

ASYMPTOTIC SOLUTION FOR UNSTEADY MAGNETOHYDRODYNAMIC LAMINAR FREE CONVECTION ON A VERTICAL PLATE

BY

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§ 1. **Introduction.** Menold and Yang [1] have obtained solutions for unsteady free convection flow of an incompressible fluid past an infinite flat plate with time dependent arbitrary surface temperature or heat flux variations. Gupta [2] studied free convection of a viscous electrically conducting fluid from a hot infinite porous flat plate maintained at constant temperature under a transverse magnetic field. However, unsteady laminar hydromagnetic free convection flow has received less attention. Recently Singh [3] studied unsteady laminar free convection flow of a viscous incompressible and conducting fluid past a vertical infinite flat plate in presence of a constant horizontal magnetic field.

The purpose of the present paper is to study the effect of magnetic field on one-dimensional unsteady laminar free convection flow of an electrically conducting fluid past a vertical infinite flat plate with time dependent arbitrary surface temperature or heat flux variations. General formulae for small time for velocity, temperature and induced magnetic field are obtained when the magnetic Prandtl number is unity.

Similar types of problems have been studied by Ballabh and Singh [4], Gupta [5, 6], Lu [7, 8], Kakutani [9] and some others.

§ 2. **Analysis.** Considering an infinite vertical flat plate and constant fluid properties, the unsteady hydromagnetic laminar free convection boundary layer equations in rationalized MKS units may be written as [1, 2];

$$(2.1) \quad \frac{\partial u}{\partial t} = G + \frac{\partial^2 u}{\partial y^2} + M \frac{\partial H}{\partial y},$$

$$(2.2) \quad \frac{\partial H}{\partial t} = M \frac{\partial u}{\partial y} + \frac{1}{R_m} \frac{\partial^2 H}{\partial y^2}$$

and

$$(2.3) \quad \frac{\partial G}{\partial t} = \frac{1}{Pr} \frac{\partial^2 G}{\partial y^2}$$

where $y = \bar{y}/L$, $t = \nu \bar{t}/L^2$, $u = \bar{u}L/\nu$, $G = g\beta L^3 (T - T_\infty)/\nu^2$, $M = \left(\frac{\mu_r}{\delta}\right)^{1/2} \frac{H_0 L}{\nu}$,

$R_m = \sigma\mu_r\nu$, $H = \frac{\bar{H}_x L}{\nu} (\mu_r/\rho)^{1/2}$ and \bar{y} is the coordinate normal to the plate,

\bar{t} , the time variable, \bar{u} the velocity component along the plate, \bar{T} , the temperature, \bar{T}_∞ , the ambient temperature, L a characteristic length, ν the kinematic viscosity, Pr the Prandtl number, g , the gravitational acceleration, β the coefficient of volumetric expansion, R_m the magnetic Prandtl number, H_0 the imposed magnetic field (normal to the plate), \bar{H}_x , the induced magnetic field (parallel to the plate), μ_r , the magnetic permeability and σ , the electrical conductivity. In writing these equations the viscous energy dissipation and Joule heat dissipation have been neglected.

The boundary conditions are

$$(2.4) \quad u(y, 0) = G(y, 0) = H(y, 0) = 0,$$

$$(2.5) \quad u(0, t) = u(\infty, t) = G(\infty, t) = H(0, t) = H(\infty, t) = 0,$$

$$(2.6) \quad G(0, t) = G_w(t), \text{ or } \frac{\partial G(0, t)}{\partial y} = -Q(t),$$

where G_w is the prescribed value of G at the surface, and $Q = \bar{q}g\beta L^4/\nu^2 K$, \bar{q} being the surface heat flux and K the thermal conductivity of the fluid.

§ 3. Solution of the equations for arbitrary surface temperature variations. Using the Laplace transform u^* of u , G^* of G and H^* of H , defined as

$$u^*(p) = \int_0^\infty u(t)e^{-pt} dt, \quad G^*(p) = \int_0^\infty G(t)e^{-pt} dt$$

and

$$H^*(p) = \int_0^\infty H(t)e^{-pt} dt,$$

equations (2.1), (2.2) and (2.3) become

$$(3.1) \quad pu^* = G^* + \frac{d^2 u^*}{dy^2} + M \frac{dH^*}{dy},$$

$$(3.2) \quad pH^* = M \frac{du^*}{dy} + \frac{1}{R_m} \frac{d^2 H^*}{dy^2},$$

$$(3.3) \quad pG^* = \frac{1}{Pr} \frac{d^2 G^*}{dy^2}.$$

Solution of equations (3.1), (3.2) and (3.3) satisfying the transformed boundary conditions, for $R_m = 1$, is given by

$$(3.4) \quad G^* = G_w^*(p) \exp\{-y \text{Pr}^{1/2} p^{1/2}\},$$

$$(3.5) \quad u^* = \frac{G_w^*(p)}{2M(\text{Pr} - 1)(p - a^2)} \left[M(e^{-ny} + e^{-hy}) + \left\{ \frac{1}{\sqrt{p \text{Pr}}} - \sqrt{\frac{p}{\text{Pr}}} - \frac{(\text{Pr} - 1)(p - a^2)}{\sqrt{p \text{Pr}}} \right\} (e^{-ny} - e^{-hy}) - 2M \exp\{-y \text{Pr}^{1/2} p^{1/2}\} \right],$$

$$(3.6) \quad H^* = \frac{G_w^*(p)}{2M(\text{Pr} - 1)(p - a^2)} \left[M(e^{-ny} - e^{-hy}) + \left\{ \frac{1}{\sqrt{p \text{Pr}}} - \sqrt{\frac{p}{\text{Pr}}} - \frac{(\text{Pr} - 1)(p - a^2)}{\sqrt{p \text{Pr}}} \right\} \{e^{-ny} + e^{-hy} - 2 \exp(-y \text{Pr}^{1/2} p^{1/2})\} \right],$$

$$\text{where } n = \frac{M + \sqrt{M^2 + 4p}}{2}, \quad h = \frac{-M + \sqrt{M^2 + 4p}}{2} \quad \text{and } a = \frac{M \text{Pr}^{1/2}}{\text{Pr} - 1}.$$

These expressions, when inversely transformed, yield G , u and H for all values of y and t . But the inversion is too complicated to be evaluated in closed form.

For small values of t , we may take p in Laplace-transform large [10], so that for values of M such that M^2 can be neglected in comparison with p , we obtain on inverting*):

$$(3.7) \quad G = \frac{y}{2} \sqrt{\frac{\text{Pr}}{\pi}} \int_0^t G_w(t - \tau) \tau^{-3/2} \exp\left(-\frac{y^2 \text{Pr}}{4\tau}\right) d\tau,$$

$$(3.8) \quad u = \frac{\cosh(My/2)}{2(\text{Pr} - 1)} \int_0^t G_w(t - \tau) \cdot e^{a\tau} \left[e^{ny} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}} + a\sqrt{\tau}\right) + e^{-ny} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}} - a\sqrt{\tau}\right) \right] d\tau -$$

*) Throughout this paper, *Tables of Integral Transforms*, Vol. 1, 1954, edited by A. Erdélyi, has been used to evaluate inverse Laplace transforms of functions involved.

$$\begin{aligned}
& - \frac{1}{2(\text{Pr} - 1)} \int_0^t G_w(t - \tau) e^{a\tau} \left[\exp(a\gamma \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{\gamma \text{Pr}^{1/2}}{2|\tau} + a|\tau \right) + \right. \\
& \quad \left. + \exp(-a\gamma \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{\gamma \text{Pr}^{1/2}}{2|\tau} - a|\tau \right) \right] d\tau - \\
& - \frac{\sinh(My/2)}{2aM \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t G_w(t - \tau) \left[(1 - a^2) e^{a\tau} \left\{ e^{-a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} - a|\tau \right) - \right. \right. \\
& \quad \left. \left. - e^{a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} + a|\tau \right) \right\} - \frac{2 \text{Pr} a}{|\pi\tau} \exp \left(-\frac{\gamma^2}{4\tau} \right) \right] d\tau, \quad (\text{Pr} \neq 1), \\
(3.9) \quad H &= \frac{\sinh(My/2)}{2(\text{Pr} - 1)} \int_0^t G_w(t - \tau) e^{a\tau} \left[e^{a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} + a|\tau \right) + \right. \\
& \quad \left. + e^{-a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} - a|\tau \right) \right] d\tau + \\
& + \frac{(1 - a^2) \cosh(My/2)}{2aM \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t G_w(t - \tau) \left[e^{a\tau} \left\{ e^{-a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} - a|\tau \right) - \right. \right. \\
& \quad \left. \left. - e^{a\gamma} \operatorname{erfc} \left(\frac{\gamma}{2|\tau} + a|\tau \right) \right\} - \frac{2a \text{Pr}}{(1 - a^2) |\pi\tau} \exp \left(-\frac{\gamma^2}{4\tau} \right) \right] d\tau - \\
& - \frac{(1 - a^2)}{2aM \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t G_w(t - \tau) \left[e^{a\tau} \left\{ \exp(-a\gamma \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{\gamma \text{Pr}^{1/2}}{2|\tau} - a|\tau \right) \right. \right. \\
& \quad \left. \left. - \exp(a\gamma \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{\gamma \text{Pr}^{1/2}}{2|\tau} + a|\tau \right) \right\} + \right. \\
& \quad \left. + \frac{2a \text{Pr}}{(a^2 - 1) |\pi\tau} \exp \left(-\frac{\gamma^2 \text{Pr}}{4\tau} \right) \right] d\tau, \quad (\text{Pr} \neq 1), \\
(3.10) \quad u &= \frac{\sinh(My/2)}{M|\pi} \int_0^t G_w(t - \tau) \tau^{-1/2} \exp \left(-\frac{\gamma^2}{4\tau} \right) d\tau, \quad (\text{Pr} = 1), \\
(3.11) \quad H &= \frac{1 - \cosh(My/2)}{M|\pi} \int_0^t G_w(t - \tau) \tau^{-1/2} \exp \left(-\frac{\gamma^2}{4\tau} \right) d\tau, \quad (\text{Pr} = 1).
\end{aligned}$$

Particular case: Stepwise variation in surface temperature. Here we have $G_w(t) = \delta_1$, a constant.

By putting $G_w(t - \tau) = \delta_1$ in (3.7) to (3.11) and integrating we get

$$(3.12) \quad G(y, t) = \delta_1 \operatorname{erfc}(\gamma \text{Pr}^{1/2}),$$

$$(3.13) \quad u(y, t) = \frac{2\delta_1 \sinh(\gamma Mt^{1/2})}{M} \left[\sqrt{\frac{t}{\pi}} e^{-\eta^2} - \gamma t^{1/2} \operatorname{erfc} \eta \right], \quad (\text{Pr} = 1),$$

$$\begin{aligned}
(3.14) \quad u &= \frac{\delta_1 \cosh(\gamma Mt^{1/2})}{2a^2 (\text{Pr} - 1)} \left[e^{a^2 t} \left\{ \exp(2\gamma a t^{1/2}) \operatorname{erfc}(\gamma + a t^{1/2}) + \right. \right. \\
& \quad \left. \left. + \exp(-2\gamma a t^{1/2}) \operatorname{erfc}(\gamma - a t^{1/2}) \right\} - 2 \operatorname{erfc} \eta \right] - \\
& - \frac{\delta_1}{2a^2 (\text{Pr} - 1)} \left[e^{a^2 t} \left\{ \exp(2\gamma a |\overline{\text{Pr}} t) \operatorname{erfc}(\gamma \text{Pr}^{1/2} + a t^{1/2}) + \right. \right. \\
& \quad \left. \left. + \exp(-2\gamma a |\overline{\text{Pr}} t) \operatorname{erfc}(\gamma \text{Pr}^{1/2} - a t^{1/2}) \right\} - 2 \operatorname{erfc}(\gamma \text{Pr}^{1/2}) \right] - \\
& - \frac{\delta_1 \sinh(\gamma Mt^{1/2}) (1 - a^2)}{2a^3 M \text{Pr}^{1/2} (\text{Pr} - 1)} e^{a^2 t} \left[\exp(-2\gamma a t^{1/2}) \operatorname{erfc}(\gamma - a t^{1/2}) - \right. \\
& \quad \left. - \exp(2\gamma a t^{1/2}) \operatorname{erfc}(\gamma + a t^{1/2}) \right] + \\
& + \frac{\delta_1 [1 + a^2 (\text{Pr} - 1)] \sinh(\gamma Mt^{1/2})}{a^2 M \text{Pr}^{1/2} (\text{Pr} - 1)} \left[2 \sqrt{\frac{t}{\pi}} \exp(-\eta^2) - 2\gamma t^{1/2} \operatorname{erfc} \eta \right], \quad (\text{Pr} \neq 1), \\
(3.15) \quad H &= \frac{2\delta_1 t^{1/2} (1 - \cosh \eta Mt^{1/2})}{M} \left\{ \frac{e^{-\eta^2}}{|\pi} - \eta \operatorname{erfc} \eta \right\}, \quad (\text{Pr} = 1), \\
(3.16) \quad H &= \frac{\delta_1 \sinh(\gamma Mt^{1/2})}{2a^2 (\text{Pr} - 1)} \left[e^{a^2 t} \left\{ \exp(2\gamma a t^{1/2}) \operatorname{erfc}(\gamma + a t^{1/2}) + \right. \right. \\
& \quad \left. \left. + \exp(-2\gamma a t^{1/2}) \operatorname{erfc}(\gamma - a t^{1/2}) \right\} - 2 \operatorname{erfc} \eta \right] + \\
& + \frac{\delta_1 (1 - a^2) \cosh(\gamma Mt^{1/2})}{2a^3 M \text{Pr}^{1/2} (\text{Pr} - 1)} e^{a^2 t} \left[\exp(-2\gamma a t^{1/2}) \operatorname{erfc}(\gamma - a t^{1/2}) - \right. \\
& \quad \left. - \exp(2\gamma a t^{1/2}) \operatorname{erfc}(\gamma + a t^{1/2}) \right] - \\
& - \frac{\delta_1 (1 - a^2)}{2a^3 M \text{Pr}^{1/2} (\text{Pr} - 1)} e^{a^2 t} \left[\exp(-2\gamma a |\overline{\text{Pr}} t) \operatorname{erfc}(\gamma \text{Pr}^{1/2} - a t^{1/2}) - \right. \\
& \quad \left. - \frac{2\delta_1 \{1 + a^2 (\text{Pr} - 1)\} t^{1/2} \cosh(My/2) \left\{ \frac{e^{-\eta^2}}{|\pi} - \eta \operatorname{erfc} \eta \right\} + \right. \\
& \quad \left. + \frac{2\delta_1 \{1 + a^2 (\text{Pr} - 1)\} t^{1/2} \left\{ \exp(-\eta^2 \text{Pr}) - \eta \text{Pr}^{1/2} \operatorname{erfc}(\eta \text{Pr}^{1/2}) \right\}}{a^2 M \text{Pr}^{1/2} (\text{Pr} - 1) |\pi} \right], \quad (\text{Pr} \neq 1),
\end{aligned}$$

where $\eta = \gamma/2 \sqrt{vt}$.

The rate of heat transfer at the plate is given by

$$(3.17) \quad \bar{q} = -K \left(\frac{\partial \bar{T}}{\partial y} \right)_{y=0} = \frac{\delta_1 K \nu^2}{g \beta L^4} \sqrt{\frac{\text{Pr}}{\pi t}},$$

and skin friction at the plate is given by

$$(3.18) \quad \tau_{\omega} = \rho \nu \left(\frac{\partial \bar{u}}{\partial y} \right)_{y=0} = \frac{\rho \nu^2}{L^2} \delta_1 \sqrt{\frac{t}{\pi}}, \quad (\text{Pr} = 1),$$

$$(3.19) \quad \tau_{\omega} = \rho \nu \left(\frac{\partial \bar{u}}{\partial y} \right)_{y=0} = \frac{\rho \nu^2}{L^2} \delta_1 \left[\frac{e^{a^2 t}}{a(1 + \text{Pr}^{1/2})} \text{erf}(at^{1/2}) \times \right. \\ \left. \times \left\{ 1 - \frac{(1 - a^2)}{2a^2 \text{Pr}^{1/2} (\text{Pr}^{1/2} - 1)} \right\} + \frac{t^{1/2} \{1 + a^2 (\text{Pr} - 1)\}}{a^2 \sqrt{\pi \text{Pr}} (\text{Pr} - 1)} \right], \quad (\text{Pr} \neq 1).$$

It is interesting to note that when $\text{Pr} = 1$, the velocity and skin friction are not affected by magnetic field. It is also seen from equation (3.15) that when $\text{Pr} = 1$, the induced magnetic field vanishes for small M and t .

The variation of velocity and induced field with η for $\delta_1 = 1$, $\text{Pr} = 1.01$ and for different values of M and t is shown in figures 1 and 2. Figure 3 gives the variation of velocity gradient at the plate with $t^{1/2}$ for $\delta_1 = 1$, $\text{Pr} = 1.01$ and for different values of M .

§ 4. Solution of the equations for arbitrary surface heat flux variations. In this case G satisfies the wall boundary condition.

$$\frac{\partial G(0, t)}{\partial y} = -Q(t).$$

Solution of equations (2.1), (2.2) and (2.3) satisfying the boundary conditions, for $R_{\infty} = 1$ and under the same assumptions as in section 3, is given by

$$(4.1) \quad G = \frac{1}{\sqrt{\pi \text{Pr}}} \int_0^t Q(t - \tau) \tau^{-1/2} \exp\left(-\frac{y^2 \text{Pr}}{4|\tau}\right) d\tau,$$

$$(4.2) \quad u = \frac{\sinh(M\nu/2)}{M} \int_0^t Q(t - \tau) \text{erfc}\left(\frac{y}{2|\tau}\right) d\tau, \quad (\text{Pr} = 1),$$

$$(4.3) \quad u = \frac{\cosh(M\nu/2)}{2a \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t Q((t - \tau) e^{a^2 \tau}) \left\{ e^{-ay} \text{erfc}\left(\frac{y}{2|\tau}\right) - a|\tau \right. \\ \left. - e^{ay} \text{erfc}\left(\frac{y}{2|\tau} + a|\tau\right) \right\} d\tau -$$

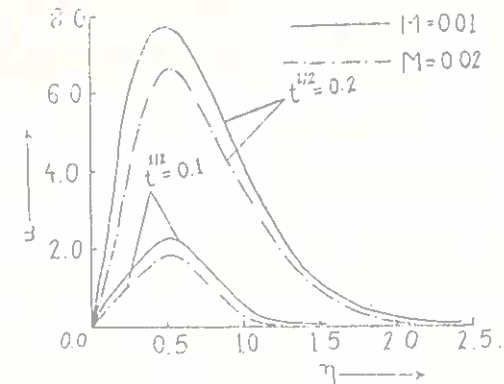


FIG 1 VELOCITY PROFILE vs η

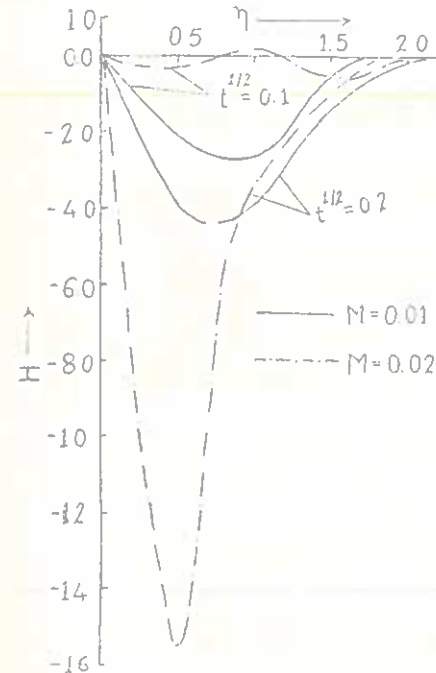


FIG 2 INDUCED MAGNETIC FIELD PROFILE vs η

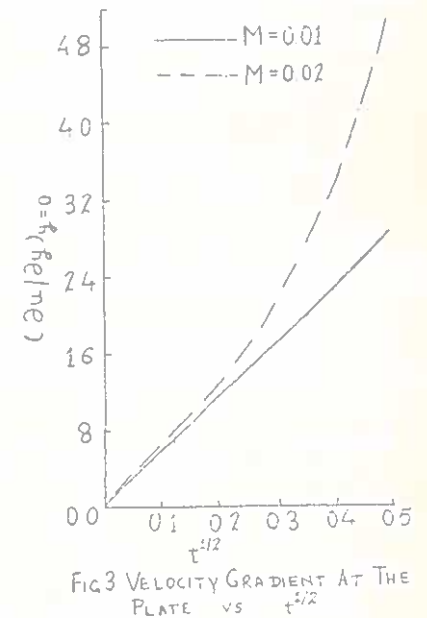


FIG 3 VELOCITY GRADIENT AT THE PLATE vs $t^{1/2}$

$$\begin{aligned}
& - \frac{1}{2a \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t Q(t - \tau) e^{a^2 \tau} \left\{ \exp(-ay \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{y \text{Pr}^{1/2}}{2\sqrt{\tau}} - a\sqrt{\tau} \right) - \right. \\
& \quad \left. - \exp(ay \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{y \text{Pr}^{1/2}}{2\sqrt{\tau}} + a\sqrt{\tau} \right) \right\} d\tau - \\
& - \frac{\sinh(My/2)(1 - a^2)}{2a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) e^{a^2 \tau} \left\{ e^{ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} + a\sqrt{\tau} \right) + \right. \\
& \quad \left. + e^{-ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} - a\sqrt{\tau} \right) \right\} d\tau + \\
& + \frac{\sinh(My/2) \{1 + a^2 (\text{Pr} - 1)\}}{a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} \right) d\tau, \quad (\text{Pr} \neq 1), \\
(4.4) \quad H & = \frac{1 - \cosh(My/2)}{M} \int_0^t Q(t - \tau) \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} \right) d\tau, \quad (\text{Pr} = 1), \\
(4.5) \quad H & = \frac{\sinh(My/2)}{2a^2 \text{Pr}^{1/2} (\text{Pr} - 1)} \int_0^t Q(t - \tau) e^{a^2 \tau} \left\{ e^{-ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} - a\sqrt{\tau} \right) - \right. \\
& \quad \left. - e^{ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} + a\sqrt{\tau} \right) \right\} d\tau + \\
& + \frac{(1 - a^2) \cosh(My/2)}{2a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) e^{a^2 \tau} \left\{ e^{ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} + a\sqrt{\tau} \right) + \right. \\
& \quad \left. + e^{-ay} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} - a\sqrt{\tau} \right) \right\} d\tau - \\
& - \frac{\{1 + a^2 (\text{Pr} - 1)\} \cosh(My/2)}{a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} \right) d\tau + \\
& + \frac{\{1 + a^2 (\text{Pr} - 1)\}}{a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) \operatorname{erfc} \left(\frac{y \text{Pr}^{1/2}}{2\sqrt{\tau}} \right) d\tau - \\
& - \frac{(1 - a^2)}{2a^2 M \text{Pr} (\text{Pr} - 1)} \int_0^t Q(t - \tau) e^{a^2 \tau} \left\{ \exp(ay \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{y \text{Pr}^{1/2}}{2\sqrt{\tau}} + a\sqrt{\tau} \right) + \right.
\end{aligned}$$

$$+ \exp(-ay \text{Pr}^{1/2}) \operatorname{erfc} \left(\frac{y \text{Pr}^{1/2}}{2\sqrt{\tau}} - a\sqrt{\tau} \right) \left. \right\} d\tau, \quad (\text{Pr} \neq 1).$$

Particular case: Stepwise variation in heat flux. Here we have $Q(t) = \delta_2$, a constant.

By putting $Q(t - \tau) = \delta_2$ in (4.1) to (4.5) and integrating we get

$$(4.6) \quad G = \frac{2\delta_2 t^{1/2}}{\text{Pr}^{1/2}} \left\{ \frac{\exp(-\eta^2 \text{Pr})}{\sqrt{\pi}} - \eta \text{Pr}^{1/2} \operatorname{erfc}(\eta \text{Pr}^{1/2}) \right\},$$

$$(4.7) \quad u = \frac{\delta_2 t \sinh(\eta M t^{1/2})}{M} \left\{ (1 + 2\eta^2) \operatorname{erfc} \eta - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} \right\}, \quad (\text{Pr} = 1),$$

$$(4.8) \quad u = \frac{\delta_2 \cosh(\eta M t^{1/2})}{a^2 \text{Pr}^{1/2} (\text{Pr} - 1)} \left[\frac{e^{a^2 t}}{2a} \left\{ \exp(-2\eta a t^{1/2}) \operatorname{erfc}(\eta - a t^{1/2}) - \right. \right. \\
\left. \left. - \exp(2\eta a t^{1/2}) \operatorname{erfc}(\eta + a t^{1/2}) \right\} - 2t^{1/2} \left\{ \frac{e^{-\eta^2}}{\sqrt{\pi}} - \eta \operatorname{erfc} \eta \right\} \right] - \\
- \frac{\delta_2}{a^2 \text{Pr}^{1/2} (\text{Pr} - 1)} \left[\frac{e^{a^2 t}}{2a} \left\{ \exp(-2\eta a \text{Pr}^{1/2} t^{1/2}) \operatorname{erfc}(\eta \text{Pr}^{1/2} - a t^{1/2}) - \right. \right.$$

$$\left. \left. - \exp(2\eta a \text{Pr}^{1/2} t^{1/2}) \operatorname{erfc}(\eta \text{Pr}^{1/2} + a t^{1/2}) \right\} - 2t^{1/2} \left\{ \frac{\exp(-\eta^2 \text{Pr})}{\sqrt{\pi}} - \right. \right. \\
\left. \left. - \eta \text{Pr}^{1/2} \operatorname{erfc}(\eta \text{Pr}^{1/2}) \right\} \right] - \frac{\delta_2 (1 - a^2) \sinh(\eta M t^{1/2})}{2a^4 M \text{Pr} (\text{Pr} - 1)} \times \\
\times \left[e^{a^2 t} \left\{ \exp(2\eta a t^{1/2}) \operatorname{erfc}(\eta + a t^{1/2}) + \right. \right. \\
\left. \left. + \exp(-2\eta a t^{1/2}) \operatorname{erfc}(\eta - a t^{1/2}) - 2 \operatorname{erfc} \eta \right\} + \right. \\
\left. + \frac{\delta_2 \{1 + a^2 (\text{Pr} - 1)\} \sinh(\eta M t^{1/2})}{a^2 M \text{Pr} (\text{Pr} - 1)} \left[t(1 + 2\eta^2) \operatorname{erfc} \eta - \eta t \frac{2}{\sqrt{\pi}} e^{-\eta^2} \right], \quad (\text{Pr} \neq 1), \right.$$

$$(4.9) \quad H = \frac{\delta_2 t (1 - \cosh \eta M t^{1/2})}{M} \left[(1 + 2\eta^2) \operatorname{erfc} \eta - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} \right], \quad (\text{Pr} = 1),$$

$$(4.10) \quad H = \frac{\delta_2 \sinh(\eta M t^{1/2})}{a^2 \text{Pr}^{1/2} (\text{Pr} - 1)} \left[\frac{e^{a^2 t}}{2a} \left\{ \exp(-2\eta a t^{1/2}) \operatorname{erfc}(\eta - a t^{1/2}) - \right. \right. \\
\left. \left. - \exp(2\eta a t^{1/2}) \operatorname{erfc}(\eta + a t^{1/2}) \right\} - 2t^{1/2} \left\{ \frac{e^{-\eta^2}}{\sqrt{\pi}} - \eta \operatorname{erfc} \eta \right\} \right] + \\
+ \frac{\delta_2 (1 - a^2) \cosh(\eta M t^{1/2})}{2a^4 M \text{Pr} (\text{Pr} - 1)} \left[e^{a^2 t} \left\{ \exp(2\eta a t^{1/2}) \operatorname{erfc}(\eta + a t^{1/2}) + \right. \right.$$

$$\begin{aligned}
 & + \exp(-2\eta at^{1/2}) \operatorname{erfc}(\eta - at^{1/2}) - 2 \operatorname{erfc} \eta] - \\
 & - \frac{\delta_2 t \{1 + a^2(\operatorname{Pr} - 1)\} \cosh(\eta Mt^{1/2})}{a^2 M \operatorname{Pr} (\operatorname{Pr} - 1)} \left[(1 + 2\eta^2) \operatorname{erfc} \eta - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} \right] + \\
 & + \frac{\delta_2 t \{1 + a^2(\operatorname{Pr} - 1)\}}{a^2 M \operatorname{Pr} (\operatorname{Pr} - 1)} \left[(1 + 2\eta^2 \operatorname{Pr}) \operatorname{erfc}(\eta \operatorname{Pr}^{1/2}) - \frac{2\eta \operatorname{Pr}^{1/2} \exp(-\eta^2 \operatorname{Pr})}{\sqrt{\pi}} \right] - \\
 & - \frac{\delta_2 (1 - a^2)}{2a^4 M \operatorname{Pr} (\operatorname{Pr} - 1)} [e^{a^2 t} \{ \exp(2\eta a \operatorname{Pr}^{1/2} t^{1/2}) \operatorname{erfc}(\eta \operatorname{Pr}^{1/2} + at^{1/2}) + \\
 & + \exp(-2\eta \operatorname{Pr}^{1/2} at^{1/2}) \operatorname{erfc}(\eta \operatorname{Pr}^{1/2} - at^{1/2}) \} - 2 \operatorname{erfc}(\eta \operatorname{Pr}^{1/2})], (\operatorname{Pr} \neq 1).
 \end{aligned}$$

The plate temperature difference is given by

$$(4.11) \quad G_w(t) = 2\delta_2 \sqrt{\frac{t}{\pi \operatorname{Pr}}},$$

where $G_w(t) = \frac{g\beta L^3}{\nu^2} (\bar{T}_w - \bar{T}_\infty)$, \bar{T}_w being the plate temperature.

The skin friction at the plate is given by

$$(4.12) \quad \tau_w = \frac{\rho \nu^2}{L^2} \delta_2 t^{3/2}, \quad (\operatorname{Pr} = 1),$$

$$(4.13) \quad \tau_w = \frac{\delta_2 \rho \nu^2}{L^2} \left[\frac{(1 - a^2)(1 - e^{a^2 t})}{2a^4 \operatorname{Pr} (\operatorname{Pr} - 1)} - \frac{(1 - e^{a^2 t})}{a^2 \operatorname{Pr}^{1/2} (\operatorname{Pr}^{1/2} + 1)} + \frac{t \{1 + a^2(\operatorname{Pr} - 1)\}}{2a^2 \operatorname{Pr} (\operatorname{Pr} - 1)} \right], (\operatorname{Pr} \neq 1).$$

Here also, for small M and t the effect of the magnetic field on skin friction and velocity vanishes when $\operatorname{Pr} = 1$. Further inspection of equation (4.9) reveals that induced magnetic field becomes zero for small M and t when $\operatorname{Pr} = 1$.

The variation of velocity and induced field with η for $\delta_2 = 1$, $\operatorname{Pr} = 1.01$ and for different values of M and t is shown in figures 4 and 5. Figure 6 gives the variation of velocity gradient at the plate with $t^{1/2}$ for $\delta_2 = 1$ and for different values of M .

§ 5. Conclusion. We thus conclude that :

(i) for a stepwise variation in surface temperature velocity, induced magnetic field and skin friction (velocity gradient at the plate) increase as time increases. It is also seen that the velocity decreases with M and the skin friction increases as M increases while the induced magnetic field shows no definite character for increasing values of M . Further as time increases the effect of M on velocity, induced magnetic field and skin

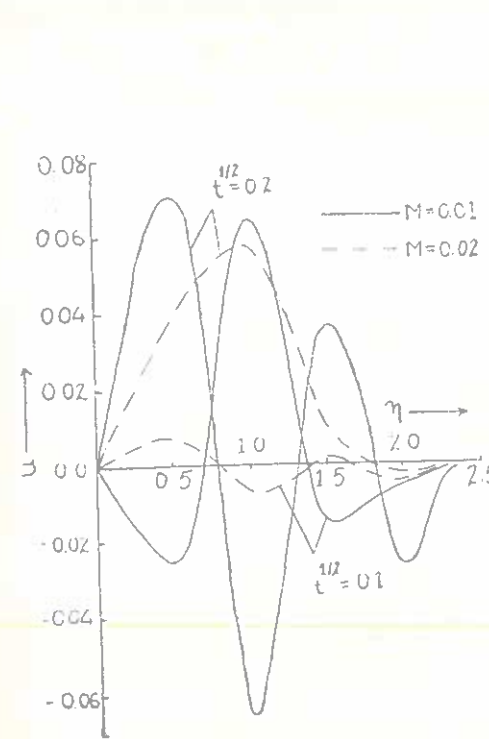


FIG 4 VELOCITY PROFILE VS η

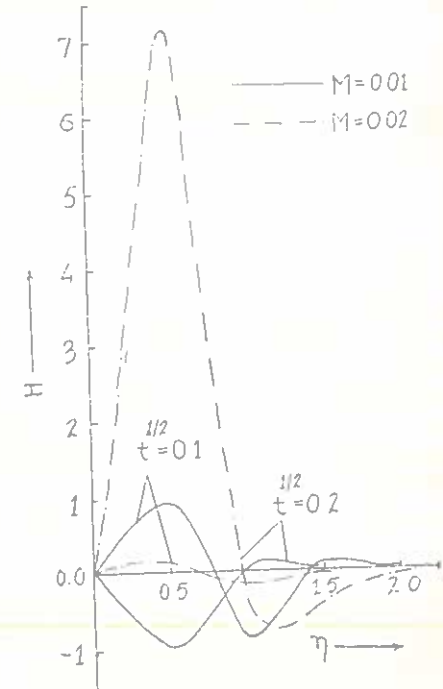


FIG 5 INDUCED MAGNETIC FIELD PROFILE VS η

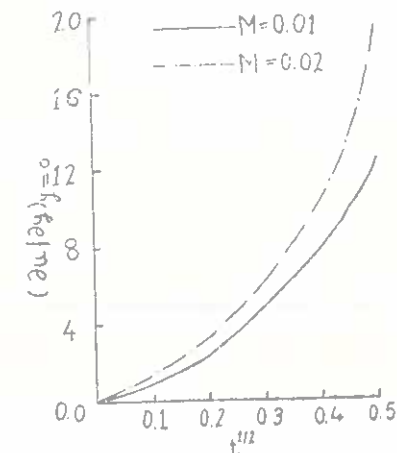


FIG 6 VELOCITY GRADIENT AT PLATE VS $t^{1/2}$

friction increases. This rate of increase of the effect of magnetic field with time is rapid on induced magnetic field and skin friction while it is slow on velocity.

(ii) For a stepwise variation in heat flux, velocity and skin friction (velocity gradient at the plate) increase as time increases while the induced magnetic field increases with time only when $M=0,02$. When $M=0,01$, it is seen that as we go above the plate the induced magnetic field first increases as time increases and then begins to decrease. The velocity of the fluid decreases with M while the skin friction increases as M increases. It is also seen in this case that for $r^{1/2}=0,1$, induced magnetic field decreases as M increases while for $r^{1/2}=0,2$ the effect of magnetic field is to increase the value of induced magnetic field. Further inspection reveals that, in this case also the effect of M on velocity, induced magnetic field and skin friction increase as time increases and this effect is rapid for induced magnetic field.

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SOLUȚIA ASIMPTOTICĂ PENTRU CONVECȚIA LAMINARĂ, LIBERĂ, NESTAȚIONARĂ, MAGNETOHIDRODINAMICĂ PE O PLACĂ VERTICALĂ

Rezumat

Se studiază convecția liberă, nestaționară, laminară a unui fluid electroconductor viscos pentru o placă verticală infinită, neconductoare și nemagnetică, cu temperatură de suprafață arbitrară sau variații ale fluxului de căldură.

Utilizînd metoda transformatei Laplace au fost obținute formule generale pentru timpi scurți, pentru viteză, cîmpul magnetic indus și temperatură, ca funcție de temperatura la suprafață sau fluxul de căldură, în următoarele condiții:

1. numărul Prandtl magnetic este egal cu unitatea,
2. parametrul magnetic adimensional M este mic.

Este studiat cazul variației în salt a temperaturii de suprafață cît și a fluxului de căldură pentru unele valori ale numărului Prandtl Pr și ale parametrului magnetic M .