3. Charnes A.-Cooper W. — *Management Models and Industrial Application of Linear Programming*, I—II. John Wiley, New York, 1961.
4. Kolumban I. — *Über die Characterisierung der Lösungen eines optimalen Approximationsproblems*, Mathematica (Cluj). 10(33), 2, 1968, 285—291.
5. Maruşciac I. — Rădulescu M. — *Un problème de la programmation quadratique à plusieurs fonctions*. Studia Univ. Babeş—Bolyai, 1, 1970, 81—89.
6. Maruşciac I. — Rădulescu M. — *Un problème général de la programmation linéaire à plusieurs fonctions économiques*. Studia Univ. Babeş—Bolyai, 2, (1970), 55—65
7. Ремез Е. Я., Штейнберг А. С. - *Про деякі екстремальні задачі, узагальненого-чебишевского типу та про метод зрівняльних спусків*. Доповіді АН УРСС, 8 (1961), стр, 983 — 987.
8. Зуховицкий С. И. - *Алгоритм для решения одной обобщенной задачи линейного программирования*. ДАН СССР, 133 № 1 (1960).
9. Зуховицкий С. И., Авдеева Л. И. - *Линейное и выпуклое программирование*. Москва, 1964.

ASUPRA MULȚIMII DE PUNCTE EFICIENTE A UNEI PROBLEME DE PROGRAMARE MATEMATICĂ

Rezumat

În lucrare se studiază o problemă generală de programare matematică în care se minimizează simultan mai multe funcții de scop pe un domeniu convex din spațiul euclidian $\mathbf{R}^n$. Astfel se consideră o vector-funcție $f = f(x)$ în care x și y sînt vectori respectiv cu n și p componente: $x = (x_1,\dots,x_n)$ $y = (y_1,\dots,y_p)$, unde $y_j = f_j(x)$ sînt funcții convexe și diferențiabile în domeniul $\Omega = \{x \in \mathbf{R}^n \mid Ax \leqslant b\}$.

Introducînd în mulțimea $y = \{y \in \mathbf{R}^p \mid y = f(x), x \in \Omega\}$ relația de ordine naturală (pe componente), un punct $x^0 \in \Omega$ se numește eficient în raport cu Y dacă nu există nici un punct $x \in \Omega$ astfel ca $y = f(x) < y^0 = f(x^0)$. După ce se arată legătura dintre această problemă și alte probleme de programare matematică în care intervin mai multe funcții de scop, se dau criterii de eficiență care permit apoi o clasificare a punctelor de eficiență. Astfel se obțin așa-numitele curbe eficiente și suprafețe de eficiență de diferite ordine. Se dă și o metodă care permite determinarea efectivă a curbelor și suprafețelor eficiente. În partea a doua a lucrării se dă un algoritm convergent care dă posibilitatea găsirii întotdeauna a unui punct de eficiență în raport cu Y. Pentru ilustrare se prezintă exemple concrete care sînt apoi rezolvate complet cu metodele propuse în lucrare.

OSCILLATORY HYDROMAGNETIC FREE CONVECTION FLOW PAST AN INFINITE VERTICAL FLAT PLATE WITH VARIABLE SUCTION

BY

SHISHIR KUMAR DUBE

1. Introduction. Singh [1] studied the effect of a constant horizontal magnetic field on one-dimensional unsteady laminar free convection flow of viscous incompressible and electrically conducting fluids past a vertical infinite flat plate on which the wall temperature or surface heat flux varies as some powers of time. Recently Ballabh and Singh [2] considered the unsteady free convection flow of an incompressible viscous fluid past a vertical infinite porous flat plate with time dependent arbitrary surface temperature or heat flux variations when suction velocity is uniform.

In the present paper an attempt has been made to study the effect of the magnetic field and oscillatory suction upon the free convection flow of an incompressible viscous electrically conducting fluid, when the plate temperature fluctuates only in magnitude but not in direction. Moreover, the surface temperature is taken to be fluctuating with the same frequency as that of the suction. The magnetic field of strength B_0' is imposed transverse to the plate. Similar forced or free convection problems have been considered by Pop [3] and Krishna Lal [4].

2. Analysis. With x'-axis along the plate and y'-axis normal to it, the time dependent equations which describe the unsteady hydromagnetic laminar free convection flow past a vertical infinite flat plate are [1]

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\theta' - \frac{\sigma B_0'^2}{\rho} u' \tag{2.1}$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} \tag{2.2}$$

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.3}$$

$$\frac{\partial \theta'}{\partial t'} + v'\frac{\partial \theta'}{\partial y'} = \frac{\nu}{\mathrm{Pr}}\frac{\partial^2 \theta'}{\partial y'^2}. \tag{2.4}$$

Here g is the gravitational acceleration, u' the velocity along the plate, t the time, p the pressure, σ the electrical conductivity of the fluid, Pr the Prandtl number, ρ the density, ν the kinematic viscosity, β the expansivity; θ is the difference between the local temperature and that at a very large distance from the plate, where temperature, T, is constant and equal to T_∞, i.e., $\theta = T - T_\infty$.

Following Singh [1], it is assumed that the induced magnetic field is negligible compared with the imposed magnetic field. Viscous dissipation and Joule heating effect have also been neglected in writting the above equations. Throughout this paper the rationalized MKS units have been used.

Since the suction velocity oscillates only in magnitude but not in direction we take

$$v' = -v_0'(1 + \varepsilon A e^{i\omega' t'}), \tag{2.5}$$

where v_0' is always positive and ε and εA are less than unity.

We introduce the non-dimensional quantities defined by

$$\begin{cases} y = \dfrac{y' v_0'}{\nu}, \quad t = \dfrac{v_0'^2 t'}{4\nu}, \quad \omega = \dfrac{4\nu\omega'}{v_0'^2}, \\[2ex] u = \dfrac{u'}{v_0'}, \quad G = \dfrac{g\beta\theta'\nu}{v_0'^3} \quad \text{and} \quad M = \dfrac{4\nu\sigma' B_0'^2}{\rho' v_0'^2}. \end{cases} \tag{2.6}$$

With (2.5) and (2.6) equations (2.1) and (2.4) takes the non-dimensional form

$$\frac{\partial^2 u}{\partial y^2} + (1 + \varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} - \frac{M}{4}u - \frac{1}{4}\frac{\partial u}{\partial t} = -G, \tag{2.7}$$

$$\frac{\partial^2 G}{\partial y^2} + \mathrm{Pr}\,(1 + \varepsilon A e^{i\omega t})\frac{\partial G}{\partial y} - \frac{\mathrm{Pr}}{4}\frac{\partial G}{\partial t} = 0, \tag{2.8}$$

with the boundary conditions

$$\begin{cases} y = 0: \quad u = 0, \quad G = (1 + \varepsilon e^{i\omega t}) = G_0(t) \\ y = \infty: \quad u = 0, \quad G = 0. \end{cases} \tag{2.9}$$

3. Solution of equations (2.7) and (2.8) when $G_0(t) = (1 + \varepsilon e^{i\omega t})$. It is clearly seen from the energy equation (2.8) that the temperature is not affected by the magnetic field.

Putting

$$G = f_1(y) + \varepsilon e^{i\omega t} f_2(y) \tag{3.1}$$

in (2.8) and comparing harmonic terms, neglecting coefficients of ε^2, we get the following systems for the determination of functions f_1 and f_2:

$$\frac{d^2 f_1}{dy^2} + \mathrm{Pr}\frac{df_1}{dy} = 0, \tag{3.2}$$

$$\frac{d^2 f_2}{dy^2} + \mathrm{Pr}\frac{df_2}{dy} - \frac{\mathrm{Pr}\, i\omega}{4} f_2 = -\mathrm{Pr}\, A\frac{df_1}{dy} \tag{3.3}$$

subjected to the boundary conditions

$$\begin{cases} f_1(0) = 1 = f_2(0), \\ f_1(\infty) = 0 = f_2(\infty). \end{cases} \tag{3.4}$$

The solution of equations (3.2) and (3.3), with the boundary conditions (3.4) is:

$$f_1(y) = \exp(-\mathrm{Pr}\, y), \quad f_2(y) = h\exp(-\mathrm{Pr}\, y) + (1 - h)\exp(-\beta y), \tag{3.5}$$

where

$$h = \frac{4Ai\,\mathrm{Pr}}{\omega}, \quad \beta = \frac{1}{2}\left(\mathrm{Pr} + \sqrt{\mathrm{Pr}^2 + i\mathrm{Pr}\,\omega}\right).$$

Then the non-dimensional temperature is given by

$$G = \exp(-\mathrm{Pr}\, y) + \varepsilon e^{i\omega t}\{h\exp(-\mathrm{Pr}\, y) + (1 - h)\exp(-\beta y)\}. \tag{3.6}$$

Again substituting relations (3.6) and $u = F_1(y) + \varepsilon e^{i\omega t} F_2(y)$ in (2.7) and comparing harmonic terms, neglecting coefficients of ε^2, we get the following system for determination of functions F_1 and F_2:

$$\begin{cases} \dfrac{d^2F_1}{dy^2} + \dfrac{dF_1}{dy} - \dfrac{M}{4}F_1 = -\exp(-\mathrm{Pr}\, y), \\[2ex] \dfrac{d^2F_2}{dy^2} + \dfrac{dF_2}{dy} - \dfrac{(M + i\omega)}{4}F_2 = -A\dfrac{dF_1}{dy} - h\exp(-\mathrm{Pr}\, y) - \\ \qquad\qquad - (1 - h)\exp(-\beta y). \end{cases} \tag{3.7}$$

The boundary conditions are:

$$\begin{cases} F_1(0) = 0 = F_2(0); \\ F_1(\infty) = 0 = F_2(\infty). \end{cases} \tag{3.8}$$

The solution of systems (3.7), with the boundary conditions (3.8) is:

$$
(3.9)\quad
\begin{cases}
F_1(y) = \dfrac{(e^{-ny} - e^{-\mathrm{Pr}\,y})}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)}, \\[2ex]
F_2(y) = \dfrac{(e^{-n_1 y} - e^{-\mathrm{Pr}\,y})}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M + i\omega}{4}\right)} \left\{ h + \dfrac{A\mathrm{Pr}}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} \right\} + \\[2ex]
\quad + \dfrac{(1-h)(e^{-n_1 y} - e^{-\beta y})}{\left(\beta^2 - \beta - \dfrac{M + i\omega}{4}\right)} - \dfrac{4Ain(e^{-n_1 y} - e^{-ny})}{\omega\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)}.
\end{cases}
$$

where

$$
n = \frac{1}{2}\left(1 + \sqrt{1 + M}\right), \quad n_1 = \frac{1}{2}\left(1 + \sqrt{1 + M + i\omega}\right).
$$

Then the velocity of hydromagnetic flow is given by

$$
(3.10)\quad
\begin{cases}
u = \dfrac{(e^{-ny} - e^{-\mathrm{Pr}\,y})}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} + \\[2ex]
\quad + \varepsilon e^{i\omega t} \left[\dfrac{(e^{-n_1 y} - e^{-\mathrm{Pr}\,y})}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M + i\omega}{4}\right)} \left\{ h + \dfrac{A\mathrm{Pr}}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} \right\} + \right. \\[2ex]
\quad \left. + \dfrac{(1-h)(e^{-n_1 y} - e^{-\beta y})}{\left(\beta^2 - \beta - \dfrac{M + i\omega}{4}\right)} - \dfrac{4Ain(e^{-n_1 y} - e^{-ny})}{\omega\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} \right].
\end{cases}
$$

Taking real part of (3.6) and (3.10), we get

$$
(3.11)\quad
\begin{cases}
G = \left(1 - \dfrac{4A\varepsilon\mathrm{Pr}}{\omega} \sin \omega t\right) \exp(-\mathrm{Pr}\,y) + \\[2ex]
\quad + \varepsilon \exp\left\{ -\left(\dfrac{\mathrm{Pr}}{2} + \dfrac{r_1}{2} \cos \dfrac{\theta}{2}\right) y \right\} \cos\left(\omega t - \dfrac{r_1 y}{2} \sin \dfrac{\theta}{2}\right) + \\[2ex]
\quad + \dfrac{4A\varepsilon\mathrm{Pr}}{\omega} \exp\left\{ -\left(\dfrac{\mathrm{Pr}}{2} + \dfrac{r_1}{2} \cos \dfrac{\theta}{2}\right) y \right\} \times \sin\left(\omega t - \dfrac{r_1 y}{2} \sin \dfrac{\theta}{2}\right).
\end{cases}
$$

and

$$
(3.12)\quad
\begin{aligned}
u = {} & \frac{(e^{-ny} - e^{-\mathrm{Pr}\,y})}{\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} + \frac{4A\varepsilon}{\omega\left(\mathrm{Pr}^2 - \mathrm{Pr} - \dfrac{M}{4}\right)} \times \\
& \times \left[\sin \omega t \{ \mathrm{Pr}\, e^{-\mathrm{Pr}\,y} - n e^{-ny} \} - (\mathrm{Pr} - n) \exp\left\{ -\frac{y}{2}\left(1 + r_2 \cos \frac{\theta_1}{2}\right) \right\} \times \right. \\
& \left. \times \sin\left(\omega t - \frac{r_2 y}{2} \sin \frac{\theta_1}{2}\right) \right] + \frac{\varepsilon}{\Delta_1}\left(T + \frac{4A\mathrm{Pr}S}{\omega}\right) \times \\
& \times \left[\exp\left\{ -\frac{y}{2}\left(1 + r_2 \cos \frac{\theta_1}{2}\right) \right\} \sin\left(\omega t - \frac{r_2 y}{2} \sin \frac{\theta_1}{2}\right) - \right. \\
& \left. - \exp\left\{ -\frac{y}{2}\left(\mathrm{Pr} + r_1 \cos \frac{\theta}{2}\right) \right\} \sin\left(\omega t - \frac{r_1 y}{2} \sin \frac{\theta}{2}\right) \right] + \\
& + \frac{\varepsilon}{\Delta_1}\left(S - \frac{4A\mathrm{Pr}T}{\omega}\right) \left[\exp\left\{ -\frac{y}{2}\left(1 + r_2 \cos \frac{\theta_1}{2}\right) \right\} \times \right. \\
& \times \cos\left(\omega t - \frac{r_2 y}{2} \sin \frac{\theta_1}{2}\right) - \exp\left\{ -\frac{y}{2}\left(\mathrm{Pr} + r_1 \cos \frac{\theta}{2}\right) \right\} \times \\
& \left. \times \cos\left(\omega t - \frac{r_1 y}{2} \sin \frac{\theta}{2}\right) \right],
\end{aligned}
$$

where

$$
r_1 = \mathrm{Pr}^{1/2} (\omega^2 + \mathrm{Pr}^2)^{1/4}, \quad \theta = \tan^{-1} \frac{\omega}{\mathrm{Pr}},
$$

$$
r_2 = \{ \omega^2 + (1 + M)^2 \}^{1/4}, \quad \theta_1 = \tan^{-1} \frac{\omega}{1 + M},
$$

$$
\Delta_1 = \left\{ \frac{\mathrm{Pr}(\mathrm{Pr} - 2)}{4} + \frac{(r_1^2 \cos \theta - M)}{4} + \frac{r_1 \cos \dfrac{\theta}{2} (2\mathrm{Pr} - 1)}{2} \right\}^2 + \left\{ \frac{(r_1^2 \sin \theta - \omega)}{4} + \frac{r_1 \sin \dfrac{\theta}{2} (2\mathrm{Pr} - 1)}{2} \right\}^2,
$$

$$
S = \left\{ \frac{(\mathrm{Pr}(\mathrm{Pr} - 2)}{2} + \frac{(r_1^2 \cos \theta - M)}{4} + \frac{r_1 \cos \dfrac{\theta}{2} (2\mathrm{Pr} - 1)}{2} \right\},
$$

$$
T = \left\{ \frac{(r_1^2 \sin \theta - \omega)}{4} + \frac{r_1 \sin \dfrac{\theta}{2} (2\mathrm{Pr} - 1)}{2} \right\}.
$$

The rate of heat transfer at the plate is given by

$$Q=\frac{g\beta\nu^2 q'}{kv_0'^4}=\Pr\left(1+\frac{1}{2}\varepsilon\cos\omega t\right)-\frac{2A\varepsilon\Pr^2}{\omega}\sin\omega t+ \tag{3.13}$$

$$+\frac{\varepsilon r_1}{2}\cos\left(\omega t+\frac{\theta}{2}\right)+\frac{2A\varepsilon\Pr r_1}{\omega}\sin\left(\omega t+\frac{\theta}{2}\right),$$

where

$$q'=-k\left(\frac{\partial\theta'}{\partial y'}\right)_{y'=0},$$

and the skin friction coefficient at the plate is given by

$$\tau_\omega^*=\frac{\tau\omega}{\rho v_0^2}=\frac{(\Pr-n)}{\left(\Pr^2-\Pr-\frac{M}{4}\right)}\left|1+\frac{2A\varepsilon}{\omega}\left(\{1-2(\Pr+n)\}\times\right.\right.$$

$$\left.\left.\times\sin\omega t+r_2\sin\left(\omega t+\frac{\theta_1}{2}\right)\right|+\frac{\varepsilon}{2\Delta_1}\left(T+\frac{4A\Pr S}{\omega}\right)\left[(\Pr-1)\sin\omega t+\right.$$

$$\left.+r_1\sin\left(\omega t+\frac{\theta}{2}\right)-r_2\sin\left(\omega t+\frac{\theta_1}{2}\right)\right]+ \tag{3.14}$$

$$+\frac{\varepsilon}{2\Delta_1}\left(S-\frac{4A\Pr T}{\omega}\right)\left[(\Pr-1)\cos\omega t+\right.$$

$$\left.+r_1\cos\left(\omega t+\frac{\theta}{2}\right)-r_2\cos\left(\omega t+\frac{\theta_1}{2}\right)\right].$$

4. Discussion. Velocity profile for different values of parameters A and M are represented in fig. 1. Velocity increases with the increasing suction velocity while it decreases as the magnetic field strength M increases

Fig. 2 shows the variation of dimensionless temperature with y for different values of A. It is found that temperature decreases with increasing suction (with increasing the constant A).

The variation of skin friction at the plate with A, for different values of M and t, is shown in fig. 3. Fig. 4 gives the variation of non-dimensional rate of heat transfer at the plate with A for $\omega t=\pi/2$ and π.

It is seen that skin friction decreases as M increases. The effect of constant suction is first to damp the skin friction but as A increases, the effect of suction becomes to increase the skin friction. Further, we see that an increase in the time causes, a dampness in that part of skin friction expression which is due to the oscillatory part of the variable suction.

Hence it is seen that when $A=0$ the effect of time is to increase the skin friction but when we include the oscillatory part of suction the effect of time becomes to damp the skin friction.

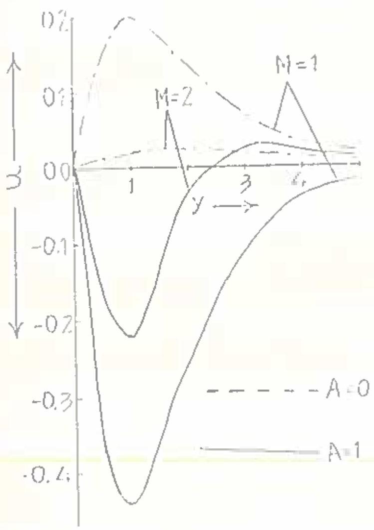

Fig. 1 — Velocity profile vs y

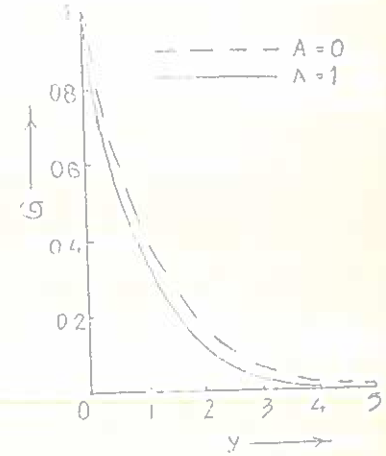

Fig. 2 — Temperature profile vs y

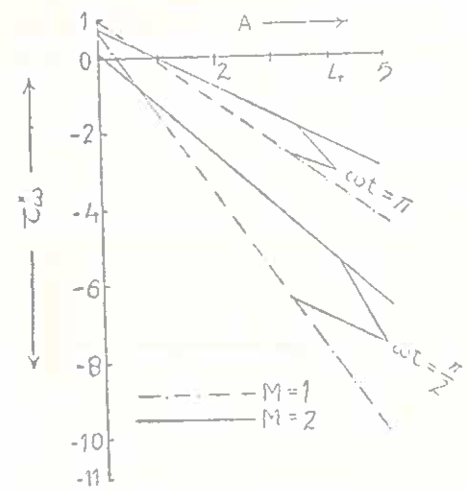

Fig. 3 — Skin friction vs. A

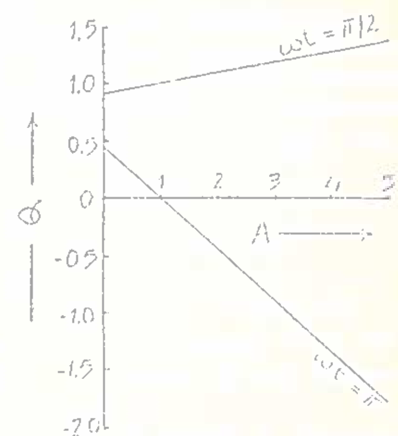

Fig. 4 — Rate of heat transfer at the plate vs A

From fig. 4 it is seen that as time increases i.e., ωt increases, the value of that part of Q, which is due to oscillatory part of the suction increases numerically. It is also seen that as time increases, the rate of heat transfer at the plate, first decreases and then increases for large values of A. Further the effect of suction is to increase the rate of heat transfer at the plate when $\omega t = \pi/2$. When $\omega t = \pi$ it is seen that as A increases, the effect of suction is first to damp Q and then to increase it.

Acknowledgements. The author has a debt of gratitude to Professor Ram Ballabh for valuable encouragement and guidance to him in connection with the present work. He is also indebted to the State Council of Scientific and Industrial Research, Uttar Pradesh for giving him financial assistance.

Department of Mathematics,
Lucknow University,
Lucknow (INDIA)

REFERENCES

1. Singh, Dilip — J. Phys. Soc. Japan, 19, (1964), 751.
2. Ballabh, R. and Dilip Singh — Ganita, 18, (1967), 39.
3. Pop, I. — J. Bul. Inst. polit. din Iaşi, 3—4, (1967), 173.
4. Lal, Krishna — Trans ASME Series E J. Appl. Mech. 36, (1969), 327.

CURGEREA HIDROMAGNETICĂ OSCILATORIE, LIBERĂ DE CONVECŢIE PESTE O PLACĂ INFINITĂ, CU SUCŢIUNE VARIABILĂ

Rezumat

Se consideră curgerea liberă de convecţie a unui fluid vîscos, incompresibil şi electric conductor peste o placă poroasă infinită verticală, în prezenţa unui cîmp magnetic orizontal constant, cînd viteza de sucţiuni are aceeaşi frecvenţă ca temperatura de suprafaţă.

CASCADES OF LINEAR BOOLEAN AUTOMATA

BY

DAN A. SIMOVICI and CORINA REISCHER

I. Introduction. The notion of Linear Boolean Automaton (LBA) was introduced and studied in [5], [6].

We shall recall some of the main notations.

A LBA is a 5-tuple $\mathcal{A} = (\mathcal{I}, \mathcal{S}, \mathcal{O}, f, g)$, where $\mathcal{I}$ (the set of inputs) is the Boolean matrix algebra $\mathcal{B}_2^{(l,1)}$, $\mathcal{S}$ (the set of states) is the Boolean matrix algebra $\mathcal{B}_2^{(n,1)}$ and $\mathcal{O}$ (the set of outputs) is the algebra of the same type $\mathcal{B}_2^{(m,1)}$. Regarding the next-state function $f: \mathcal{S} \times \mathcal{I} \to \mathcal{S}$, which gives $\mathbf{s}(t+1) = f(\mathbf{s}(t), \mathbf{u}(t))$ and the output function $g: \mathcal{S} \times \mathcal{I} \to \mathcal{O}$, which gives $\mathbf{y}(t) = g(\mathbf{s}(t), \mathbf{u}(t))$, we have

$$\mathbf{s}(t+1) = \mathbf{A} \times \mathbf{s}(t) \cup \mathbf{B} \times \mathbf{u}(t), \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C} \times \mathbf{s}(t) \cup \mathbf{D} \times \mathbf{u}(t), \tag{2}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are the characteristic matrices of the automaton. We shall denote the automaton $\mathcal{A}$ by

$$\mathcal{A} = < \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} >.$$

Now, we shall study the cascade grouping of LBA.

II. The cascade grouping of LBA. Let $\mathcal{A}_1 = (\mathcal{B}_2^{(l,1)}, \mathcal{B}_2^{(n,1)}, \mathcal{B}_2^{(m,1)}, f_1, g_1)$, $\mathcal{A}_2 = (\mathcal{B}_2^{(m,1)}, \mathcal{B}_2^{(p,1)}, \mathcal{B}_2^{(q,1)}, f_2, g_2)$ two LBA. We suppose that the outputs of the first automaton are considered as inputs of the second one (see fig. 1).

The aim of this paper is to study the system which is obtained in this manner and also, to study the set of LBA endowed with this operation of „cascade grouping".