Teorema lui R. Bredt se stabileşte fără dificultate şi în acest caz. Pentru determinarea deplasărilor avem sistemul

$$
(4.4)\quad \begin{cases} \varepsilon_{11}=\varepsilon_{22}=-\dfrac{c_{13}}{c_{11}+c_{12}}\varepsilon_{33}, \quad \varepsilon_{12}=0, \quad \varepsilon_{23}+i\varepsilon_{31}=\dfrac{1}{\sqrt{c_{11}}}w, \\ \varepsilon_{33}=\dfrac{1}{2}[(\bar{\alpha}z+\alpha\bar{z}+2\alpha_0)x_3+\beta z+\bar{\beta}\bar{z}+2\beta_0], \quad \beta=\beta_1+i\beta_2. \end{cases}
$$

Sistemul (4.4) fiind de tipul sistemului (4.5) din lucrarea citată [3], se poate integra prin aceeaşi metodă.

Observaţie. Dacă presupunem că între cei cinci coeficienţi de elasticitate există relaţiile

$$c_{33}=c_{11}, \quad c_{13}=c_{12}, \quad c_{44}=c_{11}-c_{12}$$

atunci mediul elastic este izotrop şi, punînd

$$c_{11}=\lambda+2\mu, \quad c_{12}=\lambda, \quad c_{44}=2\mu,$$

regăsim rezultatele din [3].

BIBLIOGRAFIE

1. Milne-Thomson L. M. — *Antiplane elastic systems.* Ergebnisse der Mathematik, Springer-Verlag, 1962.
2. Radu A. — *Sisteme elastice antiplane neomogene.* An. şt. Univ. Iaşi, Matematică, t. XIII (1967), p. 145–159.
3. Radu A. — *Echilibrul barelor elastice izotrope neomogene.* An. şt. Univ. Iaşi, Matematică, t. XIII (1967), p. 421–442.
4. Векуа И. Н. — *Обобщенные аналитические функции.* Москва, 1959.

LA STATIQUE DES BARRES ÉLASTIQUES MONOTROPES NON HOMOGÈNES

Résumé

On étudie l'équilibre d'une barre cylindrique monotrope (transversalement isotrope) sous l'action d'un système de forces qui agissent sur les bases, des forces appliquées sur la surface latérale et des forces de masse, pour le cas où les coefficients d'élasticité du matériel sont des fonctions de classe C^2.

Après avoir déterminé la tension normale σ_{33}, les tensions tangentielles σ_{31} et σ_{23}, on étudie séparément pour le cas où la tension σ_{33} est indépendante de x_3 et le cas où elle est une fonction de x_3.

En utilisant la méthode des fonctions analytiques généralisées de I. N. Vekua, on démontre l'existence et l'unicité de la solution du problème aux limites considéré.

ON THE RECIPROCITY THEOREM IN THE LINEAR ELASTODYNAMICS

BY

D. IEŞAN

The reciprocity theorem in the dynamic theory of linear elasticity was established [1] by Graffi using the Laplace transform.

In 1967 we gave [2] a method to obtain the reciprocity theorems without using the Laplace transform. The advantage of this method lies in the fact that it avoids extraneous analytical restrictions upon the states involved.

The results seems to be of sufficient interest to be reproduced separately with particular reference to classical theory of elasticity since it may be overlooked in a paper which is primarily concerned with other theory.

Let $\bar{V}$ be a regular (in the sense of Kellog) region of space occupied by a elastic material whose boundary is Σ. Moreover V is the interior of V and n_i are components of the unit outward normal to Σ. Throughout this paper a rectangular coordinate system $Ox_k (k=1,2,3)$ is employed.

The basic equations of the linear theory of homogeneous and anisotropic elastic solid are:

equations of motion

$$t_{ij,j}+F_i=\rho\ddot{u}_i, \tag{1}$$

constitutive equations

$$t_{ij}=C_{ijkl}\,e_{kl}, \tag{2}$$

geometrical equations

$$2e_{ij}=u_{i,j}+u_{j,i}. \tag{3}$$

In these relations we have used the following notations: u_i - components of the displacement vector, e_{ij} — components of strain tensor, t_{ij} — components of stress tensor, F_i — components of body force vector, c_{ijkl}, ρ — characteristic constants of the material, the comma denotes partial derivation with respect to the space variables x_k and a dot denotes partial derivation with respect to the time t.

We have

$$c_{ijkl} = c_{klij} = c_{jikl}. \tag{4}$$

To the system of field equations we adjoin the initial conditions

$$u_i(x,0) = a_i(x), \qquad \dot{u}_i(x,0) = b_i(x), \tag{5}$$

and the boundary conditions

$$t_{ij}n_j = t_i, \quad \text{on } \Sigma, \tag{6}$$

where a_i, b_i, t_i are prescribed functions.

Let u and v be functions defined on $\overline{V} \times [0, \infty)$, continuous on $[0, \infty$ with respect to the time t for each $x \in \overline{V}$. We denote by $u * v$ the convolution of u and v

$$u * v = \int_0^t u(x, t - \sigma)\, v(x, \tau)\, d\tau. \tag{7}$$

It is known the following properties of the convolution [3]

$$\begin{gathered} u * v = v * u, \quad u * v = 0 \text{ implies } u = 0 \text{ or } v = 0, \\ u * (v * w) = (u * v) * w = u * v * w, \quad u * (v + w) = u * v + u * w. \end{gathered} \tag{8}$$

We will denote by g the function defined on $[0, \infty)$ by

$$g(t) = t. \tag{9}$$

From (1), (7), (9), as in [4], it is easy to obtain

$$g * (t_{ij,j} + F_i) = \rho u_i - \rho[t\, \dot{u}_i(x,0) + u_i(x,0)]. \tag{10}$$

Using the initial conditions (5), the above relation can be written in the form

$$g * t_{ij,j} + f_i = \rho u_i, \tag{11}$$

where

$$f_i = g * F_i + \rho(t\, b_i + a_i). \tag{12}$$

We consider the body subjected to two different systems of elastic loadings

$$L^{(\alpha)} = \{F_i^{(\alpha)}, t_i^{(\alpha)}, a_i^{(\alpha)}, b_i^{(\alpha)}\}, \quad (\alpha = 1,2). \tag{13}$$

We denote by $u_i^{(\alpha)}, e_{ij}^{(\alpha)}, t_{ij}^{(\alpha)}$ the corresponding components of the displacement vector, strain tensor and stress tensor, respectively.

We introduce the notations

$$f_i^{(\alpha)} = g * F_i^{(\alpha)} + \rho(t\, b_i^{(\alpha)} + a_i^{(\alpha)}), \quad t_i^{(\alpha)} = t_{ij}^{(\alpha)} n_j. \tag{14}$$

On the basis of relations (4), (8), from (2) we get

$$t_{ij}^{(1)} * e_{ij}^{(2)} = t_{ij}^{(2)} * e_{ij}^{(1)}. \tag{15}$$

If we introduce the notation

$$L_{\alpha\beta} = \int_V g * t_{ij}^{(\alpha)} * e_{ij}^{(\beta)}\, dx, \tag{16}$$

from (15) we obtain

$$L_{12} = L_{21} \tag{17}$$

Using the relations (3), (6), (8), (11), (14) and the divergence theorem, we get

$$\begin{aligned} L_{\alpha\beta} &= \int_V g * t_{ij}^{(\alpha)} * e_{ij}^{(\beta)}\, dx = \int_V g * t_{ij}^{(\alpha)} * u_{i,j}^{(\alpha)}\, dx = \int_V [* g\, t_{ij}^{(\alpha)} * u_i^{(\beta)}], \\ &{}_j\, dx - \int_V g * t_{ij,j}^{(\alpha)} * u_i^{(\beta)}\, dx = \int_\Sigma g * t_i^{(\alpha)} * u_i^{(\alpha)}\, dx - \int_V [\rho u_i^{(\alpha)} - f_i^{(\alpha)}] * u_i^{(\beta)}\, dx. \end{aligned} \tag{18}$$

From (8), (17), (18), we obtain the reciprocity relation

$$\int_V f_i^{(1)} * u_i^{(2)}\, dx + \int_\Sigma g * t_i^{(1)} * u_i^{(2)}\, dx = \int_V f_i^{(2)} * u_i^{(1)}\, dx + \int_\Sigma g * t_i^{(2)} * u_i^{(1)}\, dx. \tag{19}$$

If we assume homogeneous initial conditions

$$a_i^{(\alpha)} = b_i^{(\alpha)} = 0, \quad (\alpha = 1,2),$$

from (14) we get

$$f_i^{(\alpha)} = g * F_i^{(\alpha)}.$$

Using (8) in this case, the reciprocity relation (19) becomes

$$\int_V F_i^{(1)} * u_i^{(2)}\, dx + \int_\Sigma t_i^{(1)} * u_i^{(2)}\, dx = \int_V F_i^{(2)} * u_i^{(1)}\, dx + \int_\Sigma t_i^{(2)} * u_i^{(1)}\, dx. \tag{20}$$

From (19) we can obtain Somigliana's relations.

REFERENCES

1. Graffi D. — *Sul teorema di reciprocità nella dinamica dei corpi elastici*, Mem. Accad. Sci., Bologna, 10(1946/47), p. 4.
2. Ieşan D. — *Sur la théorie de la thermoélasticité micropolaire couplée*, C.R. Acad. Sc. Paris, t. 265 A (1967), p. 271.
3. Mikusinski J. — *Operational calculus*, Pergamon, New York, 1959.
4. Ignaczak J — *A completeness problem for stress equations of motion in the linear elasticity theory*, Arch. Mech. Stos., 15(1963), p. 225.

ASUPRA TEOREMEI DE RECIPROCITATE ÎN TEORIA DINAMICĂ A ELASTICITĂŢII LINIARE

Rezumat

Se consideră ecuaţiile elasticităţii liniare în cazul dinamic pentru medii omogene şi anizotrope. Folosind metoda dată în [2] este stabilită teorema de reciprocitate fără a apela la transformata Laplace, extinzînd teorema dată în [1].

ON A BASIC THEOREM IN MINIMAL COST FLOWS

BY

IRINEL DRAGAN

Let $N = (V, A)$ be a directed network with m vertices and n arcs; denote these elements by $v_i \in V$, $(i = 1, \cdots, m)$, and $a_l \in A$, $(l = 1, \cdots, n)$, respectively. Suppose given a capacity n-vector $K \geqslant 0$, an unitar transportation costs n-vector $C \geqslant 0$ and a demand m-vector d. Then, we can state the minimal cost flow problem as: minimize

$$C(X) = C'X, \tag{1}$$

subject to

$$BX = d. \tag{2}$$

and

$$0 \leqslant X \leqslant K, \tag{3}$$

where B is the incidence matrix of the network. Of course, this is the simple case of proportional costs.

In the following we shall suppose that our network has only one source v_1 and one sink v_m, in consequence the demand vector has the form $d = (v, 0, \cdots, 0, -v)$. A vector X satisfying (2), (3), with such a vector d, will be called as habitually a flow with value v. We shall suppose also integer capacities, costs and demands.

Let us denote by $C(\mu)$ the unitar cost of a cycle μ, i. e.

$$C(\mu) = C'\mu, \tag{4}$$

where μ represents the corresponding vector cycle. A cycle μ will be called admissible cycle with respect to the flow X, when there exists at least a positive integer ε such that $\bar{X} = X + \varepsilon\mu$ be also a flow in the network. Now, we can state the central theorem in minimal cost flows: