Sunlet decomposition of tensor product graphs

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Abstract For any integer $r \geq 3$, the sunlet graph of order $2r$ is a graph consisting of a cycle of length $r$ with each vertex of the cycle adjacent to a pendant vertex. In this present article, we shall obtain the necessary and sufficient conditions for decomposing the tensor product of complete graphs and complete graph minus a 1-factor with complete graph $K_m$ (that is, $K_n \times K_m$ and $K_n - I \times K_m$ respectively) into sunlet graphs.

Keywords graph decomposition · tensor product graph · sunlet graph · complete graph · regular graphs

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1 Introduction

In this paper, we consider simple and finite graphs. Let $C_r, K_m, K_m - I$ denotes a cycle of length $r$, the complete graph with $m$ vertices and a complete graph on $m$ vertices minus a 1-factor respectively. Furthermore, we shall make use of the following definitions in the work.

Definition 1.1 Sunlet graph denoted by $L_{2r}$ is a graph that consists of a cycle $C_r$ with pendant vertex (vertex of degree one) attached to each vertex of the cycle $C_r$, as shown in Figure 1.

Definition 1.2 A graph is decomposable into graph $S$ or $S$ decomposes $G$, if $G$ can be written as the union of edge-disjoint copies of $S$, so that all edge in $G$ belongs to only one copy of $S$. Then $G = S \oplus S \oplus \ldots \oplus S$, or simply write it as $S|G$, if $G$ is decomposable into $S$. In addition, $S|G$ also means decomposition of $G$ into copies of $S$.

Definition 1.3 Let $G$ and $S$ be two graphs, the tensor product of $G$ and $S$ written as $G \times S$ has vertex set $V(G) \times V(S)$ in which two vertices $(g_1, s_1)$ and $(g_2, s_2)$ are adjacent whenever $g_1 g_2 \in E(G)$ and $s_1 s_2 \in E(S)$. See Figure 2 for an example.
Note that the tensor product of graphs is distributive over edge disjoint union of graphs, that is, if \( G = S_1 \oplus S_2 \oplus \ldots \oplus S_k \), then \( G \times S = (S_1 \times S) \oplus \ldots \oplus (S_k \times S) \). The graph \( C_r \times K_m \) is an equipartite graph, with the degree of any vertex being \( 2(n-1) \) and the total number of edges is \( rm(m-1) \). The graph \( K_m \times K_n \) is isomorphic to the complete \( n \)-partite graph in which each partite set contains \( m \) vertices. Also, the graph \( (K_m-I) \times K_n \) is an equipartite graph.

The necessary conditions for the existence of a \( r \)-cycle decomposition of a simple graph \( G \) is that \( G \) has at least \( k \) vertices, the degree of every vertex of \( G \) is even and the total number of edges in \( G \) must be divisible by \( k \). These conditions have been shown to be sufficient in the case that \( G \) is complete graph or the complete graph minus a 1-factor, \( K_n-I \) [1, 12].

The study of cycle decomposition of \( K_m \times K_n \) have been investigated. In [5], [6] and [8], the necessary and the sufficient conditions for the existence of \( C_p \) (\( p \), prime) decomposition of \( K_m \times K_n \) was given. It was shown in [7] that Hamilton cycles decomposes the tensor product of two regular complete graphs. Also, in [11], it was proved that the necessary conditions are sufficient for the decomposition of tensor product of complete graphs into a resolvable \( k \)-cycles, when \( k \) is even. In the recent work, the necessary and sufficient conditions for the decomposition of \( K_m \times K_n \) into cycles of length 4 and 10 was given [see [9], [10]].

In [[2], [3]], the necessary and sufficient conditions for the decomposition of certain equipartite graphs into sunlet graphs and sunlet graphs of order \( 2p \) was given. Decomposition of product graphs into sunlet graphs of order eight was given, in [13].

In this paper, we consider the decomposition of \( C_r \times K_n, K_m \times K_n \) (\( m \), \( n \) odd) and \( K_m-I \times K_n \) (\( m \), even and \( n \) odd) into \( L_{2r} \), where \( r \) is a positive integer. Among others, we prove the following results:

1. For positive odd integers \( n \) and \( m \), the graph \( K_m \times K_n \) is decomposable into sunlet graphs \( L_{2r} \) if and only if \( mn(m-1)(n-1) \equiv 0 \pmod{2r} \).
2. For positive even integer \( m \) and odd integer \( n \), the graph \( K_m-I \times K_n \) is decomposable into sunlet graphs \( L_{2r} \) if and only if \( mn(m-2)(n-1) \equiv 0 \pmod{2r} \).

2 Main results

We begin this section with the following Lemma.

Lemma 2.1 For \( r \geq 3 \), \( L_{2r}|C_r \times C_3 \).

Proof. Each vertex \( x_i \) in \( C_r \) is replaced by 3 independent vertices \( x_{i,1}, x_{i,2}, x_{i,3} \) and each edge \( x_i x_j, 1 \leq i, j \leq r \) is replaced by \( K_{3,3} - I \) in \( C_r \times C_3 \) which follows from the definition of tensor product graph. Next, we split the problem into the following 3 cases.
Case 1: when \( r \equiv 0(\text{mod } 3) \).
First construct the base cycles \( C_r^1, C_r^2, C_r^3 \) as follows:

\[
C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, \ldots, x_{r,s+3}
\]

where \( s = 0, 1, 2 \). Also, note that the sum on the second suffix is calculated modulo 3. Therefore, we have 3 cycles \( C_r^1, C_r^2, C_r^3 \). Define a mapping \( \rho \) by \( \rho(x_{i,1}) = x_{i+1,3}, \rho(x_{i,2}) = x_{i+1,1}, \rho(x_{i,3}) = x_{i+1,2} \), where \( 1 \leq i \leq r-1 \). Note that the sum on the first suffix is calculated modulo \( r \). The vertices \( \rho(x_{i,1}), \rho(x_{i,2}), \) and \( \rho(x_{i,3}) \) are pendant vertices. For \( 1 \leq t \leq 3 \), attach each pendant vertex \( \rho(x_{i,t}) \) to each vertex \( x_{i,t} \) in \( C_r^1, C_r^2, C_r^3 \). The cycles \( C_r^1, C_r^2, C_r^3 \) together with pendant vertices \( \rho(x_{i,t}) \) attached gives sunlet graphs of order \( 2r \).

Case 2: when \( r \equiv 1(\text{mod } 3) \).
First, we construct three base cycles \( C_r^1, C_r^2, C_r^3 \) as follows:

\[
C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, \ldots, x_{r-1,s+3}, x_{r,s+2}
\]

where \( s = 0, 1, 2 \) and the sum on the second suffix is calculated modulo 3. Hence, we have three cycles \( C_r^1, C_r^2, C_r^3 \). Next, define mapping \( \rho \) by \( \rho(x_{i,1}) = x_{i+1,3}, \rho(x_{i,2}) = x_{i+1,1}, \rho(x_{i,3}) = x_{i+1,2} \), where \( 1 \leq i \leq r-1 \). Note that the sum on the first suffix is calculated modulo \( r \).

Also, \( \rho(x_{r,1}) = x_{1,2}, \rho(x_{r,2}) = x_{1,3}, \rho(x_{r,3}) = x_{1,1} \). The vertices \( \rho(x_{i,1}), \rho(x_{i,2}), \rho(x_{i,3}) \) are pendant vertices. For \( t = 1, 2, 3 \), attach each pendant vertex \( \rho(x_{i,t}) \) to each vertex \( x_{i,t} \) in the three base cycles. The base cycles with pendant vertices attached gives the sunlet graph \( L_{2r} \).

Case 3: when \( r \equiv 2(\text{mod } 3) \).
Construct three base cycles \( C_r^1, C_r^2, C_r^3 \) as follows:

\[
C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, \ldots, x_{r-1,s+1}, x_{r,s+2}
\]

where \( s = 0, 1, 2 \) and the sum on the second suffix is calculated modulo 3. Therefore, we have three cycles \( C_r^1, C_r^2, C_r^3 \). Applying mapping \( \rho \) define in case 2 above to get the pendant vertices \( \rho(x_{i,t}), t = 1, 2, 3 \). Attach each pendant vertex \( \rho(x_{i,t}) \) to each vertex \( x_{i,t} \) in the cycles \( C_r^1, C_r^2, C_r^3 \) to get three sunlet graphs with \( 2r \) vertices. Hence, \( L_{2r} \times C_r \times C_3 \).

\[ \square \]

**Remark 2.1** For \( n \geq 3 \), \( L_{2n} \times C_n \times C_n \). This follows from the previous lemma.

**Lemma 2.2** For \( r \geq 3 \), \( L_{2r} \times L_{2r} \times C_r \).

*Proof.* Let the vertices of the sunlet graphs on \( 2r \) vertices be \( \{1, 2, \ldots, r, 1^*, 2^*, \ldots, r^*\} \) where \( i^* \) are pendant vertices adjacent to vertex \( i \) of the cycle contained in the sunlet graph \( L_{2r} \). Then \( L_{2r} \times C_r \) have the set of vertices \( \{(p, c), (p^*, c) | p, p^* = 1, 2, \ldots, r \text{ and } c = 1, 2, 3\} \). Next, form the sunlet graphs \( L_{2r}, \ldots, L_{2r} \) from \( L_{2r} \times C_r \) as follows:

If \( r \equiv 0(\text{mod } 3) \), \( C_r \) is of the form:

\[
C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, \ldots, x_{r-1,s+2}, x_{r,s+3}
\]
and
\[ *C^s_{r+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+3}, x_{r,s+2}. \]
If \( r \equiv 1 \pmod{3} \), \( C_r \) is of the form:
\[ C^s_{r+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, ..., x_{r-1,s+3}, x_{r,s+2} \]
and
\[ *C^s_{r+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+2}, x_{r,s+3}. \]
If \( r \equiv 2 \pmod{3} \), cycle \( C_r \) is of the form:
\[ C^s_{r+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, x_{4,s+1}, ..., x_{r-1,s+1}, x_{r,s+2} \]
and
\[ *C^s_{r+1} = x_{1,s+1}, x_{2,s+3}, x_{3,s+2}, x_{4,s+1}, ..., x_{r-1,s+1}, x_{r,s+3}. \]
Where \( s = 0, 1, 2 \) and the sum on the second suffix is calculated modulo 3. 
Now, we define the pendant vertices as the mapping \( \rho \) as follows: For each vertex \( x_{i,c} \) in \( C^s_{r+1} \), we have
\[ \rho(x_{i,1}) = x_{i+r,2}, \rho(x_{i,2}) = x_{i+r,3}, \rho(x_{i,3}) = x_{i+r,1}. \]
Also, for each vertex \( x_{i,c} \) in \( *C^s_{r+1} \), we have
\[ \rho(x_{i,1}) = x_{i+r,3}, \rho(x_{i,2}) = x_{i+r,1}, \rho(x_{i,3}) = x_{i+r,2}. \]
The vertices \( \rho(x_{i,c}), i = 1, 2, ..., r \) and \( c = 1, 2, 3 \) are the pendant vertices. Join each pendant vertex \( \rho(x_{i,c}) \) to each vertex \( x_{i,c} \) in the cycles \( C^s_{r+1} \) and \( *C^s_{r+1} \) to give the sunlet graphs \( L_{2r} \), which is the desired result. Therefore \( L_{2r} \times C_3 = L^1_{2r} \oplus L^2_{2r} \oplus ... \oplus L^6_{2r} \).

Next, we give the following existing result on cycle decomposition.

**Theorem 2.3** [1] For any odd integer \( t \geq 3 \), if \( n \equiv 1 \pmod{2t} \) or \( t \pmod{2t} \), then \( C_t \mid K_n \).

Now, we consider the sunlet decomposition of tensor product of sunlet graph with complete graph.

**Theorem 2.4** The graph \( L_{2r} \mid L_{2r} \times K_n \) for any positive integer \( n \equiv 1 \pmod{3} \) or \( 3 \pmod{6} \).

**Proof.** Note that for any graphs \( G, H \), if \( H \mid G \), then \( H \times T \mid G \times T \). Therefore by Theorem 2.3, \( C_3 \mid K_n \), then we have \( L_{2r} \times (C_3 \oplus C_3 \oplus ... \oplus C_3) \cong (L_{2r} \times C_3) \oplus (L_{2r} \times C_3) \oplus ... \oplus (L_{2r} \times C_3) \). Applying Lemma 2.2 give the desired result.

Next, we give the following results on the sunlet decomposition of cycle and complete graphs.

**Theorem 2.5** The graph \( C_r \times K_n \) is decomposable into sunlet graph \( L_{2r} \) for any positive odd integer \( n \equiv 1 \pmod{3} \) or \( 3 \pmod{6} \).
Proof. The graph \( K_n \) is decomposable into cycles \( C_3 \) by Theorem 2.3. Therefore, combining Theorem 2.3 and Lemma 2.1 gives the result.

\( \square \)

**Theorem 2.6** [4] If \( m \equiv 5(\text{mod } 6) \), then \( K_m \) can be decomposed into \( \frac{m(m-1)-20}{6} \) 3-cycles and 5-cycles.

**Lemma 2.7** For positive integers \( r \) and \( n \) such that \( r > n \geq 3 \) and \( r \equiv 0 \text{ or } 2(\text{mod } n) \), the graph \( L_{2r} \) decomposes \( C_r \times C_n \)

Proof. We split the problem into the following two cases.

Case 1: when \( r \equiv 0(\text{mod } n) \).

First, construct the base cycles \( C_r^1, C_r^2, \ldots, C_r^n \) by applying Lemma 2.1 (case 1) as follows:

\[ C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, \ldots, x_{r,s+r} \]

where \( s = 0, 1, 2, \ldots, n-1 \). Also, note that the sum on the second suffix is calculated modulo \( n \). Therefore, we have \( n \) cycles \( C_r^1, C_r^2, \ldots, C_r^n \).

Define the mapping \( \rho \) by \( \rho(x_{i,j}) = x_{i+1,j-1}, 1 \leq i \leq r \) and \( 1 \leq j \leq n \). Note that the sum on the first and second suffixes are calculated modulo \( r \) and \( n \) respectively. The vertices \( \rho(x_{i,j}) \) are pendant vertices. Attach each pendant vertex \( \rho(x_{i,j}) \) to vertex \( x_{i,j} \) in the base cycles to give the sunlet graphs \( L_{2r} \).

Case 2: when \( r \equiv 2(\text{mod } n) \).

Apply Lemma 2.1 (case 3) to construct \( n \) base cycles:

\[ C_r^{s+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, \ldots, x_{r-2,s+n}, x_{r-1,s+1}, x_{r,s+2} \]

where \( s = 0, 1, 2, \ldots, n-1 \). The sum on the second suffix is calculated modulo \( n \).

Next, define mapping \( \rho \) on vertices \( x_{i,j}, 1 \leq i \leq r, 1 \leq j \leq n \) to get pendant vertices as follows:

\[ \rho(x_{i,j}) = x_{i+1,j-1} \text{ for } 1 \leq i \leq r - 1 \text{ and for } i = r, \text{ define } \rho(x_{i,j}) = x_{1,j+1}. \]

Note that the sum on the first and second suffixes are calculated modulo \( r \) and \( n \) respectively. Attach each pendant vertex \( \rho(x_{i,j}) \) to each vertex in the base cycles to give the sunlet graph \( L_{2r} \). Hence the proof.

\( \square \)

**Theorem 2.8** Let \( n \equiv 5(\text{mod } 6) \), the graph \( C_3 \times K_n \) is decomposable into sunlet graphs \( L_{2r} \), for \( r = 3, 5 \) and \( n \).

Proof. We split the problem into the following two cases:

Case 1: when \( r = 3, 5 \).

Recall by Theorem 2.6 that \( K_n \) is decomposable into copies of cycles \( C_3 \) and \( C_5 \). Therefore, we have \( C_3 \times (C_3 \oplus C_3 \oplus \ldots \oplus C_5) = (C_3 \times C_3) \oplus \ldots \oplus (C_3 \times C_3) \oplus (C_3 \times C_5) \oplus \ldots \oplus (C_3 \times C_5) \). Now, apply Lemma 2.1 to \( C_3 \times C_3 \) and Lemma 2.7 to \( C_3 \times C_5 \), this gives the desired result.

Case 2: when \( r = n \).

The graph \( K_n \) is decomposable into Hamilton cycles \( C_n \). Therefore we have
$C_3 \times K_n = C_3 \times (C_n \oplus C_n \oplus ... \oplus C_n)$. By Lemma 2.1, we have $L_{2r}|C_3 \times K_n$. Hence the proof.

\[ \square \]

**Corollary 2.9** Let $n \equiv 5(\text{mod } 6)$ and $r \equiv 0 \text{ or } 2(\text{mod } 5)$, then the graph $C_r \times K_n$ is decomposable into sunlet graphs $L_{2r}$.

**Proof.** Recall that $K_n$ is decomposable into copies of cycles $C_3$ and $C_5$ by Theorem 2.6. Apply Lemma 2.1 to each $C_r \times C_3$ and Lemma 2.7 to each $C_r \times C_5$ gives the result.

\[ \square \]

**Lemma 2.10** Let $m$ be a positive even integer and $n$ a positive integer, the graph $C_m \times C_n$ is decomposable into sunlet graphs $L_{2m}$.

**Proof.** First, we construct the base cycles $C_m$ from $C_m \times C_n$ as follows:

$$C_m^{n+1} = x_{1,s+1}, x_{2,s+2}, x_{3,s+3}, ..., x_{\frac{m}{2},s+\frac{m}{2}}, x_{\frac{m}{2}+1,s+\frac{m}{2}+1}, x_{\frac{m}{2}+2,s+\frac{m}{2}+2},$$

where $t = \frac{m}{2} + 2 \leq n$ and $s = 0, 1, 2, ..., n - 1$. The sum on the second suffix is calculated modulo $n$. Hence, we have $n$ $m$-cycles.

Next, we define the pendant vertices as the mapping $\rho$ as follows:

$$\rho(x_{i,j}) = x_{i+1,j-1}, \quad \text{for } 1 \leq i \leq \frac{m}{2}$$

$$\rho(x_{i,j}) = x_{i+1,j+1}, \quad \text{for } \frac{m}{2} + 1 \leq i \leq \frac{m}{2}.$$  

Note that the sum on the first and second suffixes are calculated modulo $m$ and $n$ respectively. Attach each pendant vertex $\rho(x_{i,j})$ to vertex $x_{i,j}$ to give the sunlet graph $L_{2m}$.

\[ \square \]

**Theorem 2.11** For any odd integer $n$ and positive integer $m$, the graph $C_m \times K_n$ is decomposable into sunlet graphs $L_{2r}$ if and only if $mn(n-1) \equiv 0(\text{mod } 2r)$.

**Proof.** First, we prove the necessary condition. Assume that the graph $C_m \times K_n$ is decomposable into sunlet graph $L_{2r}$. This implies that the number of edges in $C_m \times K_n$ is divisible by $2r$, i.e., $mn(n-1) \equiv 0(\text{mod } 2r)$.

Next, we prove the sufficient condition. Assume that $mn(n-1) \equiv 0(\text{mod } 2r)$. Then we split the problem into the following 2 cases:

Case 1: $r|m$; We only prove the case for when $r = m$. By Theorem 2.3, $C_3 \mid K_n$, for $n \equiv 1 \text{ or } 3(\text{mod } 6)$. Next, apply Lemma 2.1 gives the desired result. Also, whenever $n \equiv 5(\text{mod } 6)$ and $r \equiv 0 \text{ or } 2(\text{mod } 5)$. Combining Theorem 2.6, Lemma 2.1 and Theorem 2.7 gives the result.

Case 2: $r | n(n-1)$ The graph $K_n$ is decomposable into cycle $C_r$ by Theorem 2.3. Therefore, we have $C_m \times K_n = (C_m \times C_r) \oplus ... \oplus (C_m \times C_r)$.  

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We split the problem into the following two subcases:
Subcase 1: when \( r \) is even and \( m \) a positive integer. Note that \( C_m \times C_r \equiv C_r \times C_m \). Applying Lemma 2.10 gives the result.
Subcase 2: when \( r \) is odd and \( m \) a positive integer. If \( m = 3 \), apply Lemma 2.1 to give the result. Also, if \( r > m \) and \( r \equiv 0 \mod m \), apply Lemma 2.7 to have \( L_{2r} \). Hence the proof.

\[ \Box \]

Next, we give the previous results on cycle decomposition of complete graphs.

**Theorem 2.12** [12] Let \( n \) be an even integer and \( m \) be an odd integer with \( 3 \leq m \leq n \). The graph \( K_n - I \) can be decomposed into cycles of length \( m \) whenever \( m \) divides the number of edges in \( K_n - I \).

**Theorem 2.13** [1] For positive even integers \( m \) and \( n \) with \( 4 \leq m \leq n \), the graph \( K_n - I \) can be decomposed into cycles of length \( m \) if and only if the number of edges in \( K_n - I \) is a multiple of \( m \).

**Theorem 2.14** [12] Let \( n \) be an odd integer and \( m \) be an even integer with \( 3 \leq m \leq n \). The graph \( K_n \) can be decomposed into cycles of length \( m \) whenever \( m \) divides the number of edges in \( K_n \).

The next theorem gives the necessary and sufficient conditions for sunlet decomposition of tensor product of complete graphs.

**Theorem 2.15** For positive odd integers \( n \) and \( m \), the graph \( K_m \times K_n \) is decomposable into sunlet graphs \( L_{2r} \) if and only if \( mn(m-1)(n-1) \equiv 0 \mod 2r \).

*Proof.* The necessary condition is obvious, we will only prove the sufficient condition. Next, we split the problem into the following two cases:
Case 1: \( r|m(m-1) \)
By Theorem 2.3 and 2.14, \( C_r|K_m \) whenever \( r \) divides the number of edges in \( K_m \). Therefore, we have
\[
K_m \times K_n = (C_r \times K_n) \oplus (C_r \times K_n) \oplus \ldots \oplus (C_r \times K_n).
\]
Each \( C_r \times K_n \) decomposes into sunlet graphs \( L_{2r} \) by Theorem 2.11. Hence the result.
Case 2: \( r|m(n-1) \)
Using the same argument in case 1 above and interchanging \( m \) and \( n \) gives the desired result.

\[ \Box \]

**Theorem 2.16** For positive even integer \( m \) and odd integer \( n \), the graph \( K_m - I \times K_n \) is decomposable into sunlet graphs \( L_{2r} \) if and only if \( mn(m-2)(n-1) \equiv 0 \mod 2r \).
Proof. The necessary condition is obvious since the number of edges in $K_m - I \times K_n$ is \( \frac{mn(m-2)(n-1)}{2} \). We only prove the sufficient condition. Next, we split the problem into the following two cases:

Case 1: \( r \mid m(m-2) \)

By Theorems 2.12 and 2.13, $K_m - I$ is decomposable into cycles $C_r$ whenever $r$ divides the number of edges in $K_m - I$. Therefore, we have

$$K_m - I \times K_n = (C_r \oplus C_r \oplus \ldots \oplus C_r) \times K_n \cong (C_r \times K_n) \oplus \ldots \oplus (C_r \times K_n).$$

Applying Theorem 2.11 to each $C_r \times K_n$ gives the result.

Case 2: \( r \mid n(n-1) \)

By Theorems 2.3, 2.6 and 2.14, $K_n$ decomposes into cycle $C_r$. Also, by Theorems 2.13 and 2.12, $C_t | K_m - I$, for any positive integer $t \leq m$. Therefore, we have $C_t \times C_r$. Note that $C_t \times C_r \cong C_r \times C_t$. Apply Lemma 2.10 to $C_r \times C_t$ to have $L_{2r}$ decomposition, for any positive even integer $r$. If $r$ is odd, apply Lemma 2.1 and Lemma 2.7 to have the result. Hence $L_{2r}$ decomposes $K_m - I \times K_n$.

\[ \square \]

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References

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Fig. 1 Sunlet graph on 10 vertices

Fig. 2 $C_3 \times C_5$