g, \kappa_j, \nu_j (j = 1, 2, \ldots, n) \) such that
\[
|f_j(x_j(t))| \leq M_f, \quad |g_j(x_j(t))| \leq M_g, \quad |f_j(u) - f_j(v)| \leq \kappa_j|u - v|, \quad |g_j(u) - g_j(v)| \leq \nu_j|u - v|.
\]

(A5) \( J_{ik} \in C(\mathbb{R}, \mathbb{R}) \) and there exist positive constants \( \rho_{ik} \) such that
\[
|J_{ik}(u) - J_{ik}(v)| \leq \rho_{ik}|u - v| \quad \text{for all} \quad u, v \in \mathbb{R}, \quad k \in \mathbb{N}, \quad i = 1, 2, \ldots, n.
\]

The organization of this paper is as follows. In Section 2, we introduce some definitions and lemmas. In Section 3, by using coincidence degree, we establish sufficient conditions for the existence of periodic solutions of system (1.1). In Section 4, by constructing a Lyapunov functional, we shall derive sufficient conditions for the globally exponential stability of the periodic solutions of system (1.1). An example is given to demonstrate the effectiveness of our results in Section 5. Conclusions are drawn in Section 6.

**2 Preliminaries**

In this section, we shall first recall some basic definitions, lemmas which are used in what follows.

Let \( T \) be a nonempty closed subset (time scale) of \( \mathbb{R} \). The forward and backward jump operators \( \sigma, \rho: T \to T \) and the graininess \( \mu: T \to \mathbb{R}^+ \) are defined, respectively, by \( \sigma(t) = \inf\{s \in T: s > t\} \), \( \rho(t) = \sup\{s \in T: s < t\} \), \( \mu(t) = \sigma(t) - t \).

A point \( t \in T \) is called left-dense if \( t > \inf T \) and \( \rho(t) = t \), left-scattered if \( \rho(t) < t \), right-dense if \( t < \sup T \) and \( \sigma(t) = t \), and right-scattered if \( \sigma(t) > t \). If \( T \) has a left-scattered maximum \( m \), then \( T^k = T \{m\} \), otherwise \( T^k = T \). If \( T \) has a right-scattered minimum \( m \), then \( T_k = T \{m\} \), otherwise \( T_k = T \).

A function \( f: T \to R \) is right-dense continuous provided it is continuous at right-dense point in \( T \) and its left-side limits exist at left-dense points in \( T \). If \( f \) is continuous at each right-dense point and each left-dense point, then \( f \) is said to be a continuous function on \( T \). 

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For $y : T \rightarrow \mathbb{R}$ and $t \in T^k$, we define the delta derivative of $y(t), y^\Delta(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood $U$ of $t$ such that $|y(\sigma(t)) - y(s) - y^\Delta(t)(\sigma(t) - s)| < \varepsilon|\sigma(t) - s|$ for all $s \in U$. If $y$ is continuous, then $y$ is right-dense continuous, and if $y$ is delta differentiable at $t$, then $y$ is continuous at $t$. Let $y$ be right-dense continuous, if $Y^\Delta(t) = y(t)$, then we define the delta integral by $\int_a^t y(s)\Delta s = Y(t) - Y(a)$.

**Definition 2.1** ([3]) If $a \in T, \sup T = \mathbb{R},$ and $f$ is rd-continuous on $[0, \infty)$, then we define the improper integral by

$$\int_a^\infty f(t)\Delta t = \lim_{b \to \infty} \int_a^b f(t)\Delta t$$

provided this limit exists, and we say that the improper integral converges in this case. If this limit does not exist, then we say that the improper integral diverges.

**Definition 2.2** ([14]) For each $t \in T$, let $N$ be a neighborhood of $t$, then, for $V \in C_{rd}[T \times \mathbb{R}^n, \mathbb{R}^+]$, define $D^+V^\Delta(t, x(t))$ to mean that, given $\varepsilon > 0$, there exists a right neighborhood $N_\varepsilon \subset N$ of $t$ such that

$$\frac{V(\sigma(t), x(\sigma(t))) - V(s, x(\sigma(t))) - \mu(t, s)f(t, x(t))}{\mu(t, s)} < D^+V^\Delta(t, x(t)) + \varepsilon,$$

for each $s \in N_\varepsilon, s > t$, where $\mu(t, s) = \sigma(t) - s$. If $t$ is rd and $V(t, x(t))$ is continuous at $t$, this reduces to

$$D^+V^\Delta(t, x(t)) = \frac{V(\sigma(t), x(\sigma(t))) - V(t, x(\sigma(t)))}{\sigma(t) - t}.$$

**Definition 2.3** ([14]) We say that a time scale $T$ is periodic if there exists $p > 0$ such that if $t \in T$, then $t \pm p \in T$. For $T \neq \mathbb{R}$, the least positive $p$ is called the period of the time scale. Let $T \neq \mathbb{R}$ be a periodic time scale with period $p$. We say that the function $f : T \rightarrow \mathbb{R}$ is $\omega$-periodic if there exists a natural number $n$ such that $\omega = np, f(t + \omega) = f(t)$ for all $t \in T$ and $\omega$ is the least number such that $f(t + \omega) = f(t)$. If $T = \mathbb{R}$, we say that $f$ is $\omega > 0$ periodic if $\omega$ is the least positive number such that $f(t + \omega) = f(t)$ for all $t \in T$.

A function $r : T \rightarrow R$ is called regressive if $1 + \mu(t)r(t) \neq 0$, for all $t \in T^k$.

If $r$ is a regressive function, then the generalized exponential function $e_r$ is defined by $e_r(t, s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta \tau\right\}, s, t \in T$, with the cylinder transformation

$$\xi_{h(z)} = \begin{cases} \frac{\log(1 + hz)}{h}, & h \neq 0, \\ z, & h = 0. \end{cases}$$

Let $p, q : T \rightarrow \mathbb{R}$ be two regressive functions, we define $p \oplus q := p + q + \mu pq; \ p \ominus q := p \oplus (-q); \ p := \frac{p}{1 + pq}$. 

**Lemma 2.4** ([5]) Let $p, q$ be regressive functions on $T$. Then:

(i) $e_0(t, s) = 1$ and $e_p(t, t) = 1; \ 
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(ii) \( e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s) \);
(iii) \( e_p(t, s)e_p(s, r) = e_p(t, r) \);
(iv) \( e^\Delta_p(\cdot, s) = pe_p(\cdot, s) \).

Lemma 2.5 ([16]) Assume that \( f, g : \mathbb{T} \rightarrow \mathbb{R} \) are delta differentiable at \( t \in \mathbb{T}^k \), then \( (fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)) \).

Lemma 2.6 ([4]) Let \( t_1, t_2 \in [0, \omega]_T \). If \( x : \mathbb{T} \rightarrow \mathbb{R} \) is \( \omega \)-periodic, then \( x(t) \leq x(t_1) + \int_0^\omega |x^\Delta(s)|\Delta s \), and \( x(t) \geq x(t_2) - \int_0^\omega |x^\Delta(s)|\Delta s \).

Lemma 2.7 ([16]) Let \( a, b \in \mathbb{T} \), for rd-continuous functions \( f, g : [a, b]_T \rightarrow \mathbb{R} \), we have
\[
\int_a^b |f(t)||g(t)|\Delta t \leq \left( \int_a^b |f(t)|^2\Delta t \right)^{1/2} \left( \int_a^b |g(t)|^2\Delta t \right)^{1/2}.
\]

Lemma 2.8 ([16]) Let \( \mathbb{T} \) be an \( \omega \)-periodic time scale, then \( \sigma(t + \omega) = \sigma(t) + \omega \), for all \( t \in \mathbb{T} \).

Lemma 2.9 ([16]) Let \( \mathbb{T} \) be an \( \omega \)-periodic time scale, then \( \mu(t) \) is an \( \omega \)-periodic function.

Lemma 2.10 ([26]) Suppose \( x \) and \( y \) are two states of system (1.1), then we have
\[
\left| \bigwedge_{j=1}^n \alpha_{ij}(t)g_j(x) - \bigwedge_{j=1}^n \alpha_{ij}(t)g_j(y) \right| \leq \sum_{j=1}^n |\alpha_{ij}(t)||g_j(x) - g_j(y)|,
\]
\[
\left| \bigvee_{j=1}^n \beta_{ij}(t)g_j(x) - \bigvee_{j=1}^n \beta_{ij}(t)g_j(y) \right| \leq \sum_{j=1}^n |\beta_{ij}(t)||g_j(x) - g_j(y)|.
\]

Definition 2.11 The periodic solution \( x^*(t) = (x^*_1(t), x^*_2(t), \ldots, x^*_n(t))^T \) of system (1.1) with initial value \( \varphi^*(t) = (\varphi^*_1(t), \varphi^*_2(t), \ldots, \varphi^*_n(t))^T \) is said to be globally exponentially stable if there exist constants \( M \geq 1 \) and \( \varepsilon > 0 \) such that, for any other solution \( x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \) with initial value \( \varphi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T \) and for every \( \eta \in \mathbb{T} \), \( ||x_i(t) - x^*_i(t)|| \leq Me^{-\varepsilon t}||\varphi - \varphi^*|| \), where \( ||\varphi - \varphi^*|| = \sup_{\eta \in (-\tau, 0)} \max_{1 \leq i \leq n} |\varphi_i(\eta) - \varphi^*_i(\eta)| \).

3 Existence of a periodic solution

In this section, based on Mawhin’s continuation theorem, we shall study the existence of at least one periodic solution of system (1.1). To do so, we shall make some preparations.

Let \( \mathbb{X}, \mathbb{Y} \) be two Banach spaces, \( L : \text{Dom} L \subset \mathbb{X} \rightarrow \mathbb{Y} \) be a linear mapping and \( \mathcal{N} : \mathbb{X} \rightarrow \mathbb{Y} \) be a continuous mapping. Then \( L \) will be called a Fredholm mapping of index zero if \( \dim \text{Ker} L = \text{codim} \text{Im} L < +\infty \) and \( \text{Im} L \) is closed in \( \mathbb{Y} \). If \( L \) is a Fredholm mapping of index zero and there exist continuous projectors \( P : \mathbb{X} \rightarrow \mathbb{X} \) and \( Q : \mathbb{Y} \rightarrow \mathbb{Y} \) such that \( \text{Im} P = \text{Ker} L, \text{Ker} Q = \text{Im} (I - Q) \), it follows that the
mapping \( L|\text{Dom}L \cap \text{Ker}P : (I - P)X \to \text{Im} L \) is invertible. We denote the inverse of that mapping by \( K_P \). If \( P \) is an open bounded subset of \( X \), the mapping \( N \) will be called \( L \)-compact on \( \Omega \) if \( QN(\Omega) \) is bounded and \( K_P(I - Q)N : \Omega \to X \) is compact. Since \( \text{Im} Q \) is isomorphic to \( \text{Ker} L \), there exists an isomorphism \( J : \text{Im} Q \to \text{Ker} L \).

**Lemma 3.1** ([21]) Let \( X, Y \) be two Banach spaces, \( \Omega \subset X \) be open bounded. Suppose that \( L : \text{Dom} L \subset X \to Y \) is a linear Fredholm operator of index zero with \( \text{Dom} L \cap \Omega \neq \emptyset \) and \( N : \Omega \to Y \) is \( L \)-compact. Furthermore suppose that:

(a) for each \( \lambda \in (0, 1), x \in \partial \Omega \cap \text{Dom} L, Lx \neq \lambda Nx \);
(b) for each \( x \in \partial \Omega \cap \text{Ker} L, QNx \neq 0 \);
(c) \( \deg\{JQN, \partial \Omega \cap \text{Ker} L, 0\} \neq 0 \), then the equation \( Lx = Nx \) has at least one solution in \( \Omega \), where \( \Omega \) is the closure to \( \Omega \), \( \partial \Omega \) is the boundary of \( \Omega \).

**Definition 3.2** A real matrix \( A = (a_{ij})_{n \times n} \) is said to be a non-singular \( M \)-matrix if \( a_{ij} \leq 0, i, j = 1, 2, \ldots, n, i \neq j \), and all successive principal minors of \( A \) are positive.

**Theorem 3.3** Under condition (A1) – (A5), and \( E = (e_{ij})_{n \times n} \) is a non-singular \( M \)-matrix, where

\[
e_{ij} = \begin{cases} 
    a_i \varrho_i \omega(1 - a_i \delta_i \omega) - (1 + a_i \varrho_i \omega) \sum_{k=1}^{q} \rho_{ik} & \text{if } i = j, \\
    (1 + a_i \varrho_i \omega)(\overline{e}_{ij} \kappa_j + \overline{e}_{ij} \nu_j) - \overline{e}_{ij} \varphi_i \omega & \text{if } i \neq j.
\end{cases}
\]

then system (1.1) has at least one \( \omega \)-periodic solution \( x^* = (x_1^* , x_2^* , \ldots , x_n^*)^T \).

**Proof.** Let \( PC(\mathbb{T}) = \{ x : \mathbb{T} \to \mathbb{R} | x \in C(t_k, t_{k+1}], \exists x(t_k^-) = x(t_k), x(t_k^+) \neq x(t_k), k \in \mathbb{N}, X = \{ x = (x_1, x_2, \ldots , x_n)^T \in PC(\mathbb{T}) | x(t + \omega) = x(t), t \in \mathbb{T} \}, Y = X \times \mathbb{R}^{n \times (q+1)}, \) with the norm defined by \( \| x \|_X = \sum_{i=1}^{n} \| x_i \|, \) where \( \| x_i \| = \max_{t \in [0, \omega]_T} |x_i(t)|, \| x \|_Y = \| x \|_X + \| y \|, \) where \( x \in X, y \in \mathbb{R}^{n \times (q+1)}, \) \( \| \cdot \| \) is any norm of \( \mathbb{R}^{n \times (q+1)}, \) then \( X, Y \) are Banach space.

Set \( L : \text{Dom} L \cap X \to Y, x \to (x^A, \Delta x(t_1), \ldots , \Delta x(t_q), 0), \) and \( N : X \to \mathbb{Y}, \) where

\[
Lx = \begin{pmatrix} 
    (A_1(x)) & \Delta x_1(t_1) & (\Delta x_1(t_2)) & \ldots & (\Delta x_1(t_q)) \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    (A_n(x)) & \Delta x_n(t_1) & (\Delta x_n(t_2)) & \ldots & (\Delta x_n(t_q))
\end{pmatrix},
\]

\[
A_i(t) = -a_i(x_i(t)) \left[ b_i(x_i(t)) - \sum_{j=1}^{n} \sigma_{ij}(t)f_j(x_j(t)) \right].
\]
\[
- \sum_{j=1}^{n} \alpha_{ij}(t)g_j(x_j(t - \tau_{ij}(t))) \\
- \sum_{j=1}^{n} \beta_{ij}(t)g_j(x_j(t - \tau_{ij}(t))) - I_i(t)
\]

It is easy to see that
\[
\ker L = \{ x \in \mathbb{X} : x = h \in \mathbb{R}^n \}, \\
\text{im } L = \left\{ z = (f,c_1,\ldots,c_q,d) \in \mathbb{Y} : \int_0^\omega f(s)\Delta s + \sum_{k=1}^q C_k + d = 0 \right\}.
\]

Thus \( \dim \ker L = \text{codim im } L = n \). So, \( \text{im } L \) is closed in \( \mathbb{Y} \), \( L \) is a Fredholm mapping of index zero. Define the projectors \( P \) and \( Q \) as
\[
P x = \frac{1}{\omega} \int_0^\omega x(t)\Delta t, x \in \mathbb{X}, \\
Q z = Q(f,c_1,\ldots,c_q,d) = \left( \frac{1}{\omega} \int_0^\omega f(s)\Delta s + \sum_{k=1}^q C_k + d, 0,\ldots,0 \right), z \in \mathbb{Y}.
\]

Obviously, \( P \) and \( Q \) are continuous projectors and satisfy \( \text{im } P = \ker L \), \( \text{im } L = \ker Q = \text{im } (I - Q) \). Denoting by \( L_P^{-1} = L|_{\text{Dom } L \cap \ker P} \) and the generalized inverse \( K_P = L_P^{-1} \)
\[
(K_P z)(t) = \int_0^t f(s)\Delta s + \sum_{l>t_k} C_k - \frac{1}{\omega} \int_0^\omega \int_0^t f(s)\Delta s\Delta t - \sum_{k=1}^q C_k.
\]

Similar to \([18]\), it is not difficult to show that \( QN(\Omega), K_P(I - Q)N(\Omega) \) are relatively compact for any open bounded set \( \Omega \subset \mathbb{X} \). Therefore, \( N \) is \( L \)-compact on \( \Omega \) for any open bounded set \( \Omega \subset \mathbb{X} \).

Now, we only need to search for an appropriate open bounded subset \( \Omega \) for the application of Lemma 3.1. Corresponding to the operator equation \( L x = \lambda Nx, \lambda \in (0,1) \), we have
\[
x_i^\Delta(t) = \lambda \left\{ -a_i(x_i(t)) \left[ b_i(x_i(t)) - \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) \right] \\
- \sum_{j=1}^{n} \alpha_{ij}(t) x_j(t - \tau_{ij}(t)) \\
- \sum_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) - I_i(t) \right\}, t \in T^+, t \neq t_k, \\
\Delta_i(x_i(t_k)) = \lambda J_{ik}(x_i(t_k)) = \lambda (x_i(t_k^+) - x_i(t_k^-)), k \in \mathbb{N}, i = 1,2,\ldots,n.
\]

(3.1)
Suppose that \( x = (x_1, x_2, \ldots, x_n)^T \) is a solution of system (3.1) for a certain \( \lambda \in (0, 1) \), set \( t_0 = t_0^+ = t_q = 0, t_{q+1} = \omega \), we get

\[
\int_0^\omega |x_i^\Delta(t)| \Delta t = \sum_{k=1}^{q+1} \int_{t_{k-1}}^{t_k} |x_i^\Delta(t)| \Delta t + \sum_{k=1}^q |J_{ik}(x_i(t_k))|
\leq \bar{a}_i \int_0^\omega \left| b_i(x_i(t)) - \sum_{j=1}^n c_{ij}(t) f_j(x_j(t - \tau_{ij}(t))) - \bigwedge_{j=1}^n a_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) \right| \Delta t + \sum_{k=1}^q |J_{ik}(x_i(t_k))|
\leq \bar{a}_i \left[ \int_0^\omega |b_i(x_i(t)) - b_i(0)| \Delta t + \sum_{j=1}^n \int_0^\omega |c_{ij}(t)| |f_j(x_j(t - \tau_{ij}(t))) - f_j(0)| \Delta t \right.
\left. + \int_0^\omega \left( \bigwedge_{j=1}^n a_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) - \bigwedge_{j=1}^n a_{ij}(t) g_j(0) \right) \Delta t \right] + \bar{a}_i \bar{T}_i \omega
\]

Applying Lemma 2.10, we have

\[
\int_0^\omega |x_i^\Delta(t)| \Delta t \leq \bar{a}_i \left[ \int_0^\omega |b_i(x_i(t)) - b_i(0)| \Delta t \right.
\left. + \sum_{j=1}^n \int_0^\omega |c_{ij}(t)| |f_j(x_j(t - \tau_{ij}(t))) - f_j(0)| \Delta t \right.
\left. + \sum_{j=1}^n \int_0^\omega |a_{ij}(t)| |g_j(x_j(t - \tau_{ij}(t))) - g_j(0)| \Delta t \right.
\left. + \sum_{j=1}^n \int_0^\omega |\beta_{ij}(t)| |g_j(x_j(t - \tau_{ij}(t))) - g_j(0)| \Delta t \right] + \bar{a}_i \bar{T}_i \omega + \sum_{k=1}^q |J_{ik}(x_i(t_k)) - J_{ik}(0)| + \sum_{k=1}^q |J_{ik}(0)|
\leq \bar{a}_i \delta \omega |x_i| + \sum_{k=1}^q a_t |x_i| + \sum_{j=1}^n \bar{a}_i \bar{\epsilon}_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \bar{a}_i \bar{\alpha}_{ij} \nu_j \omega |x_j|
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\[ + \sum_{j=1}^{n} a_i \beta_{ij} \nu_j \omega |x_j| + \sum_{k=1}^{q} |J_{ik}(0)| + \overline{a}_i I_i \omega, \quad i = 1, 2, \ldots, n. \]

Integrating both sides of (3.1) from 0 to \( \omega \), we have

\[
\left| \int_0^{\omega} a_i(x_i(t)) x_i(t) \Delta t \right| = \frac{1}{\varrho_i} \left| \int_0^{\omega} a_i(x_i(t)) b_i(x_i(t)) \Delta t \right|
\]

\[
= \frac{1}{\varrho_i} \left| \int_0^{\omega} a_i(x_i(t)) \left[ \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t - \tau_j(t))) + \sum_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) \right. \right.
\]

\[
+ \sum_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t - \tau_{ij}(t))) + I_i(t) \left. \left. \right| \Delta t + \sum_{k=1}^{q} J_{ik}(x_i(t_k)) \right|
\]

\[
\leq \frac{1}{\varrho_i} \left[ \sum_{k=1}^{q} \rho_{ik} |x_i| + \sum_{j=1}^{n} \overline{a}_j \overline{\alpha}_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^{n} \overline{a}_i \overline{\alpha}_{ij} \mu_j \omega |x_j| \right.
\]

\[
+ \sum_{j=1}^{n} \overline{a}_i \beta_{ij} \mu_j \omega |x_j| + \sum_{k=1}^{q} |J_{ik}(0)| + \overline{a}_i I_i \omega , \quad i = 1, 2, \ldots, n.
\]

For any \( t_1, t_2 \in [0, \omega]_\tau \), \( i = 1, 2, \ldots, n \), we have

\[
\int_0^{\omega} a_i(x_i(t)) x_i(t) \Delta t
\]

\[
\leq \int_0^{\omega} a_i(x_i(t)) x_i(t_1) \Delta t + \int_0^{\omega} a_i(x_i(t)) \left( \int_0^{\omega} |x_i^\Delta(s)| \Delta s \right) \Delta t \quad (3.2)
\]

and

\[
\int_0^{\omega} a_i(x_i(t)) x_i(t) \Delta t
\]

\[
\geq \int_0^{\omega} a_i(x_i(t)) x_i(t_2) \Delta t - \int_0^{\omega} a_i(x_i(t)) \left( \int_0^{\omega} |x_i^\Delta(s)| \Delta s \right) \Delta t. \quad (3.3)
\]

Dividing by \( \int_0^{\omega} a_i(x_i(t)) \Delta t \) both sides of (3.2) and (3.3), respectively, we obtain

\[
x_i(t_1) \geq \frac{1}{\int_0^{\omega} a_i(x_i(t)) \Delta t} \int_0^{\omega} a_i(x_i(t)) x_i(t) \Delta t
\]

\[
- \int_0^{\omega} |x_i^\Delta(s)| \Delta s, \quad i = 1, 2, \ldots, n \quad (3.4)
\]

and

\[
x_i(t_2) \leq \frac{1}{\int_0^{\omega} a_i(x_i(t)) \Delta t} \int_0^{\omega} a_i(x_i(t)) x_i(t) \Delta t
\]

\[
+ \int_0^{\omega} |x_i^\Delta(s)| \Delta s, \quad i = 1, 2, \ldots, n. \quad (3.5)
\]
Let $\bar{t}_I, \bar{t}_I \in [0, \omega] \mathbb{T}$ such that $x_i(\bar{t}_I) = \max_{t \in [0, \omega] \mathbb{T}} x_i(t)$, $x_i(\bar{t}_I) = \min_{t \in [0, \omega] \mathbb{T}} x_i(t)$, from (3.4) and (3.5), we have
\[
x_i(\bar{t}_I) \geq \frac{1}{\alpha_i \omega} \left[ \int_0^\omega a_i(x_i(t)) x_i(t) \Delta t \right] - \int_0^\omega |x_t^A(s)| \Delta s
\]
\[
\geq - \frac{1}{\alpha_i \delta_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
= - \left[ \frac{1}{\alpha_i \delta_i \omega} [\sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j|] \right]
\]
\[
\leq - \frac{1}{\alpha_i \delta_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
\geq \frac{1}{\alpha_i \delta_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
+ \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q |J_{ik}(0)| + \bar{\alpha}_i \bar{I}_i \omega \right]
\]
\[
+ \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q |J_{ik}(0)| + \bar{\alpha}_i \bar{I}_i \omega \right]
\]
\[
\geq \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
+ \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q |J_{ik}(0)| + \bar{\alpha}_i \bar{I}_i \omega \right]
\]
\[
\geq \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
+ \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q |J_{ik}(0)| + \bar{\alpha}_i \bar{I}_i \omega \right]
\]
Therefore, we can obtain that
\[
|x_i| \leq \frac{1}{\alpha_i \delta_i \omega} \left[ \sum_{k=1}^q \rho_{ik} |x_i| + \sum_{j=1}^n \alpha_i \beta_{ij} \kappa_j \omega |x_j| + \sum_{j=1}^n \alpha_i \omega |x_j| \right]
\]
\[
+ \frac{1}{\alpha_i \omega} \left[ \sum_{k=1}^q |J_{ik}(0)| + \bar{\alpha}_i \bar{I}_i \omega \right]
\]
\[ \begin{aligned}
\alpha_i \delta_i x_k + \sum_{k=1}^{q} \rho_{ik} |x_i| + \sum_{j=1}^{n} \alpha_j c_{ij} k_j \omega |x_j| + \sum_{j=1}^{n} \alpha_j c_{ij} \nu_j \omega |x_j|
\end{aligned} \]
\[ + \sum_{j=1}^{n} \alpha_i \beta_{ij} \nu_j \omega |x_j| + \sum_{k=1}^{q} |J_k(0)| + \pi_i \omega \]
that is,
\[ \left[ a_i \xi(1 - a_i \delta_i \omega) - (1 + a_i \xi) \sum_{k=1}^{q} \rho_{ik} \right] |x_i| - (1 + a_i \xi) \]
\[ \times \sum_{j=1}^{n} (c_{ij} k_j + \alpha_j c_{ij} \nu_j \omega |x_j| \leq Y_i \]
where \( Y_i = (1 + a_i \xi)(\sum_{k=1}^{q} |J_k(0)| + \pi_i \omega) \), \( i = 1, 2, ..., n \). Denote \( |x| = (|x_1|, |x_2|, \ldots, |x_n|)^T \) and \( Y = (Y_1, Y_2, \ldots, Y_n)^T \), then (3.6) can be written in the matrix form \( E|x| \leq Y \). From the conditions of Theorem 3.3, \( E \) is a nonsingular \( M \)-matrix, so
\[ |x| \leq E^{-1} Y = (D_1, D_2, \ldots, D_n)^T. \]
Let \( D = \sum_{i=1}^{n} D_i + D_0 \), where \( D_0 \) is a positive constant. Take \( \Omega = \{ x \in \mathbb{R}^n ||x||_\infty < D \} \). Obviously \( \Omega \) satisfies the condition (a) of Lemma 3.1. When \( x(t) \in \partial \Omega \cap \text{Ker} L = \partial \Omega \cap \mathbb{R}^n \), \( x \) is a constant vector with \( ||x|| = D \). Furthermore, take \( J : \text{Im} Q \to \text{Ker} L \). Then
\[ \begin{aligned}
JQN(x_i) = -a_i(x_i) \left[ b_i(x_i) - \sum_{j=1}^{n} c_{ij} f_j(x_j) - \sum_{j=1}^{n} d_{ij} g_j(x_j) - n \sum_{j=1}^{q} J_k(0_i) \right] + \frac{1}{\omega} \sum_{k=1}^{q} J_k(x_i), i = 1, 2, \ldots, n.
\end{aligned} \]
We can let \( D \) be greater such that
\[ x^T JQ N x \leq - \sum_{i=1}^{n} \left[ x_i a_i(x_i) \left[ b_i(x_i) - \sum_{j=1}^{n} c_{ij} f_j(x_j) - \sum_{j=1}^{n} d_{ij} g_j(x_j) - n \sum_{j=1}^{q} J_k(0_i) \right] + \frac{1}{\omega} \sum_{j=1}^{q} J_k(x_i) \right] < 0. \]
Hence for any \( x \in \partial \Omega \cap \text{Ker} L \), \( QN x \neq 0 \), namely, the condition (b) in Lemma 3.1 is satisfied.

Furthermore, let \( \psi(\gamma; x) = -\gamma x + (1 - \gamma) Q N x \), then for any \( x \in \partial \Omega \cap \text{Ker} L \), \( x^T \psi(\gamma; x) < 0 \), we get \( \text{deg} \{ JQ N, \Omega \cap \text{Ker} L, 0 \} = \text{deg} \{ -I, \Omega \cap \text{Ker} L, 0 \} \neq 0 \). This shows that condition (c) in Lemma 3.1 is satisfied. Thus, by Lemma 3.1, we conclude that \( L x = N x \) has at least one solution in \( X \), that is, (1.1) has at least one \( \omega \)-periodic solution. This completes the proof. \( \square \)
4 Global exponential stability of periodic solutions

Suppose that \( x^*(t) = (x^*_1(t), x^*_2(t), \ldots, x^*_n(t))^T \) is an \( \omega \)-periodic solution of system (1.1) with the initial conditions \( x^*_i(s) = \varphi^*_i(s), s \in (-\infty, 0], i = 1, 2, \ldots, n \). We will construct some suitable Lyapunov functions to prove the globally exponential stability of this periodic solution.

**Theorem 4.1** Assume that all conditions of Theorem 3.3 are satisfied, suppose further that:

(A6) The impulsive operators \( J_{ik}(x_i(t_k)) = -\gamma_{ik}(x_i(t_k)), 0 < \gamma_{ik} < 2, i = 1, 2, \ldots, n, k \in \mathbb{N} \).

(A7) There exist positive constants \( l_j \) such that \( |a_i(u) - a_i(v)| \leq l_j|u - v|, u, v \in \mathbb{R}, i, j = 1, 2, \ldots, n \).

(A8) There exist \( L_i > 0 \), such that \( |(a_i b_i)(u) - (a_i b_i)(v)| \geq L_i|u - v|, [a_i b_i)(u) - (a_i b_i)(v)](u - v) \geq 0, \) for all \( u, v \in \mathbb{R}, i = 1, 2, \ldots, n \).

(A9) There exists \( \xi \) such that \( \Theta_i = \xi - (1 + \bar{m} \xi)[L_i - \sum_{j=1}^n \xi_i + \bar{m}_i M_j + \bar{\beta}_{ij} M_j - \sum_{j=1}^n |a_j c_{ji} + \xi_j, 0| - \sum_{j=1}^n \bar{\beta}_{ij} v_j c_{ji} + \bar{\beta}_{ij} v_j c_{ji}] \leq 0, i = 1, 2, \ldots, n \). Then the \( \omega \)-periodic solution of system (1.1) is globally exponentially stable.

**Proof.** According to Theorem 3.3, we know that system (1.1) has an \( \omega \)-periodic solution \( x^*(t) = (x^*_1(t), x^*_2(t), \ldots, x^*_n(t))^T \) with the initial value \( \varphi^*(t) = (\varphi^*_1(t), \varphi^*_2(t), \ldots, \varphi^*_n(t))^T \). Let \( x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \) be an arbitrary solution of system (1.1) with initial value \( \varphi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T \). Set \( y(t) = x(t) - x^*(t) \), then from (1.1), we have

\[
y^A(t) = -[a_i(x_i(t))b_i(x_i(t)) - a_i(x^*_i(t))b_i(x^*_i(t))]
+ \left[ a_i(x_i(t)) \sum_{j=1}^n c_{ij}(t)f_j(x_j(t-\tau_{ij}(t))) - a_i(x^*_i(t)) \sum_{j=1}^n c_{ij}(t)f_j(x^*_j(t-\tau_{ij}(t))) \right]
+ \left[ a_i(x_i(t)) \bigg\wedge_{j=1}^n f_j(x_j(t-\tau_{ij}(t))) - a_i(x^*_i(t)) \bigg\wedge_{j=1}^n f_j(x^*_j(t-\tau_{ij}(t))) \right]
+ \left[ a_i(x_i(t)) \bigg\vee_{j=1}^n f_j(x_j(t-\tau_{ij}(t))) - a_i(x^*_i(t)) \bigg\vee_{j=1}^n f_j(x^*_j(t-\tau_{ij}(t))) \right]
\leq -[a_i(x_i(t))b_i(x_i(t)) - a_i(x^*_i(t))b_i(x^*_i(t))]
+ [a_i(x_i(t)) - a_i(x^*_i(t))] \sum_{j=1}^n c_{ij}(t)f_j(x^*_j(t-\tau_{ij}(t)))
+ a_i(x_i(t)) \sum_{j=1}^n c_{ij}(t)[f_j(x_j(t-\tau_{ij}(t))) - f_j(x^*_j(t-\tau_{ij}(t)))]
+ [a_i(x_i(t)) - a_i(x^*_i(t))] \sum_{j=1}^n \alpha_{ij}(t)g_j(x^*_j(t-\tau_{ij}(t)))
\]
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\[ + a_i(x_i(t)) \sum_{j=1}^{n} \alpha_{ij}(t)[g_j(x_j(t - \tau_{ij}(t))) - g_j(x_j^*(t - \tau_{ij}(t)))] \]

\[ + [a_i(x_i(t)) - a_i(x_i^*)] \sum_{j=1}^{n} \beta_{ij}(t)g_j(x_j^*(t - \tau_{ij}(t))] \]

\[ + a_i(x_i(t)) \sum_{j=1}^{n} \beta_{ij}(t)[g_j(x_j(t - \tau_{ij}(t))) - g_j(x_j^*(t - \tau_{ij}(t)))] \].

Hence from the conditions of Theorem 4.1, we can get

\[ D^+ |y_k^t(t)| \leq -L_i |x_i(t) - x_i^*(t)| + \sum_{j=1}^{n} l_i \alpha_{ij} M_f |x_i(t) - x_i^*(t)| \]

\[ + \sum_{j=1}^{n} \overline{\alpha}_{ij} \kappa_j |x_j(t - \tau_{ij}(t)) - x_j^*(t - \tau_{ij}(t))| \]

\[ + \sum_{j=1}^{n} l_i \overline{\alpha}_{ij} \mu_j |x_i(t) - x_i^*(t)| \]

\[ + \sum_{j=1}^{n} \overline{\alpha}_{ij} \overline{\beta}_{ij} \nu_j |x_j(t - \tau_{ij}(t)) - x_j^*(t - \tau_{ij}(t))| \]

\[ + \sum_{j=1}^{n} l_i \overline{\beta}_{ij} M_g |x_i(t) - x_i^*(t)| + \sum_{j=1}^{n} \overline{\alpha}_{ij} \overline{\beta}_{ij} \nu_j |x_j(t - \tau_{ij}(t)) - x_j^*(t - \tau_{ij}(t))| \]

for \( i = 1, 2, \ldots, n \). From (A6), we have that \( |x_i(t_k^+^+) - x_i^*(t_k^+)| = |1 - \gamma_{ik}| |x_i(t_k) - x_i^*(t_k)| \), \( i = 1, 2, \ldots, n, k \in \mathbb{N} \). For any \( \delta \in (-\infty, 0] \), we consider the Lyapunov functional \( V(t) = \sum_{i=1}^{n} e_x(t, \delta) |x_i(t) - x_i^*(t)| \), for \( t > 0, t \neq t_k, k \in \mathbb{N}, i = 1, 2, \ldots, n \), from (4.1), we get

\[ D^+ V^t(t) \leq \sum_{i=1}^{n} \{ e_x(t, \delta) |x_i(t) - x_i^*(t)| + e_x(\sigma(t), \delta)([-L_i \]

\[ + \sum_{j=1}^{n} l_i (\tau_{ij} M_f + \overline{\alpha}_{ij} M_g + \overline{\beta}_{ij} M_g) |x_i(t) - x_i^*(t)| \]

\[ + \sum_{j=1}^{n} (\alpha_{ij} \kappa_j + \overline{\alpha}_{ij} \mu_j + \alpha_{ij} \overline{\beta}_{ij} \nu_j) |x_j(t - \tau_{ij}(t)) - x_j^*(t - \tau_{ij}(t))| \}

\[ \leq \sum_{i=1}^{n} \{ e_x(t, \delta) |x_i(t) - x_i^*(t)| + (1 + \mu(t) e_x(t, \delta))([-L_i \]

\[ + \sum_{j=1}^{n} l_i (\tau_{ij} M_f + \overline{\alpha}_{ij} M_g + \overline{\beta}_{ij} M_g) |x_i(t) - x_i^*(t)| \]
\[ + \sum_{j=1}^{n} \left( \prod_{i,j} \alpha_{ij} \beta_{ij} \nu_{ij} + \prod_{i,j} \beta_{ij} \gamma_{ij} \nu_{ij} \right) + \prod_{i,j} \nu_{ij} \right) \left( x_j(t - \tau_{ij}(t)) - x_i^*(t - \tau_{ij}(t)) \right) \).

By assumption (A9), we conclude that
\[ D^+ V^D(t) \leq \sum_{i=1}^{n} \left\{ \varepsilon + (1 + \mu(t) \varepsilon) \left[ -L_i + \sum_{j=1}^{m} \left( \prod_{i,j} \alpha_{ij} \beta_{ij} \nu_{ij} + \prod_{i,j} \beta_{ij} \gamma_{ij} \nu_{ij} \right) \sum_{n=1}^{m} \left( \prod_{i,j} \nu_{ij} \right) \right] \right\} \times e^{-\theta(t)} |x_i(t) - x_i^*(t)| \leq 0, \]

Also, we have
\[ V(t_k) = \sum_{i=1}^{n} e^{-\theta(t_k)} |x_i(t_k) - x_i^*(t_k)| \leq \sum_{i=1}^{n} e^{-\theta(t_k)} |x_i(t_k)| - |x_i^*(t_k)| = V(t_k), \]

On the other hand, we have
\[ V(0) = \sum_{i=1}^{n} e^{-\theta(0)} |x_i(0) - x_i^*(0)| \leq \sum_{i=1}^{n} e^{-\theta(0)} \sup_{s \in (-\infty, 0]} |x_i(s) - x_i^*(s)| \leq M e^{-\theta(0)} \sup_{s \in (-\infty, 0]} |x_i(s) - x_i^*(s)|, \]

It follows that
\[ \sum_{i=1}^{n} e^{-\theta(t)} |x_i(t) - x_i^*(t)| \leq \sup_{s \in (-\infty, 0]} |x_i(s) - x_i^*(s)|, \]

so it follows that
\[ \sum_{i=1}^{n} |x_i(t) - x_i^*(t)| \leq M e^{-\theta(t)} \sum_{i=1}^{n} \sup_{s \in (-\infty, 0]} |x_i(s) - x_i^*(s)| = M e^{-\theta(t)} \sup_{s \in (-\infty, 0]} |x_i(s) - x_i^*(s)|. \]

From Definition 2.11, the \( \omega \)-periodic solution of system (1.1) is globally exponentially stable. This completes the proof. \( \square \)
5 An example

Example 5.1 Consider the following impulsive fuzzy Cohen-Grossberg neural networks with time-varying delays on time scales:

\[
\begin{cases}
    x_{ij}^+(t) = -a_i(x_i(t)) \left[ b_i(x_i(t)) - \sum_{j=1}^{2} c_{ij}(t)f_j(x_j(t) - \tau_{ij}(t)) \right] \\
    \Delta x_i(t_k) = -0.03x_i(t_k), \quad i = 1, 2,
\end{cases}
\]

(5.1)

where \( T \) is a \( \frac{1}{0} \)-periodic time scale, \([0, \frac{1}{0}] \cap \{ t : t \in \mathbb{Z} \} = \{ t_1 \} \), \( a_1(u) = \frac{1}{3} (\frac{2}{9} \tan |u| + 1) \), \( a_2(u) = \frac{1}{3} (\frac{2}{9} \tan |u| + 1) \), \( b_1(u) = \frac{2}{3} u \), \( b_2(u) = \frac{2}{3} u \), \( c_{11}(t) = 0.005 \sin 8\pi t \), \( c_{12}(t) = 0.005 \sin 8\pi t \), \( f_1(u) = g_1(u) = \frac{1}{7} (|u| + 1 - |u - 1|) \), \( \alpha_{11}(t) = 0.002 \cos 8\pi t \), \( \alpha_{12}(t) = \alpha_{21}(t) = 0 \), \( \alpha_{22}(t) = 0.003 \sin 8\pi t \), \( \beta_{11}(t) = 0.004 \sin 8\pi t \), \( \beta_{12}(t) = 0 \), \( \beta_{22}(t) = 0.005 \sin 8\pi t \), \( \tau_{ij}(t) = \frac{1}{8} \sin 8\pi t \), \( \kappa_j = \nu_j = 1 \).

By calculating, we have \( \omega = \frac{1}{3}, \alpha_1 = \frac{1}{2}, \alpha_2 = \frac{1}{2}, \beta_1 = \frac{1}{2}, \beta_2 = \frac{1}{4}, \tilde{\tau}_{11} = 0.005, \tilde{\tau}_{22} = 0.004, \tilde{\tau}_{12} = \tilde{\tau}_{21} = 0, \tilde{\tau}_{11} = 0.004, \tilde{\tau}_{22} = 0.003, \tilde{\tau}_{12} = \tilde{\tau}_{21} = 0, \tilde{\beta}_{11} = 0.004, \tilde{\beta}_{22} = 0.005, \tilde{\beta}_{12} = \tilde{\beta}_{21} = 0, L_1 = \frac{1}{2}, L_2 = \frac{1}{4} \).

Take \( \xi = 0.1 \), it is easy to compute \( E = \begin{pmatrix} 0.0641 & 0 \\ 0.0468 & 0 \end{pmatrix} \) and \( \Theta_1(\xi, t) = \Theta_1(0.1, t) \approx -0.0719, \Theta_2(\xi, t) = \Theta_2(0.1, t) \approx -0.0248 \). Hence we have \( E = (\epsilon_{ij})_{2 \times 2} \) is a nonsingular \( M \)-matrix, and (49) holds. From Theorem 3.3 and Theorem 4.1, we know that system (5.1) has at least one \( \frac{1}{4} \)-periodic solution, which is globally exponentially stable.

6 Conclusion

In this paper, we have studied the existence, global exponential stability of the periodic solution for impulsive fuzzy Cohen-Grossberg neural networks with time delays on time scales. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and time delays of the system (1.1). The obtained criteria can be applied to design globally exponentially stable periodic fuzzy Cohen-Grossberg neural networks.

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