(ii) In (2.3), substituting $p = r = 0$, $q = s = \rho = \sigma = 1$, $u = w + 1$ and $c'_1 = w$, we have a results involving Appel's function F_2[1, p. 14], with $\lambda = b_1$ and employing the formula [4, p. 105(3)], we have after little simplification, an interesting formula,

$$\sum_{n=0}^{\infty} \frac{(b_1)_n}{n!} F_2(c_1, -n, b_1, +n, b_1, \beta_1; x, y) . z^n = (1-z)^{c_1-b_1} . (1 + xz - \tag{2.5}$$
$$- y - z)^{-c_1} . H_3\left(c_1, \beta_1 - b_1; \beta_1; \frac{-xyz}{(1+xz-y-z)^2}, \frac{-y}{1+xz-y-z}\right)$$

where H_3 is the Horn's function [4, p. 225, (15)].

(iii) In (2.4), substituting $p = r = 0$, $q = s = \rho = \sigma = 1$, $c'_1 = w$, and $u = w + 1$, we have a result involving Appell's function F_2.

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ASUPRA UNOR FORMULE DE SUMARE CUPRINZÎND FUNCŢIILE LUI KAMPÉ DE FÉRIET

Rezumat

Obiectul articolului este stabilirea unor formule de sumare pentru funcţia lui Kampé de Fériet. Pentru obţinerea rezultatelor se utilizează tehnica operaţională şi metoda inducţiei finite. Unele din cazurile particulare sînt menţionate.

ON A RELATION INVOLVING HARDY'S TRANSFORM

BY

S. L. KALLA

1. Let $f(t)$ denote any function of a prescribed class of functions defined on a given interval (a, b). Further let $k(t, p)$ denote a definite function of t on that interval for each value of p, a parameter whose domain is prescribed; then an integral transform $T\{f(t); p\}$ of $f(t)$ is defined by means of the integral equation:

$$T\{f(t); p\} = p \int_0^{\infty} k(t, p) f(t) \, dt, \tag{1}$$

where the class of functions and the domain of parameters are so prescribed that the integral has sense. The function $k(t, p)$ is known as the kernel of the transform. Here we shall restrict our attention to the case in which the kernel $k(t, p)$ is a function of tp alone, $k(t, p) = k(p, t) = k(tp)$ as this case has the widest applications.

By taking a suitable Special Function for the kernel $k(t, p)$ in (1), various integral transforms have been defined and developed by many authors including Erdélyi [4], Meijer [9], Varma [15], Bora, Kalla & Saxena [2] and Gupta and Mittal [6].

By taking a kernel $t F_{\nu, \alpha}(pt)$, where

$$F_{\nu,\alpha}(t) = \sum_{k=0}^{\infty} \frac{(-1)^k [(1/2)t]^{\nu+2\alpha+2k}}{\Gamma(\alpha+1+k)\Gamma(\alpha+1+\nu+k)}$$

Hardy [7, p. 62] has given a generalization of Hankel's transform

$$H_\nu\{f(t);p\}=\int_0^\infty t\,f(t)\,J_\nu(pt)\,dt,\quad t>0 \tag{3}$$

by means of the integral equation:

$$H_{\nu,\alpha}\{f(t);p\}=\int_0^\infty t\,F_{\nu,\alpha}(pt)\,f(t)\,dt. \tag{4}$$

If we set $\alpha=0$, (4) reduces to (3).

In the last decade a number of papers [1, 10, 11, 12, 13, 14] have been published on obtaining a relation between Hardy's transform and other integral transforms, having a particular special function for the kernel $k(t,p)$ in (1). In the present note, we wish to unify all these results concerning relations between Hardy's transform and other integral transforms, by establishing a relation between Hardy's transform and the general integral transform (1).

A few particular cases are also mentioned.

2. Theorem : *If*

$$T\{t^\mu f(t);p\}=\int_0^\infty k(t,p)\,t^\mu f(t)\,dt \tag{5}$$

and

$$H_{\nu,\alpha}\{f(t);s\}=\int_0^\infty t\,f(t)\,F_{\nu,\alpha}(st)\,dt \tag{6}$$

then

$$T\{t^\mu f(t);p\}=\operatorname{cosec}(\nu\pi)\Big[\sin(\alpha+\nu)\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,\Phi_1(s,p)\,ds-$$

$$-\sin(\alpha\pi)\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,\Phi_2(s,p)\,ds\Big], \tag{7}$$

where

$$\Phi_1(s,p)=\int_0^\infty k(t,p)\,t^\mu J_\nu(st)\,dt\quad\text{and}\quad \Phi_2(s,p)=\int_0^\infty k(t,p)\,t^\mu J_{-\nu}(st)\,dt$$

provided

(A) $\alpha>-1$, $(\alpha+\nu)>1$, $(\alpha+2\nu)<3/2$, $|\nu|<3/2$, $t^\sigma f(t)$ *be integrable over* $(0,\delta)$ *where* $\delta>0$ *and* $\sigma=\min(1+\nu+2\alpha, 1/2)$, $t^{1/2}f(t)$ *be integrable over* (δ,∞) *and*

(B) $R(\mu+\nu+\xi+1)>0$ *where* $k(pt)\sim O(|t^\xi|)$ *for small* t *and*

(i) if $c<0$, *then* $R(a+\mu+1)<0$,

(ii) if $c>0$ *then* $R(b)>0$,

where $k(pt)$ *behaves as* $t^a e^{-bt^c}$ *for large values of* t *and the integrals in* (7) *are absolutely convergent.*

Proof. Let the conditions under (A) be satisfied; then by the inversion theorem for the Hardy's transform as given by Cooke [3], we have

$$f(t)=\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,M_{\nu,\alpha}(st)\,ds \tag{8}$$

where

$$M_{\nu,\alpha}(t)=\cos(\alpha\pi)\,J_\nu(t)+\sin(\alpha\pi)\,Y_\nu(t). \tag{9}$$

From (5) and (8) we have

$$T\{t^\mu f(t);p\}=\int_0^\infty k(t,p)\,t^\mu\,dt\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,M_{\nu,\alpha}(st)\,ds=$$

$$=\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,ds\int_0^\infty k(t,p)\,t^\mu M_{\nu,\alpha}(st)\,dt=$$

$$=\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,ds\int_0^\infty k(t,p)\,t^\mu[\cos(\alpha\pi)\,J_\nu(st)+$$

$$+\sin(\alpha\pi)\,Y_\nu(st)]\,dt=$$

$$=\operatorname{cosec}(\nu\pi)\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,ds\int_0^\infty k(t,p)\,t^\mu[\sin(\alpha+\nu)\pi\,J_\nu(st)-$$

$$-\sin(\alpha\pi)\,J_{-\nu}(st)]\,dt;$$

on changing the order of integration, which is justified by the absolute convergence of the integrals, under the conditions stated by the theorem,

$$= \operatorname{cosec}(\nu\pi)\left[\sin(\alpha+\nu)\pi\int_0^\infty sH_{\nu,\alpha}\{f(t)\,s\}\,\Phi_1(s,p)\,ds\,-\right.$$

$$\left.-\sin(\alpha\pi)\int_0^\infty s\,H_{\nu,\alpha}\{f(t);s\}\,\Phi_2(s,p)\,ds\right].$$

Particular cases: If we set, $k(pt)=e^{-1/2\,pt}\,pt^{-\lambda-\frac{1}{2}}\,W_{k+\frac{1}{2},m}(pt)$, then the transform (1) takes the form of a generalized Laplace transform as given by Mainra [8, p. 23] and consequently our theorem reduces to a relation between Mainra transform and Hardy's transform as obtained by Nigam [10, p. 285], which itself is a generalization of many results given earlier by Singh [11, 12], Srivastava and Bhonsle [13], Varma [14], and Bhonsle [1].

If we take Fox's H-function for the kernel $k(pt)$ in (1), then it reduces to the H-function transform of Gupta and Mittal [6] and consequently our theorem becomes a relation between this transform and Hardy's transform. Similarly many other relations between Hardy's transform and other integral transform can be established by using our theorem.

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ASUPRA UNEI TRANSFORMĂRI CARE CUPRINDE TRANSFORMAREA LUI HARDY

Rezumat

În lucrare se unifică mai multe rezultate referitoare la relația dintre transformarea lui Hardy și alte transformări integrale, stabilindu-se o legătură între transformarea lui Hardy și transformarea generală (1). Din această relație se deduc diverse cazuri particulare cunoscute.