From fig. 4 it is seen that as time increases i.e., ωt increases, the value of that part of Q, which is due to oscillatory part of the suction increases numerically. It is also seen that as time increases, the rate of heat transfer at the plate, first decreases and then increases for large values of A. Further the effect of suction is to increase the rate of heat transfer at the plate when $\omega t = \pi/2$. When $\omega t = \pi$ it is seen that as A increases, the effect of suction is first to damp Q and then to increase it.

Acknowledgements. The author has a debt of gratitude to Professor Ram Ballabh for valuable encouragement and guidance to him in connection with the present work. He is also indebted to the State Council of Scientific and Industrial Research, Uttar Pradesh for giving him financial assistance.

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CURGEREA HIDROMAGNETICĂ OSCILATORIE, LIBERĂ DE CONVECŢIE PESTE O PLACĂ INFINITĂ, CU SUCŢIUNE VARIABILĂ

Rezumat

Se consideră curgerea liberă de convecţie a unui fluid vîscos, incompresibil şi electric conductor peste o placă poroasă infinită verticală, în prezenţa unui cîmp magnetic orizontal constant, cînd viteza de sucţiuni are aceeaşi frecvenţă ca temperatura de suprafaţă.

CASCADES OF LINEAR BOOLEAN AUTOMATA

BY

DAN A. SIMOVICI and CORINA REISCHER

I. Introduction. The notion of Linear Boolean Automaton (LBA) was introduced and studied in [5], [6].

We shall recall some of the main notations.

A LBA is a 5-tuple $\mathcal{A} = (\mathcal{J}, \mathcal{S}, \mathcal{O}, f, g)$, where $\mathcal{J}$ (the set of inputs) is the Boolean matrix algebra $\mathcal{B}_2^{(l,1)}$, $\mathcal{S}$ (the set of states) is the Boolean matrix algebra $\mathcal{B}_2^{(n,1)}$ and $\mathcal{O}$ (the set of outputs) is the algebra of the same type $\mathcal{B}_2^{(m,1)}$. Regarding the next-state function $f: \mathcal{S} \times \mathcal{J} \to \mathcal{S}$, which gives $\mathbf{s}(t+1) = f(\mathbf{s}(t), \mathbf{u}(t))$ and the output function $g: \mathcal{S} \times \mathcal{J} \to \mathcal{O}$, which gives $\mathbf{y}(t) = g(\mathbf{s}(t), \mathbf{u}(t))$, we have

$$\mathbf{s}(t+1) = \mathbf{A} \times \mathbf{s}(t) \cup \mathbf{B} \times \mathbf{u}(t), \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C} \times \mathbf{s}(t) \cup \mathbf{D} \times \mathbf{u}(t), \tag{2}$$

where **A**, **B**, **C**, **D** are the characteristic matrices of the automaton. We shall denote the automaton $\mathcal{A}$ by

$$\mathcal{A} = < \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} >.$$

Now, we shall study the cascade grouping of LBA.

II. The cascade grouping of LBA. Let $\mathcal{A}_1 = (\mathcal{B}_2^{(l,1)}, \mathcal{B}_2^{(n,1)}, \mathcal{B}_2^{(m,1)}, f_1, g_1)$, $\mathcal{A}_2 = (\mathcal{B}_2^{(m,1)}, \mathcal{B}_2^{(p,1)}, \mathcal{B}_2^{(r,1)}, f_2, g_2)$ two LBA. We suppose that the outputs of the first automaton are considered as inputs of the second one (see fig. 1).

The aim of this paper is to study the system which is obtained in this manner and also, to study the set of LBA endowed with this operation of „cascade grouping".

For, let us consider

$$\mathcal{A}_1 = \langle \mathbf{A}_1, \mathbf{B}_1, \mathbf{C}_1, \mathbf{D}_1 \rangle, \mathcal{A}_2 = \langle \mathbf{A}_2, \mathbf{B}_2, \mathbf{C}_2, \mathbf{D}_2 \rangle,$$

where

$$\mathbf{A}_1 \in \mathcal{B}_2^{(n,n)}, \ \mathbf{B}_1 \in \mathcal{B}_2^{(n,l)}, \ \mathbf{C}_1 \in \mathcal{B}_2^{(m,n)}, \ \mathbf{D}_1 \in \mathcal{B}_2^{(m,l)},$$

$$\mathbf{A}_2 \in \mathcal{B}_2^{(p,p)}, \ \mathbf{B}_2 \in \mathcal{B}_2^{(p,m)}, \ \mathbf{C}_2 \in \mathcal{B}_2^{(q,p)}, \ \mathbf{D}_2 \in \mathcal{B}_2^{(q,m)}.$$

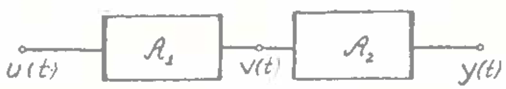

Fig. 1

Hence, the behaviour of these two automaton will be described by the equations:

$$\mathbf{s}_1(t+1) = \mathbf{A}_1 \times \mathbf{s}_1(t) \cup \mathbf{B}_1 \times \mathbf{u}(t), \tag{1'}$$

$$\mathbf{v}(t) = \mathbf{C}_1 \times \mathbf{s}_1(t) \cup \mathbf{D}_1 \times \mathbf{u}(t), \tag{2'}$$

and

$$\mathbf{s}_2(t+1) = \mathbf{A}_2 \times \mathbf{s}_2(t) \cup \mathbf{B}_2 \times \mathbf{v}(t), \tag{1''}$$

$$\mathbf{y}(t) = \mathbf{C}_2 \times \mathbf{s}_2(t) \cup \mathbf{D}_2 \times \mathbf{v}(t). \tag{2''}$$

It is natural to consider the set of states of the system formed by coupling the automata $\mathcal{A}_1$ and $\mathcal{A}_2$ as the cartesian product $\mathcal{B}_2^{(n,1)} \times \mathcal{B}_2^{(p,1)} \simeq \simeq \mathcal{B}_2^{(n+p,1)}$, whose elements will be denoted by $\begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix} \in \mathcal{B}_2^{(n+p,1)}$, where $\mathbf{s}_1 \in \mathcal{B}_2^{(n,1)}$ and $\mathbf{s}_2 \in \mathcal{B}_2^{(p,1)}$.

We can write

$$\mathbf{s}_1(t+1) = (\mathbf{A}_1 : \mathbf{O}) \times \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix}(t) \cup \mathbf{B}_1 \times \mathbf{u}(t), \tag{3}$$

where $\mathbf{O} \in \mathcal{B}_2^{(n,p)}$ is a null matrix, and also

$$\begin{aligned} \mathbf{s}_2(t+1) &= \mathbf{A}_2 \times \mathbf{s}_2(t) \cup \mathbf{B}_2 \times (\mathbf{C}_1 \times \mathbf{s}_1(t) \cup \mathbf{D}_1 \times \mathbf{u}(t)) = \\ &= \mathbf{A}_2 \times \mathbf{s}_2(t) \cup \mathbf{B}_2 \times \mathbf{C}_1 \times \mathbf{s}_1(t) \cup \mathbf{B}_2 \times \mathbf{D}_1 \times \mathbf{u}(t) = \\ &= (\mathbf{B}_2 \times \mathbf{C}_1 : \mathbf{A}_2) \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix} \cup \mathbf{B}_2 \times \mathbf{D}_2 \times \mathbf{u}(t). \end{aligned} \tag{4}$$

The last two equations can by summarized into the following equality:

$$\begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix}(t+1) = \begin{pmatrix} \mathbf{A}_1 & \mathbf{O} \\ \mathbf{B}_2 \times \mathbf{C}_1 & \mathbf{A}_2 \end{pmatrix} \times \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix}(t) \cup \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \times \mathbf{D}_1 \end{pmatrix} \times \mathbf{u}(t). \tag{5}$$

Afterwards:

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{C}_2 \times \mathbf{s}_2(t) \cup \mathbf{D}_2 \times (\mathbf{C}_1 \times \mathbf{s}_1(t) \cup \mathbf{D}_1 \times \mathbf{u}(t)) = \\ &= (\mathbf{D}_2 \times \mathbf{C}_1 : \mathbf{C}_2) \times \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix}(t) \cup \mathbf{D}_2 \times \mathbf{D}_1 \times \mathbf{u}(t). \end{aligned} \tag{6}$$

By examining the last two equations we conclude that the „cascade" formed by the automata $\mathcal{A}_1$, $\mathcal{A}_2$ behaves exactly as a LBA. Denoting this automaton by $\mathcal{A}_1 \to \mathcal{A}_2$, we have:

$$\mathcal{A}_1 \to \mathcal{A}_2 = \left\langle \begin{pmatrix} \mathbf{A}_1 & \mathbf{O} \\ \mathbf{B}_2 \times \mathbf{C}_1 & \mathbf{A}_2 \end{pmatrix}, \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \times \mathbf{D}_1 \end{pmatrix}, (\mathbf{D}_2 \times \mathbf{C}_1 : \mathbf{C}_2), \quad \mathbf{D}_2 \times \mathbf{D}_1 \right\rangle. \tag{7}$$

Theorem 1. *The set of LBA can be considered as a partial semi-group*) if the cascade coupling, „→" is considered as the semigroup operation.*

Proof. We need only to prove the associativity of the operation. For, let $\mathcal{A}_i = \langle \mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i \rangle$, $i = 1, 2, 3$, three composable automata (under the operation „→"). Then

$$(\mathcal{A}_1 \to \mathcal{A}_2) \to \mathcal{A}_3 = \left\langle \begin{pmatrix} \mathbf{A}_1 & \mathbf{O} & \mathbf{O} \\ \mathbf{B}_2 \times \mathbf{C}_1 & \mathbf{A}_2 & \mathbf{O} \\ \mathbf{B}_3 \times \mathbf{D}_2 \times \mathbf{C}_1 & \mathbf{B}_3 \times \mathbf{C}_2 & \mathbf{A}_3 \end{pmatrix}, \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \times \mathbf{D}_1 \\ \mathbf{B}_3 \times \mathbf{D}_2 \times \mathbf{D}_1 \end{pmatrix}, \right.$$

$$\left. (\mathbf{D}_3 \times \mathbf{D}_2 \times \mathbf{C}_1 : \mathbf{D}_3 \times \mathbf{C}_2 \ \mathbf{C}_3), \quad \mathbf{D}_3 \times \mathbf{D}_2 \times \mathbf{D}_1 \right\rangle = \mathcal{A}_1 \to (\mathcal{A}_2 \to \mathcal{A}_3).$$

III. The redundance index of the automaton $\mathcal{A}_1 \to \mathcal{A}_2$. The redundance index of a LBA is the number of null components of the vector $\zeta = \bigcup_{\omega \in \mathcal{Z}} \omega$, where $\mathcal{Z}$ is the null ideal of the LBA (see [5], [6]). It was proved in [6] that this number is the same with the number of null columns of the matrix $\mathbf{C} \times \hat{\mathbf{A}}$, where $\hat{\mathbf{A}} = \mathbf{E} \cup \mathbf{A} \cup \ldots \cup \mathbf{A}^e$ (also [6] th. 5). Here e is the characteristic exponent of the matrix $\mathbf{E} \cup \mathbf{A}$ (see [2], [3]).

Regarding the redundance index of the automaton $\mathcal{A}_1 \to \mathcal{A}_2$ we have:

Theorem 2. *The redundance index of the automaton $\mathcal{A}_1 \to \mathcal{A}_2$ is at least equal with the sum of the redundance indices of the automata $\mathcal{A}_1$ and $\mathcal{A}_2$.*

Proof. Let $\mathcal{A}_i = \langle \mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i \rangle$, $i = 1, 2$, and $\mathcal{A}_1 \to \mathcal{A}_2 = \langle \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} \rangle$.

*) A partial semigroup is a semigroup whose operation is not defined between any two elements.

We intend to find out an inferior bound for the number of null columns of the matrix $\mathbf{C}\times\widehat{\mathbf{A}}$. Because

$$\mathbf{A}=\begin{pmatrix}\mathbf{A}_1 & \mathbf{O}\\ \mathbf{B}_2\times\mathbf{C}_1 & \mathbf{A}_2\end{pmatrix}$$

is a quasitriangular matrix, its powers will be also quasitriangular matrices, namely:

$$\begin{pmatrix}\mathbf{A}_1 & \mathbf{O}\\ \mathbf{B}_2\times\mathbf{C}_1 & \mathbf{A}_2\end{pmatrix}^k=\begin{pmatrix}\mathbf{A}_1^k & \mathbf{O}\\ \mathbf{G}_k & \mathbf{A}_2^k\end{pmatrix}.$$

It is easy to see, regarding the new introduced matrix $\mathbf{G}_k$, we have the reccurence formula:

$$\mathbf{G}_{k+1}=\mathbf{B}_2\times\mathbf{C}_1\times\mathbf{A}_1^k\cup\mathbf{A}_2\times\mathbf{G}_k. \tag{8}$$

Applying repeatedly this relation we obtain:

$$\mathbf{G}_{k+1}=\mathbf{B}_2\times\mathbf{C}_1\times\mathbf{A}_1^k\cup\mathbf{A}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\mathbf{A}_1^{k-1}\cup\ldots\cup\mathbf{A}_2^m\times\mathbf{B}_2\times\mathbf{C}_1\times\mathbf{A}_1^{k-m}\cup\ldots$$

$$\ldots\cup\mathbf{A}_2^k\times\mathbf{B}_2\times\mathbf{C}_1=(\mathbf{A}_2^k\times\mathbf{B}_2:\ \mathbf{A}_2^{k-1}\times\mathbf{B}_2\ldots:\ \mathbf{A}_2\times\mathbf{B}_2:\ \mathbf{B}_2)\times$$

$$\times\begin{pmatrix}\mathbf{C}_1\\ \mathbf{C}_1\times\mathbf{A}_1\\ \vdots\\ \mathbf{C}_1\times\mathbf{A}_1^k\end{pmatrix}={}^2\mathbf{K}_k\times{}^1\mathbf{H}_k,$$

where ${}^2\mathbf{K}_k=(\mathbf{A}_2^{k-1}\times\mathbf{B}_2\ :\ \mathbf{A}_2^{k-2}\times\mathbf{B}_2\ :\ \ldots\ :\ \mathbf{B}_2)$ is the *k-control matrix* of the automaton $\mathcal{A}_2$ and

$${}^1\mathbf{H}_k=\begin{pmatrix}\mathbf{C}_1\\ \mathbf{C}_1\times\mathbf{A}_1\\ \vdots\\ \mathbf{C}_1\times\mathbf{A}_1^k\end{pmatrix}$$

is a submatrix of the *diagnostic matrix* of the automaton $\mathcal{A}_1$. Because $\mathbf{C}=(\mathbf{D}_2\times\mathbf{C}_1:\ \mathbf{C}_2)$ we have

$$\mathbf{C}\times\mathbf{A}^k=(\mathbf{D}_2\times\mathbf{C}_1:\ \mathbf{C}_2)\times\begin{pmatrix}\mathbf{A}_1^k & \mathbf{O}\\ \mathbf{G}_k & \mathbf{A}_2^k\end{pmatrix}=(\mathbf{D}_2\times\mathbf{C}_1\times\mathbf{A}_1^k:\ \mathbf{C}_2\times\mathbf{G}_k:\ \mathbf{C}_2\times\mathbf{A}_2^k)=$$

$$=(\mathbf{D}_2\times\mathbf{C}_1\times\mathbf{A}_1^k:\ \mathbf{C}_2\times\mathbf{A}_2^k)\cup(\mathbf{C}_2\times\mathbf{G}_k:\ \mathbf{O})=$$

$$=(\mathbf{D}_2\times\mathbf{C}_1\times\mathbf{A}_1^k:\ \mathbf{C}_2\times\mathbf{A}_2^k)\cup(\mathbf{C}_2\times{}^2\mathbf{K}_k\times{}^1\mathbf{H}_k:\ \mathbf{O}).$$

Due to the isotony of the Boolean matricial product, [1, p. 17] we can give superior bound for the matrix $\mathbf{G}_k$.

$$\mathbf{G}_k={}^2\mathbf{K}_k\times{}^1\mathbf{H}_k=(\mathbf{A}_2^{k-1}\times\mathbf{B}_2:\ \ldots\ :\ \mathbf{A}_2\times\mathbf{B}_2:\ \mathbf{B}_2)\times\begin{pmatrix}\mathbf{C}_1\\ \mathbf{C}_1\times\mathbf{A}_1\\ \vdots\\ \mathbf{C}_1\times\mathbf{A}_1^{k-1}\end{pmatrix}=$$

$$=\bigcup_{r=0}^{k-1}\mathbf{A}_2^r\times\mathbf{B}_2\times\mathbf{C}_1\times\mathbf{A}_1^{k-1-r}\leqslant\bigcup_{r=0}^{k-1}\widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1=\widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1.$$

Hence

$$\mathbf{A}^k\leqslant\begin{pmatrix}\mathbf{A}_1^k & \mathbf{O}\\ \widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1 & \mathbf{A}_1^k\end{pmatrix}\leqslant\begin{pmatrix}\widehat{\mathbf{A}}_1 & \mathbf{O}\\ \widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1 & \widehat{\mathbf{A}}_2\end{pmatrix}.$$

Then

$$\widehat{\mathbf{A}}\leqslant\begin{pmatrix}\widehat{\mathbf{A}}_1 & \mathbf{O}\\ \widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1 & \widehat{\mathbf{A}}_2\end{pmatrix}.$$

Now, we can infer that

$$\mathbf{C}\times\widehat{\mathbf{A}}=(\mathbf{D}_2\times\mathbf{C}_1:\ \mathbf{C}_2)\times\widehat{\mathbf{A}}\leqslant$$

$$\leqslant(\mathbf{D}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1\cup\mathbf{C}_2\times\widehat{\mathbf{A}}_2\times\mathbf{B}_2\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1;\ \mathbf{C}_2\times\widehat{\mathbf{A}}_2)=$$

$$=((\mathbf{D}_2\cup\mathbf{C}_2\times\widehat{\mathbf{A}}_2\times\mathbf{B}_2)\times\mathbf{C}_1\times\widehat{\mathbf{A}}_1;\ \mathbf{C}_2\times\widehat{\mathbf{A}}_2).$$

Here, the number of null columns of the matrix $\mathbf{C}\times\widehat{\mathbf{A}}$ is equal at least to the sum of the number of null columns of the matrices $\mathbf{C}_1\times\widehat{\mathbf{A}}_1$ and $\mathbf{C}_2\times\widehat{\mathbf{A}}_2$.

IV. The cascade power of a LBA. Let $\mathcal{A}=(\mathcal{B}_2^{(m,1)},\mathcal{B}_2^{(n,1)},\mathcal{B}_2^{(m,1)},f,g)=$ $=\langle\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}\rangle$, where $\mathbf{A}\in\mathcal{B}_2^{(n,n)}$, $\mathbf{B}\in\mathcal{B}_2^{(n,m)}$, $\mathbf{C}\in\mathcal{B}_2^{(m,n)}$, $\mathbf{D}\in\mathcal{B}_2^{(m,m)}$. The *k cascade power* of the automaton $\mathcal{A}$ will be the automaton:

$$\mathcal{A}^{(k)}=\underbrace{\mathcal{A}\to\mathcal{A}\to\ldots\to\mathcal{A}}_{k\ \text{times}}.$$

If $\mathcal{A}^{(k)}=\langle\mathbf{A}^{(k)},\mathbf{B}^{(k)},\mathbf{C}^{(k)},\mathbf{D}^{(k)}\rangle$, then

$$\mathbf{A}^{(k)} = \begin{pmatrix} \mathbf{A}^{(k-1)} & \mathbf{O} \\ \mathbf{B} \times \mathbf{C}^{(k-1)} & \mathbf{A} \end{pmatrix}, \qquad \mathbf{B}^{(k)} = \begin{pmatrix} \mathbf{B}^{(k-1)} \\ \mathbf{B} \times \mathbf{D}^{(k-1)} \end{pmatrix}, \tag{9}$$

$$\mathbf{C}^{(k)} = (\mathbf{D} \times \mathbf{C}^{(k-1)} \times \mathbf{C}), \qquad \mathbf{D}^{(k)} = \mathbf{D} \times \mathbf{D}^{(k-1)}.$$

Aplying repeatedly the equalities (9) we have:

$$\mathbf{D}^{(k)} = \mathbf{D}^{k}$$
$$\mathbf{C}^{(k)} = (\mathbf{D}^{k-1} \times \mathbf{C} \vdots\ \mathbf{D}^{k-2} \times \mathbf{C} \vdots \ldots \vdots\ \mathbf{D} \times \mathbf{C} \vdots\ \mathbf{C}).$$

The matrices $\mathbf{A}^{(k)}$ and $\mathbf{B}^{(k)}$ can be obtained immediately from the last equations. Regarding the redundance index of the automaton $\mathcal{A}^{(k)}$, denoted by $i^{(k)}$ we have:

$$ki \leqslant i^{(k)} \leqslant kn,$$

where i is the redundance index of the automaton $\mathcal{A}$. It is clear that, by an appropriate renumbering of the delayers (see [6]), it is possible to carry the units of the state vector $\zeta = \bigcup_{\omega \in \mathcal{Z}} \omega$ into the last k positions (instead of first k positions as in [6]). Then the states of the null ideal will have the aspect $\sigma = \begin{pmatrix} \mathbf{o} \\ \mu \end{pmatrix}$, $\mu \in \mathcal{B}_2^{(k,1)}$, $\mathbf{o} \in \mathcal{B}_2^{(n-k,1)}$.

Then the state $\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix}$ will be equivalent to the state $\begin{pmatrix} \sigma_1 \\ \mathbf{o} \end{pmatrix}$ where $\sigma_1 \in \mathcal{B}_2^{(n-k,1)}$, because

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} \oplus \begin{pmatrix} \sigma_1 \\ \mathbf{o} \end{pmatrix} = \begin{pmatrix} \mathbf{o} \\ \sigma_2 \end{pmatrix} \in \mathcal{Z}.$$

Following a similar development as in [6], we can conclude that under the permutation which acts on the delayers in the way which was described earlier, the characteristic matrices of the automaton become:

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}' & \mathbf{O} \\ \mathbf{A}'' & \mathbf{A}''' \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} \mathbf{B}' \\ \mathbf{B}'' \end{pmatrix},$$

$$\mathbf{C} = (\mathbf{C}' \vdots\ \mathbf{O}), \qquad \mathbf{D} = \mathbf{D},$$

where $\mathbf{A}'' \in \mathcal{B}_2^{(k,k)}$, $\mathbf{C}' \in \mathcal{B}_2^{(m,n-k)}$ a.s.o.

Now, we notice that it holds:

Theorem 3. *Every LBA, $\mathcal{A}$ can be regarded as a cascade of two automata iff the system of Boolean matricial equations:*

$$\begin{aligned} \mathbf{A}'' &= \mathbf{Z} \times \mathbf{U}, & \mathbf{B}'' &= \mathbf{Z} \times \mathbf{Y}, \\ \mathbf{D} &= \mathbf{X} \times \mathbf{Y}, & \mathbf{C}' &= \mathbf{X} \times \mathbf{U}, \end{aligned} \tag{10}$$

has at least a solution

Proof. Let us suppose that the system (10) has a solution $(\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{U})$. Then, let us consider the automata

$$\mathcal{A}_1 = \langle \mathbf{A}', \mathbf{B}', \mathbf{U}, \mathbf{Y} \rangle,$$
$$\mathcal{A}_2 = \langle \mathbf{A}''', \mathbf{Z}, \mathbf{O}, \mathbf{X} \rangle.$$

Their cascade-product is the automaton

$$\mathcal{A}_1 \to \mathcal{A}_2 = \left\langle \begin{pmatrix} \mathbf{A}' & \mathbf{O} \\ \mathbf{Z} \times \mathbf{U} & \mathbf{A}''' \end{pmatrix}, \begin{pmatrix} \mathbf{B}' \\ \mathbf{Z} \times \mathbf{Y} \end{pmatrix}, (\mathbf{X} \times \mathbf{U} \vdots\ \mathbf{O}), \mathbf{X} \times \mathbf{Y} \right\rangle.$$

Taking into account the equations (10) we obtain the relation

$$\mathcal{A}_1 \to \mathcal{A}_2 = \left\langle \begin{pmatrix} \mathbf{A}' & \mathbf{O} \\ \mathbf{A}'' & \mathbf{A}''' \end{pmatrix}, \begin{pmatrix} \mathbf{B}' \\ \mathbf{B}'' \end{pmatrix}, (\mathbf{C}' \vdots\ \mathbf{O}), \mathbf{D} \right\rangle,$$

which we had to prove. It is easy to see that the second automaton is in fact a combinatorial device because $\mathbf{C}_2 = \mathbf{O}$. Its outputs are influenced only by its inputs and not by the internal state.

We can also infer that, if there exists a square root of an automaton (in the sens of the cascade-composition) then the automaton and its square root must be combinatorial devices.

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CASCADE DE AUTOMATE LINEARE BOOLEENE

Rezumat

În mulțimea automatelor liniare booleene, care au fost introduse în [5,6] se consideră operația „→", de grupare în cascadă a două automate, față de care mulțimea ALB formează un semigrup în sens larg.

Cunoscîndu-se matricele caracteristice ale ALB, $\mathcal{A}_1$ şi $\mathcal{A}_2$ se găsesc matricele caracteristice ale ALB, $\mathcal{A}_1 \rightarrow \mathcal{A}_2$. Se introduce apoi puterea k în cascadă, a unui ALB pentru care se găsesc matricele caracteristice. Pentru indicele de redundanţă al automatului $\mathcal{A}_1 \rightarrow \mathcal{A}_2$ se demonstrează teorema: indicele de redundanţă al automatului $\mathcal{A}_1 \rightarrow \mathcal{A}_2$ este cel puţin egal cu suma indicilor de redundanţă ai automatelor $\mathcal{A}_1$ şi $\mathcal{A}_2$. Un ultim rezultat indică o condiţie necesară şi suficientă de decompozabilitate a unui ALB într-o cascadă de două ALB.

RECENZII

ACTES DU CONGRÈS INTERNATIONAL DES MATHEMATICIENS, 1/10 septembre 1970 (Nice) France Gauthier-Villars, Paris 1971, 3 volume 532+968+375 pp.

Ultimul Congres al matematicienilor, organizat din patru în patru ani de Uniunea internaţională a matematicienilor, a avut loc în septembrie 1970 la Nisa sub înaltul patronaj al preşedintelui Republicii Franceze G. Pompidou şi al primului ministru J. Chaban-Delmas. Cele trei volume cuprind documentele congresului de la şedinţa inaugurală pînă la şedinţa de închidere, iar publicarea lor s-a realizat prin grija profesorilor: K. Berger, J. Dieudonne, J. Leray, J. L. Lions, P. Malliavin şi J. P. Serre. În şedinţa inaugurală profesorul H. Cartan prezintă raportul comisiei, însărcinată cu decernarea medaliilor Fields, care se acordă unor matematicieni valoroşi sub 40 ani cu ocazia fiecărui congres organizat de U.I.M. Cu ocazia congresului de la Nisa au fost medaliaţi: Alan Baker pentru importante contribuţii în teoria numerelor transcendente, Heisuke Hironaka pentru rezultate deosebite în domeniul rezoluţiei singularităţilor, Serghei Novikov pentru remarcabile rezultate în topologia geometrică şi algebrică şi John G. Thompson pentru lucrări în teoria grupurilor finite şi aplicaţii. Prima parte a volumului I este rezervată rapoartelor asupra lucrărilor medaliaţilor Fields prezentate de către P. Turán, Al. Grothendieck, M. F. Atiyah şi R. Brauer; în continuare sînt prezentate conferinţele generale susţinute de către matematicieni de reputaţie mondială. În continuare vom enumera titlurile şi autorii acestor conferinţe: Metode efective în teoria numerelor de A. Baker, Obstrucţii topologice în cadrul procesului de integrare de R. Bott, Varietăţi şi teoria homotopiei de W. Browder, Geometria diferenţială, trecutul şi prezentul ei de S. Chern, Situaţii curente în teoria grupurilor simple finite de W. Feit, Cohomologia algebrelor Lie infinit dimensionale: unele probleme de geometrie integrală de I. M. Gelfand, O metodă transcendentă în geometria algebrică de P. A. Griffiths, Operatori diferenţiali liniari de Lara Hörmander, Teoria skatteringului şi a perturbării spectrului continuu de T. Kato, Teoria modelelor de S. Keisler, Probleme de metode şi calcul în matematică de G. I. Marčhuk, Jocuri diferenţiale liniare de L. Pontryagin, Unele probleme de analiză armonică sugerate de spaţiile simetrice şi grupuri semisimple de E. M. Stein, K-teoria algebrică de R. G. Shan, Simboli în aritmetică de J. Tate, Topologie geometrică: varietăţi şi structuri de G. T. C. Wall.

Comunicările la acest congres sînt clasificate, în cadrul acestor trei volume, în următoarele şase grupe: grupa A — logică matematică, grupa B — algebră, grupa C — geometrie şi topologie, grupa D — analiză, grupa E — matematică aplicată, grupa F — istoria matematicii şi învăţămînt.

În cele ce urmează vom cita din cadrul grupelor, enumerate mai sus, unele lucrări care prin conţinutul lor dovedesc a avea o importanţă deosebită. Grupa B este divizată în următoarele subgrupe: B_1 — algebră generală, categorii, B_2 — algebră omologică, B_3 — grupuri finite, B_4 — corpuri locale şi analiză p- adică