Polinomul caracteristic vectorial al matricei A este

$$C_A(\Lambda) = \Lambda^p - \Sigma_1 \Lambda^{p-1} + \dots + (-1)^{p-1} \Sigma_{p-1} \Lambda + (-1)^p \Sigma_p, \tag{12}$$

unde Σ_i sînt elemente ale algebrei cu baza $e_1, e_2, \dots, e_q$, funcţii simetrice elementare de valorile proprii ale matricei A. Evident, $\Sigma_p = \det A$, $\Sigma_1 = S(A) = S(A_1)e_1 + \dots + S(A_q)e_q$, $S(A_j)$ fiind urma matricei A_j.

Pentru o matrice tridimensională pătratică A este adevărată o teoremă analogă cu aceea a lui Cayley-Hamilton din cazul bidimensional şi anume:

$$C_A(A) = 0.$$

Demonstraţia se face prin calcul, aplicînd teorema lui Cayley-Hamilton la feţele $A_j (1 \leqslant j \leqslant q)$ şi reprezentînd matricea A prin (1).

Se verifică de asemenea că, dacă $\Lambda_h (1 \leqslant h \leqslant p)$ sînt valorile proprii ale matricei A, atunci matricea A^r va avea valorile proprii Λ_h^r, ţinînd seamă de tabla de înmulţire din algebra cu baza $(e_1, e_2, \dots, e_q)$.

Se pot introduce de asemenea factorii invarianţi şi divizorii elementari, deci se poate ajunge la forma normală Jordan pentru o matrice tridimensională pătratică.

Folosind acelaşi simbolism, forma Jordan pentru matricea A va f

$$\mathcal{J} = \mathcal{J}_1 e_1 + \dots + \mathcal{J}_q e_q \tag{13}$$

unde $\mathcal{J}_j$ este forma Jordan a feţei $A_j (1 \leqslant j \leqslant q)$.

Dacă considerăm transformările elementare pentru matricele tridimensionale pătratice, date cu ajutorul unor matrice de forma

$$T = T_1 e_1 + T_2 e_2 + \dots + T_q e_q, \tag{14}$$

$T_j (1 \leqslant j \leqslant q)$ fiind matricele transformărilor elementare pentru matricele bidimensionale $A_j (1 \leqslant j \leqslant q)$ de tip $p \times p$, atunci matricele A şi $\mathcal{J}$ sînt legate prin relaţia

$$A = P \mathcal{J} P^{-1}, \tag{15}$$

unde P este un produs de matrice al transformărilor elementare.

BIBLIOGRAFIE

1. Соколов Н. Н. — *Пространственные матрицы и их приложения*. Гос. изд. физ.-мат. лит. Москва, 1960.
2. Соколов Н. Н. — *Об операциях над многомерными матрицами*. ДАН СССР, 163 (6), 1965, стр. 1322 — 1325.

О ТРЕХМЕРНЫХ МАТРИЦАХ

Резюме

Приводится представление трехмерных символов, имеющие простой закон умножения. Это позволяет расширение одных результатов из теории двухмерных матриц и упрощает формулы и исчисления с трехмерными матрицами.

COMPARISON PRINCIPLE AND LYAPUNOV'S SECOND METHOD

BY

V. LAKSHMIKANTHAM and S. LEELA

1. It is widely known that the comparison principle has been one of the important approaches to Lyapunov's second method, which provides a convenient means of discussing the behaviour of solutions of a given differential system in terms of the properties of the Lyapunov function and the solutions of the scalar differential equation. The advantage of the comparison technique is in obtaining stability criteria under rather less restrictive conditions than those demanded in the classical theorems of Lyapunov's second method. In particular, it often helps in deducing asymptotic stability without the stronger requirement that the derivative of the Lyapunov function be negative definite, which, in many practical applications, is difficult to achieve. Moreover, the comparison method is readily applicable to the questions of conditional stability and boundedness. [4].

Recently, Brauer [1] has proved a result which apparently suggests that, in the case of stability, theorems stated in terms of differential inequality and the behaviour of the solutions of the corresponding scalar differential equation are not really more general than the classical theorems.

The purpose of the present paper is to substantiate the generality of the comparison technique. It is shown that whenever a Lyapunov function exists satisfying the conditions of the classical stability theorems, it is possible to find another Lyapunov function which fulfills the conditios of the stability theorems in terms of differential inequalities. On the other hand, starting with the stability theorems in terms of differential inequalities, we can arrive at the classical stability theorems, provided some additional conditions are assumed, namely, in the case of stability (simple or uniform) and equiasymptotic stability the assumption (a_2) [See (Theorem 2] and in the case of uniform asymptotic stability assumptions (a_2) and (3.9) [See Theorem 7].

2. Let I denote the interval $0 \leqslant t < \infty$, R^n the euclidean n-space and R^+ the half-line $0 \leqslant r < \infty$. Let $\| x \|$ be any convenient norm of $x \in R^n$. We shall consider the differential system

$$\frac{dx}{dt} = f(t, x),$$

where the function f is a continuous mapping of $I \times R^n$ into R^n and $f(t, 0) = 0$, $t \in I$.

The following definitions and notations are useful in our subsequent discussion.

(i) A function $\varphi(r)$ belongs to class K ($\varphi \in K$), if it is defined and continuous for $r \in R^+$, strictly monotone increasing in r, $\varphi(0) = 0$ and $\varphi(r) \to \infty$ as $r \to \infty$.

(ii) A function $\lambda(t, r)$ is said to belong to class KK ($\lambda \in KK$), if it is defined and continuous for $(t, r) \in I \times R^+$, belongs to class K with respect to r, for each fixed t and is positive, monotone increasing with respect to t and $\lambda(t, r) \to \infty$ as $t \to \infty$, for each fixed $r > 0$.

(iii) A function $\sigma(t)$ belongs to class L ($\sigma \in L$), if it is defined and continuous for $t \in I$, monotone decreasing in t and $\sigma(t) \to 0$ as $t \to \infty$.

(iv) A function $h(t, x) \geqslant 0$ is said to belong to class E ($h \in E$) if it is defined and continuous on $I \times R^n$, $h(t, 0) = 0$, $t \in I$, and has continuous partial derivatives with respect to t and the components of x.

(v) For any function $h(t, x) \in E$, the total derivative $h'(t, x)$, in virtue of the differential system (2.1), is given by

$$h'(t, x) = \frac{\partial h(t, x)}{\partial t} + \frac{\partial h(t, x)}{\partial x} \cdot f(t, x),$$

where $\cdot$ denotes the usual scalar product of vectors.

(vi) A function $h(t, x)$, with $h(t, 0) \equiv 0$, is said to be positive definite (negative definite) if there exists a function $\varphi(r) \in K$, such that the relation

$$h(t, x) \geqslant \varphi(\| x \|), \quad (h(t, x) \leqslant -\varphi(\| x \|))$$

is satisfied for $(t, x) \in I \times R^n$.

(vii) A function $h(t, x) \geqslant 0$ is said to be decrescent if a function $\psi(r) \in K$ exists such that

$$h(t, x) \leqslant \psi(\| x \|), \quad (t, x) \in I \times R^n.$$

(viii) A function $h(t, x)$ with $h(t, 0) \equiv 0$ is said to be strongly positive definite, if there exists a function $\lambda(t, r) \in KK$ such that

$$h(t, x) \geqslant \lambda(t, \| x \|), \quad (t, x) \in I \times R^n.$$

3. We shall say that a function $g(t, r)$ satisfies the hypothesis H_1, if it is defined and continuous, together with the first partial derivative $(\partial/\partial r)[g(t, r)]$, for $(t, r) \in I \times R^+$, $g(t, 0) = 0$, $t \in I$ and the solution $r(t, 0, r_0)$ of the scalar differential equation

$$\frac{dr}{dt} = g(t, r) \tag{3.1}$$

satisfies the estimate

$$\beta_1(r_0) \leqslant r(t, 0, r_0) \leqslant \beta_2(r_0), \quad t \in I, \quad \beta_1, \beta_2 \in K. \tag{3.2}$$

The function $g(t, r)$ is said to fulfill the hypothesis H_2, if the hypothesis H_1 holds with the replacement of (3.2) by the one-sided estimate

$$r(t, 0, r_0) \leqslant \beta_2(r_0)\,\sigma(t), \quad t \in I, \quad \beta_2 \in K, \quad \sigma \in L. \tag{3.3}$$

From the assumption that $(\partial/\partial r)[g(t, r)]$ exists and is continuous for $(t, r) \in I \times R^+$, it follows [2] that the solutions $r(t, t_0, r_0)$ of (3.1) through every point (t_0, r_0) are unique and are differentiable functions with respect to the initial values. Further $(\partial/\partial r_0)\, r(t, t_0, r_0)$ and $(\partial/\partial t_0)\, r(t, t_0, r_0)$ are solutions of the linear equation

$$y' = \frac{\partial g(t, r(t, t_0, r_0))}{\partial r} y$$

such that

$$\frac{\partial r(t_0, t_0, r_0)}{\partial r_0} = 1 \quad \text{and} \quad \frac{\partial r(t_0, t_0, r_0)}{\partial t_0} = -g(t_0, r_0)$$

respectively. Also

$$\frac{\partial r(t, t_0, r_0)}{\partial t_0} = -\frac{\partial r(t, t_0, r_0)}{\partial r_0} g(t_0, r_0). \tag{3.3'}$$

Concerning the stability criteria, we have the following two theorems.

Theorem 1. *Let there exist a positive definite function* $V(t, x) \in E$ *such that* $V'(t, x) \leqslant 0$. *Then, there exists a positive definite function* $U(t, x) \in E$, *which satisfies the differential inequality*

$$U'(t, x) \leqslant g(t, U(t, x)), \quad (t, x) \in I \times R^n, \tag{3.4}$$

where g *is any given function satisfying the hypothesis* H_1. *If, in addition,* $V(t, x)$ *is decrescent,* $U(t, x)$ *is also decrescent.*

Proof. Let the assumption of the theorem hold. Let $g(t, r)$ be any given function fulfilling the hypothesis H_1. Now define

$$U(t, x) = r(t, 0, V(t, x)). \tag{3.5}$$

In view of the hypothesis H_1, the solutions $r(t, t_0, r_0)$ of (3.1), through every point (t_0, r_0), are differentiable functions with respect to the initial values. Moreover (3.1) has identically zero solution. This fact, together with $V(t, x) \in E$ implies that $U(t, x) \in E$. Thus,

$$U'(t, x) = \frac{\partial U}{\partial t}(t, x) + \frac{\partial U}{\partial x}(t, x) . f(t, x) =$$

$$= r'(t, 0, V(t, x)) + \frac{\partial r'}{\partial r_0}(t, 0, V(t, x)) \left[\frac{\partial V}{\partial t}(t, x) + \frac{\partial V}{\partial x}(t, x) . f(t, x)\right] \leqslant$$

$$\leqslant g(t, U(t, x)) + \frac{\partial r}{\partial r_0}(t, 0, V(t, x)) . V'(t,x).$$

The fact that $V'(t,x) \leqslant 0$ and $\frac{\partial r}{\partial r_0}(t,0,V(t,x))$ is positive proves (3.4).

Usind the lower estimate in (3.2), the positive definiteness of $V(t,x)$ and the definiton of $U(t,x)$, we get

$$U(t, x) = r(t, 0, V(t, x)) \geqslant \beta_1(V(t, x)) \geqslant \beta_1(\varphi(\| x \|)) \equiv b(\| x \|),$$

Clearly $b \in K$, since both β_1 and φ belong to class K.

Further, if $V(t, x)$ is decrescent, we can use the upper estimate in (3.2) and the definition (3.5) to obtain

$$U(t, x) = r(t, 0, V(t, x)) \leqslant \beta_2(V(t, x)) \leqslant \beta_2(\psi(\| x \|)) \equiv a(\| x \|).$$

where β_2, ψ and hence a, all belong to class K, thus proving that $U(t, x)$ is decrescent. The proof of the theorem is complete.

Theorem 2. *Suppose that*

a_1) *there exists a positive definite function $U(t, x) \in E$ satisfying the differential inequality* (3.4), *for any given function g which fulfills the hypothesis* H_1;

a_2) *the solutions $r(t, t_0, r_0)$ of* (3.1) *through every point (t_0, r_0) can be continued backwards up to $t = 0$. Then, there exists a positive definite function $V(t, x) \in E$ such that $V'(t x) \leqslant 0$. Further, if $U(t, x)$ is decrescent, $V(t, x)$ is also decrescent.*

Proof. Let the assumptions (a_1) (a_2) hold. In view of the assumption (a_2), the solution of (3.1) through the point $(t, U(t, x))$ can be continued back up to $t = 0$. Define

$$V(t, x) = r(0, t, U(t, x)). \tag{3.6}$$

As the function g obeys the hypothesis H_1, the solution $r(t, t_0, r_0)$ is a differentiable function with respect to its arguments and (3.1) has identically zero solution. Hence $V(t, x) \in E$ and

$$V'(t, x) = \frac{\partial V}{\partial t}(t, x) + \frac{\partial V}{\partial x}(t, x) \cdot f(t, x) =$$

$$= r_{t_0}(0, t, U(t, x)) + r_{r_0}(0, t, U(t, x)) \left| \frac{\partial U}{\partial t}(t, x) + \frac{\partial U}{\partial x}(t, x) \cdot f(t, x) \right| \leqslant$$

$$\leqslant r_{t_0}(0, t, U(t, x)) + r_{r_0}(0, t, U(t, x)) \cdot g(t, U(t, x)),$$

using (3.4). Now $V'(t,x) \leqslant 0$ follows from (3.3*).

Since the equation (3.1) has a unique solution through every point, the definition (3.6) implies that

$$U(t, x) = r(t, 0, V(t, x)),$$

which, by using the upper estimate in (3.2), yields

$$U(t, x) = r(t, 0, V(t, x)) \leqslant \beta_2(V(t, x)), \quad \beta_2 \in K.$$

Thus

$$V(t, x) \geqslant \beta_2^{-1}(U(t, x)), \quad \beta_2^{-1} \in K,$$

and since $U(t, x)$ is positive definite, we get

$$V(t, x) \leqslant \beta_2^{-1}(\varphi(\| x \|)) \equiv b(\| x \|), \quad \varphi, b \in K.$$

Similarly, if $U(t, x)$ is decrescent, using the lower estimate in (3.2), we have

$$V(t, x) \leqslant \beta_1^{-1}(U(t, x)) \leqslant \beta_1^{-1}(\psi(\| x \|)) \equiv a(\| x \|), \quad a \in K,$$

which proves the decrescent nature of $V(t, x)$ and the theorem is established.

Remark. Note that the stability of the null solution of (3.1) is expressed as an inequality in terms of monotone functions. The lower estimate in (3.2) is compatible with the stability of the solution $r = 0$ of (3.1). Theorem 2 is a refinement of Brauer's result [1] and dispenses with the assumption $g(t, r) \geqslant 0$.

Regarding equi-asymptotic stability, we have the following results.

Theorem 3. *Assume that there exists a strongly positive definite function $V(t, x) \in E$ such that $V'(t, x) \leqslant 0$. Then, there exists a strongly positive definite function $U(t, x) \in E$ obeying the differential inequality* (3.4), *where g is any given function satisfying the hypothesis* H_1.

Proof. Let the assumption of the theorem hold and $g(t, r)$ be any given function which satisfies the hypothesis H_1. Using the definition (3.5) and employing the arguments used in the proof of Theorem 1, it is easy to see that inequality (3.4) holds.

Making use of the strongly positive definiteness of $V(t, x)$, the lower estimate in (3.2) and the definition (3.5), we get

$$U(t, x) = r(t, 0, V(t, x)) \geqslant \beta_1(V(t, x)),$$

$$\geqslant \beta_1(\lambda(t, \| x \|)) \equiv \lambda_1(t, \| x \|),$$

$$\lambda \in KK, \beta_1 \in K, \lambda_1 \in KK$$

and the proof is complete.

Theorem 4. *Let the assumption* (a_2) *of Theorem 2 hold. Suppose further that there exists a strongly positive definite function* $U(t,x) \in E$ *such that the differential inequality* (3.4) *holds, for any given function* g *satisfying the hypothesis* H_1. *Then there exists a function* $V(t,x) \in E$, *which is strongly positive definite and* $V'(t,x) \leqslant 0$.

Proof. By using the definition (3.6) and proceeding as in the proof of Theorem 2, it can be shown that $V'(t,x) \leqslant 0$. Further, the uniqueness of solutions of (3.1) and the definition (3.6) imply that

$$U(t,x) = r(t,0,V(t,x)),$$

which, together with the upper estimate in (3.2) and the strongly positive definiteness $U(t,x)$ shows that $V(t,x)$ is strongly positive definite. We omit the details.

The hypothesis H_2 can be used in place of hypothesis H_1 in the above theorem to get the following variant of Theorem 4.

Theorem 5. *Let the assumption* (a_2) *of Theorem 2 hold. Let there exist a positive definite function* $U(t,x) \in E$ *satisfying* (3.4) *for any given function* g *which obeys the hypotesis* H_2. *Then, there exists a strongly positive definite function* $V(t,x) \in E$ *such that* $V'(t,x) \leqslant 0$.

Proof. The definition (3.6) and the reasoning similar to that in Theorem 2 imply that there exists a $V(t,x) \in E$ such that $V'(t,x) \leqslant 0$.

Now, as before, we have

$$U(t,x) = r(t,0,(V(t,x)),$$

which, by using the estimate (3.3), yields

$$U(t,x) \leqslant \beta_2(V(t,x))\,\sigma(t).$$

Hence,

$$V(t,x) \geqslant \beta_2^{-1}\left(\frac{U(t,x)}{\sigma(t)}\right) \geqslant \beta_2^{-1}\left(\frac{\varphi(\|x\|)}{\sigma(t)}\right),$$

thus proving that $V(t,x)$ is strongly positive definite. This proves the theorem.

Remark. It has been shown by Hahn [3] that if a strongly positive definite function exists, having a non-positive derivative, the null-solution is equi-asymptotically stable. The estimate (3.3) is equivalent to the asymptotic stability of the null solution of (3.1).

In case of uniform asymptotic stability, the following theorems are in order.

Theorem 6. *Let there exist a positive definite, decrescent function* $V(t,x) \in E$ *such that* $V'(t,x)$ *is negative definite. Then, there exists a function* $U(t,x) \in E$, *which is positive definite and decrescent, satisfying*

$$U'(t,x) \leqslant -\varphi(U(t,x)), \quad \varphi \in K, \quad (t,x) \in I \times R^n. \tag{3.7}$$

Proof. Let $V(t,x) \in E$ be such that

$$b(\|x\|) \leqslant V(t,x) \leqslant a(\|x\|), \quad b, a \in K, \quad (t,x) \in I \times R^n.$$

Since, by assumption, $V'(t,x)$ is negative definite,

$$V'(t,x) \leqslant -c(\|x\|), \quad c \in K, \quad (t,x) \in I \times R^n.$$

Define

$$\mathcal{J}(u) - \mathcal{J}(u_0) = \int_{u_0}^{u} \frac{dr}{\gamma(r)}, \qquad (u_0 > 0),$$

where $\gamma(r) = c(a^{-1}(r))$, and $U(t,x) = \mathcal{J}^{-1}(V(t,x))$. Since $\gamma \in K$, $\mathcal{J}(u)$ is an increasing function and this yields, in view of the definition of $U(t,x)$, that

$$\mathcal{J}'(U(t,x))\,U'(t,x) = V'(t,x) \leqslant -c(\|x\|).$$

From this inequality, we deduce, using the decrescent character of $V(t,x)$ and the definition of $\mathcal{J}(u)$, that

$$\begin{aligned} U'(t,x) &\leqslant -\gamma(V(t,x))\,\gamma(U(t,x)) = \\ &= -\gamma(\mathcal{J}(U(t,x)))\,\gamma(U(t,x)) \\ &\equiv -\varphi(U(t,x)). \end{aligned}$$

This proves (3.7).

Further, the positive definiteness and the decrescentness of $V(t,x)$ imply

$$U(t,x) = \mathcal{J}^{-1}(V(t,x)) \geqslant \mathcal{J}^{-1}(b(\|x\|)) \equiv b_1(\|x\|), \quad b_1 \in K$$

and

$$U(t,x) = \mathcal{J}^{-1}(V(t,x)) \leqslant \mathcal{J}^{-1}(a(\|x\|)) \equiv a_1(\|x\|), \quad a_1 \in K,$$

respectively. The proof is complete.

Theorem 7. *Let the assumption* (a_2) *of Theorem 2 hold. Let there exist a positive definite, decrescent function* $U(t,x) \in E$, *such that* (3.4) *holds, for any given function* g *satisfying the hypothesis* H_2. *Suppose further that*

$$r(t,0,r_0) \geqslant \beta_1(r_0)\,\sigma(t), \quad t \in I, \quad \beta_1 \in K, \quad \sigma \in L \tag{3.8}$$

and

$$R(t,x) \equiv \frac{\partial r(t,0,U(t,x)\sigma(t))}{\partial r_0}\,[-g(t,U(t,x)\sigma(t)) + g(t,U(t,x))\,\sigma(t) + U(t,x)\,\sigma'(t)] \tag{3.9}$$

is negative definite.

Then, there exists a positive definite, decrescent function $V(t,x) \in E$ *such that* $V'(t,x)$ *is negative definite.*

Proof. Let the assumptions of the theorem hold. Define

$$V(t,x) = r(0,t,U(t,x)\,\sigma(t)). \tag{3.10}$$

Since (a_2) holds, the function $V(t, x)$ is defined for $t \geqslant 0$. In view of the fact that $U(t, x) \in E$ and the hypothesis H_2 it follows that $V(t,x) \in E$. Also,

$$V'(t, x) = \frac{\partial r(0, t, U(t, x)\sigma(t))}{\partial t_0} + \frac{\partial r(0, t, U(t, x)\sigma(t))}{\partial r_0} \times$$

$$\times [U'(t, x)\sigma(t) + U(t, x)\sigma'(t)]$$

which, in virtue of (3.3*) and (3.9), is negative definite.

The uniqueness of the solutions of (3.1), together with the definition (3.10) and the estimates (3.3), (3.8), implies that

$$\beta_1(V(t, x))\sigma(t) \leqslant U(t, x)\sigma(t) = r(t, 0, V(t, x)) \leqslant \beta_2(V(t, x))\sigma(t),$$

whence,

$$\beta_1(V(t, x)) \leqslant U(t, x) \leqslant \beta_2(V(t, x)),$$

which yields, in view of the positive definiteness and decrescentness of $U(t, x)$, that

$$V(t, x) \geqslant \beta_2^{-1}(U(t, x)) \geqslant \beta_2^{-1}(b(\|x\|)) \equiv b_1(\|x\|), \quad \beta_2^{-1},\ b,\ b_1 \in K$$

and

$$V(t, x) \leqslant \beta_1^{-1}(U(t, x)) \leqslant \beta_1^{-1}(a(\|x\|)) \equiv a_1(\|x\|), \quad \beta_1^{-1},\ a,\ a_1 \in K$$

respectively. The theorem is proved.

Remarks. (1) If the solutions of (3.1) are exponentially asymptotically stable, it is easy to verify that $V(t, x) = r(0, t, U(t, x)e^{-\gamma t})$, $\gamma > 0$ is positive definite, decrescent and $V'(t, x)$ is negative definite.

(2) Observe that, in general, it is not necessary to demand that $\sigma \in L$, in the estimates which guarantee asymptotic stability of the null solution of (3.1). It is enough even if $\sigma(t)$ decreases after $t \geqslant t_1 > 0$ and tends to zero as $t \to \infty$, for example $\sigma(t) = \exp\left[\frac{-(t-1)^2}{2}\right]$.

With this remark, consider the following example, which is illustrative.

$$r' = -(t-1)r,$$

whose solution through $(0, r_0)$ is $r(t, 0, r_0) = r_0 \exp\left[\frac{1-(t-1)^2}{2}\right]$. This shows that $r = 0$ is asymptotically stable. However, we observe that, in this case, $R(t, x)$ is $\gtreqless 0$ acording as $t \lesseqgtr 1$, $x \in R^n$.

This means, that even in the simpler case when $g(t, r)$ is linear in r, $V'(t, x)$ need not be strictly negative, let alone being negative definite.

REFERENCES

1. Brauer, F. — *The use of comparison theorems for ordinary differential equations*. Proceedings of Nato Advanced Study Institute, Padua, Italy, 29–50 (1966).
2. Coddington, E. A and Levinson, N. — *Theory of ordinary differential equations*. NewYork (1955).
3. Hahn, W. — *On the general concept of stability and Lyapunov's direct method*. MRC Technical Summary Report. 485 (1964).
4. Lakshmikantham, V. — *Vector Lyapunov functions and conditional stability*. Jour. Math. Anal. Appl. 10 (1965), 368—377.

University of Rhode Island
Kingston, Rhode Island

PRINCIPIUL COMPARAŢIEI ŞI METODA A DOUA A LUI LIAPUNOV

Rezumat

În această lucrare, autorul demonstrează cîteva teoreme de existenţă a funcţiei Liapunov pentru sistemul $\dot{x} = f(t, x)$.

Condiţiile clasice referitoare la derivata funcţiei Liapunov în baza sistemului sînt înlocuite prin inegalităţi de forma (3.4), sugerate de principiul comparaţiei.

Se arată că existenţa unei funcţii Liapunov, cu proprietăţi cunoscute din teoremele clasice de stabilitate, atrage existenţa unei alte funcţii Liapunov cu proprietăţi cerute de principiul comparaţiei.