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FUNCŢII ŞI ŞIRURI ASIMPTOTIC APROAPE-PERIODICE CU VALORI ÎNTR-UN SPAŢIU BANACH

Rezumat

Nota de faţă are drept scop prezentarea unor rezultate cu privire la funcţiile şi şirurile asimptotic-aproape-periodice (prescurtat a.p.p.) cu valori într-un spaţiu Banach X, stabilirea unor proprietăţi de legătură între funcţiile şi şirurile a.p.p. precum şi introducerea noţiunilor de funcţie şi şir slab asimptotic-aproape-periodic (prescurtat f.a.p.p.). După o scurtă introducere, în care sînt indicate unele notaţii şi noţiuni cu caracter general, Nota conţine 3 paragrafe.

În § 1, după ce se face o scurtă trecere în revistă a principalelor proprietăţi ale funcţiilor şi şirurilor a.p.p. cu valori în X, se insistă mai mult asupra proprietăţilor topologice atît ale spaţiului $AP^{\infty}(X)$ al funcţiilor a.p.p. cît şi ale spaţiului $ap^{\infty}(X)$ al şirurilor a.p.p.

În § 2 se demonstrează două teoreme care permit pe de o parte să se caracterizeze funcţiile a.p.p. cu ajutorul şirurilor a.p.p. iar pe de altă parte şirurile a.p.p. cu ajutorul funcţiilor a.p.p.

Ultimul paragraf e dedicat introducerii noţiunilor de funcţie şi şir f.a.p.p. cu principalele lor proprietăţi.

FORMULAE FOR JACOBI POLYNOMIALS

BY

B. L. SHARMA

1. Introduction: This paper is in continuation to my previous papers [2, 3, 5, 6, 7]; in these papers the author has obtained some interesting generating functions of Jacobi polynomials. The object of this paper is to give new generating functions of Jacobi polynomials. The results obtained are believed to be new.

We require the following formulae [2], [5] and [8] in the investigation.

$$\sum_{n=0}^{\infty} \binom{m+n}{m} P_{n+m}^{(\alpha-n,\,\beta-n)}(x)\, t^n = \tag{1.1}$$

$$= \left[1 + \frac{1}{2}(x+1)t\right]^{\alpha}\left[1 + \frac{1}{2}(x-1)t\right]^{\beta} P_m^{(\alpha,\beta)}\left[x + \frac{1}{2}(x^2-1)t\right],$$

$$\sum_{n=0}^{\infty} \frac{(m+n)!\,(\lambda)_n}{n!\,(\mu)_n} P_{m+n}^{(\alpha-m-n,\,\beta-m-n)}(x)\, t^n = (-\alpha-\beta)_m \left(\frac{x-1}{x+1}\right)^{m-\alpha}$$

$$F_2\left[m-\alpha-\beta, -\alpha, \lambda; -\alpha-\beta, \mu; \frac{2}{1+x}, \frac{x-1}{2}t\right] \tag{1.2}$$

and

$$F_E[a_1, a_1, a_1, b_1, b_2, b_2; c_1, c_2, c_3; x, y, z] =$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} \frac{(a_1)_{m+n+p}\,(b_1)_m\,(b_2)_{n+p}}{(c_1)_m\,(c_2)_n\,(c_3)_p\, m!\, n!\, p!} x^m y^n z^p, \tag{1.3}$$

$$F_G[a_1, a_1, a_1, b_1, b_2, b_3; c_1, c_2, c_2; x, y, z] =$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} \frac{(a_1)_{m+n+p}\,(b_1)_m\,(b_2)_n\,(b_3)_p}{(c_1)_m\,(c_2)_{n+p}\, m!\, n!\, p!} x^m y^n z^p. \tag{1.4}$$

For definition and properties of Jacobi polynomials, see [4, Chapter 16].
2. The formula to be proved is

$$\sum_{n=0}^{\infty} F_2[-n, \gamma, \delta ; \lambda, \mu ; y, z]\, P_n^{(\alpha-n,\, \beta-n)}(x)\, t^n =$$

$$(2.1)\quad = \left(\frac{x+1}{x-1}\right)^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\alpha+\beta} \times F_A\Big[-\alpha-\beta, -\alpha, \gamma, \delta ; -\alpha-\beta, \lambda, \mu ;$$

$$\frac{2(x+1)^{-1}}{1+\frac{1}{2}(x-1)t}, \frac{-zt(x-1)}{1+\frac{1}{2}(x-1)t}, \frac{-yt(x-1)(x+1)^{-1}}{\left(1+\frac{1}{2}(x-1)t\right)^2}\Big]$$

for Lauricella's function F_A, see [1].

Proof: To prove (2.1), we start with the left-hand side of (2.1)

$$\sum_{n=0}^{\infty} F_2[-n, \gamma, \delta ; \lambda, \mu ; y, z]\, P_n^{(\alpha-n,\, \beta-n)}(x)\, t^n =$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{n \geq m+p} \sum_{p=0} \frac{n!\,(\gamma)_m (\delta)_p (-y)^m (-z)^p}{(-n-m-p)!\, m!\, p!\, (\lambda)_m (\mu)_p} P_n^{(\alpha-n,\, \beta-n)}(x)\, t^n$$

$$= \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \frac{(\gamma)_m (\delta)_p}{(\lambda)_m (\mu)_p} \frac{(m+p)!}{m!\,p!} (-yt)^m (-zt)^p \times$$

$$\times \sum_{n=0}^{\infty} \binom{n+m+p}{m+p} P_{n+m+p}^{(\alpha-n-m-p,\, \beta-n-m-p)}(x)\, t^n$$

$$= \left[1+\frac{1}{2}(x+1)t\right]^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\beta} \sum_{m=0}^{\infty} \frac{(\gamma)_m}{(\lambda)_m\, m!} \left(\frac{-yt}{\left(1+\frac{1}{2}xt\right)^2 - \frac{1}{4}t^2}\right)^m \times$$

$$\sum_{p=0}^{\infty} \frac{(\delta)_p}{(\mu)_p} \frac{(m+p)}{p!} \left(\frac{-zt}{\left(1+\frac{1}{2}xt\right)^2 \frac{1}{4} - t^2}\right)^p P_{m+p}^{\alpha-n-p,\, \beta-m-p}\left[x+\frac{1}{2}(x^2-1)t\right] \text{ by } (1.1)$$

$$= \left(\frac{x+1}{x-1}\right)^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\alpha+\beta} \sum_{m=0}^{\infty} \frac{(\gamma)_m (-\alpha-\beta)_m}{(\lambda)_m\, m!} \left(\frac{-yt(x-1)(x+1)^{-1}}{\left(1+\frac{1}{2}(x-1)t^2\right)}\right)^m \times$$

$$F_2\Big[m-\alpha-\beta, -\alpha, \delta ; -\alpha-\beta, \mu ; \frac{2(x+1)^{-1}}{1+\frac{1}{2}(x-1)t}, \frac{-zt(x-1)}{1+\frac{1}{2}(x-1)t}\Big] \text{ by } (1.2)$$

$$= \left(\frac{x+1}{x-1}\right)^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\alpha+\beta} \times$$

$$F_A\Big[-\alpha-\beta, -\alpha, \gamma, \delta : -\alpha-\beta, \lambda, \mu ; \frac{2(x+1)^{-1}}{1+\frac{1}{2}(x-1)t}, \frac{-zt(x-1)}{1+\frac{1}{2}(x-1)t},$$

$$\frac{-yt(x-1)(x+1)^{-1}}{\left(1+\frac{1}{2}(x-1)t\right)^2}\Big].$$

This completes the proof of (5).

3. **Further generating Functions**: Proceeding on the same lines and using the formulae (1.1), (1.2), (1.3) and (1.4), we obtain the following generating functions.

$$\sum_{n=0}^{\infty} F_1(-n, \lambda, \mu ; \delta ; y, z)\, P_n^{(\alpha-n,\, \beta-n)}(x)\, t^n - \left(\frac{x+1}{x-1}\right)^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\alpha+\beta}$$

$$F_G\Big[-\alpha-\beta, -\alpha-\beta, -\alpha-\beta, -\alpha, \mu, \lambda ; -\alpha-\beta, \delta, \delta ;$$

$$(3.1)\quad \frac{2(x+1)^{-1}}{1+\frac{1}{2}(x-1)t}, \frac{-zt(x-1)}{1+\frac{1}{2}(x-1)t}, \frac{-yt(x-1)(x+1)^{-1}}{\left(1+\frac{1}{2}(x-1)t\right)^2}\Big].$$

$$\sum_{n=0}^{\infty} F_4[-n, \gamma ; \lambda, \mu ; y, z]\, P_n^{(\alpha-n,\, \beta-n)}(x)\, t^n = \left(\frac{x+1}{x-1}\right)^{\alpha} \left[1+\frac{1}{2}(x-1)t\right]^{\alpha+\beta}$$

$$F_E\Big[-\alpha-\beta, -\alpha-\beta, -\alpha-\beta, -\alpha, \gamma, \gamma ; -\alpha-\beta, \mu, \lambda ;$$

$$(2.3)\quad \frac{2(x+1)^{-1}}{1+\frac{1}{2}(x-1)t}, \frac{-zt(x-1)}{1+\frac{1}{2}(x-1)t}, \frac{-yt(x-1)(x+1)^{-2}}{\left(1+\frac{1}{2}(x-1)t\right)^2}\Big].$$

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FORME PENTRU POLINOAME JACOBI

Rezumat

Se arată că funcţiile (2.1), (3.1), (3.2) sînt funcţii generatoare pentru polinoame Jacobi.

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A NEW EXPANSION FORMULA FOR G-FUNCTION

BY

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1. Introduction: In my previous paper [7] I have proved the expansion formula for a generalised hypergeometric function of two variables in a series of product of generalised hypergeometric functions of two variables and Meijer's G-function.

$$x^{\lambda} F\left[\begin{matrix}(a_1); (c_{p_1}); (e_{p_2}); -ax, -bx \\ (b_m); (d_{q_1}); (f_{q_2});\end{matrix}\right] = \frac{\prod_{j=p+1}^{s} \Gamma(1-\beta_j-\lambda) \prod_{j=q+1}^{r} \Gamma(\alpha_j+\lambda)}{\prod_{j=1}^{p} \Gamma(\beta_j+\lambda) \prod_{j=1}^{q} \Gamma(1-\alpha_j-\lambda)} \times$$

$$\times \sum_{n=0}^{\infty} \frac{(2\lambda+2n)\Gamma(2\lambda+n)}{n!} G_{r+2,s}^{p,q+1}\left(\frac{1}{z}\left|\begin{matrix}1-\lambda-n, \alpha_1, \ldots, \alpha_r, 1+\lambda+n \\ \beta_1, \ldots, \beta_s\end{matrix}\right.\right) \times$$

$$(1) \quad \times F\left|\begin{matrix}-n, 2\lambda+n, \alpha_r+\lambda, (a_1); (c_{p_1}); (e_{p_2}); (-1)^{p+q-s}a, (-1)^{p+q-s}b \\ \beta_s+\lambda, (b_m); (d_{q_1}); (f_{q_2});\end{matrix}\right|$$

provided that $1+p_1 = m+q+1$ or $1+p_1 < m+q_1$, $1+p_2 = 1+m+q_2$ or $1+p_2 < m+q_2$, $p+q > \frac{1}{2}(r+s)$, $|\arg x| < \left(p+q-\frac{1}{2}r-\frac{1}{2}s\right)\pi$, and the notation for the double hypergeometric function is due to Burchnall and Chaundy [2, p. 112] in preference for the sake of brevity, to an earlier one introduced by Kampé de Feriet [1, p. 150].

The object of this paper is to give the inverse of (1). The result obtained is believed to be new.