$$
\left.\begin{matrix} :\{(bq_1,\ \beta q_1)\} \\ (1:1)(n+r+2,\ 1):\ \frac{3}{2}+\frac{n}{2},1\Big)\left(1+\frac{n}{2},1\right),\ \{(dq_2,\ \delta q_2)\}\ (1,1) \\ (1,1)(m+s+2,1):\left(\frac{3}{2}+\frac{m}{2},1\right)\left(1+\frac{m}{2},1\right)\{(fq_3,\ \zeta q_3)\}\ (1,1) \end{matrix}\right| \begin{matrix} x \\ \\ y \end{matrix} \Bigg]
$$

provedid that,

$$
0 \leqslant \theta \leqslant \pi,\quad 0 \leqslant \varphi \leqslant \pi,\quad \frac{3}{2}+\frac{n}{2} > 0,\ \frac{3}{2}+\frac{m}{2} > 0,
$$

$$
T > 0: |\arg x| < \frac{T\pi}{2},\ T' > 0;\ |\arg y| < \frac{T'\pi}{2},
$$

$$
Q \leqslant 0,\quad Q' \leqslant 0
$$

where T and T' are given by (1.2) and Q, Q' are given by (1.ii).

Many more results can be obtained by specialising the parameters. I am thankfull to Dr. H. M. Srivastava for his kind guidance during the preparation of the paper.

Department of Mathematics
Lucknow University
Lucknow (India)

REFERENCES

1. Richard A. — Proc. Amer. Math. Soc. 16 (1969), 1 191—1 194.
2. Fox C. — Trans. Amer. Math. Soc. 98 (1961), 395—425.
3. Keserwani R. N. — Compo. Math. 17 (1966), 149—151.
4. Parihar C. L. — Proc. Nat. Ins. Sci. A (35, 1969.)
5. Pathak R. S. — Bull. Cal. Math. Soc. (2) 62 (1970), 97—106.
6. Rainvielle E. D. — *special functions*. 1964.

SERII FOURIER PENTRU FUNCȚIA H DE DOUĂ VARIABILE

Rezumat

Utilizînd definiția funcției H de două variabile dată de R. S. Pathak [5] și forma ei integrală dată de Mellin-Barne, autorul stabilește formula (1.1) de reprezentare printr-o serie Fourier.

SOME INFINITE SERIES FOR THE H-FUNCTION OF TWO VARIABLES

BY

S. D. GUPTA

§ 1. Macrobert [2] in 1959 has summed few series for the Macrobert E-function. In this paper an attempt has been made to sum the series of similar types with the H-function of two variables defined by Pathak [1].

For the definition of H-function of two variables see Pathak 1] and for other notations and results see Macrobert [2].

§ 2. **Infinite Series.** In this section we shall obtain following series

Series 1:

$$
\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{\alpha^m\ \beta^n}\frac{(xt)^m}{m!}\frac{(yz)^n}{n!}
$$

$$
P\left[\begin{matrix} \left|\begin{matrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{matrix}\right| \\ \begin{pmatrix} m_2+1,\ n_2 \\ p_2-m_2,\ q_2-n_2 \end{pmatrix} \\ \begin{pmatrix} m_3+1,\ n_3 \\ p_3-m_3,\ q_3-n_3 \end{pmatrix} \end{matrix}\right. \left|\begin{matrix} \{(ap_1-(m+n)\alpha p_1,\ \alpha p_1)\}:\ \{(bq_1-(m+n)\beta q_1,\beta q_1)\} \\ (1,1)\{(cp_2+m\gamma p_2,\ \gamma p_2)\}:\ \{(dq_2+m\delta q_2,\ \delta q_2)\} \\ (1,1)\{(ep_3+n\varepsilon p_3,\ \varepsilon p_3)\}:\ \{(fq_3+n\zeta q_3,\ \zeta q_3)\} \end{matrix}\right| \left.\begin{matrix} \frac{\alpha}{x^2} \\ \\ \frac{\beta}{y^2} \end{matrix}\right]
$$

$$
=P\left[\begin{matrix} \left|\begin{matrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{matrix}\right| \\ \begin{pmatrix} m_2+1,\ n_2 \\ p_2-m_2,\ q_2-n_2 \end{pmatrix} \\ \begin{pmatrix} m_3+1,\ n_3 \\ p_3-m_3,\ q_3-n_3 \end{pmatrix} \end{matrix}\right. \left|\begin{matrix} \{(ap_1,\ \alpha p_1)\};\quad \{(bq_1,\ \beta q_1)\} \\ (1,1)\{(cp_2,\ \gamma p_2)\};\quad \{(dq_2,\ \delta q_2)\} \\ (1,1)\{(ep_3,\ \varepsilon p_3)\};\quad \{(fq_3,\ \zeta q_3)\} \end{matrix}\right| \left.\begin{matrix} \frac{\alpha}{(x^2-tx)} \\ \\ \frac{\beta}{(y^2-yz)} \end{matrix}\right]
$$

Provided,

$|\operatorname{amp} t| < \pi/2$; $|\operatorname{amp} z| < \pi/2$,

$|\operatorname{amp}(x-t)| < \dfrac{\pi}{2}$; $|\operatorname{amp}(z-y)| < \dfrac{\pi}{2}$, $\left|\dfrac{t}{x}\right| < 1$; $\left|\dfrac{z}{y}\right| < 1$.

$$T=\sum_{\tau=1}^{m_1}\alpha_\tau+\sum_{\tau=1}^{m_2}\gamma_\tau+\sum_{\tau=1}^{n_2}\delta_\tau+\sum_{\tau=m_1+1}^{p_1}\alpha_\tau-\sum_{\tau=1}^{q_1}\beta_\tau-\sum_{\tau=m_2+1}^{p_2}\gamma_\tau-\sum_{\tau=n_2+1}^{q_2}\delta_\tau>0 ;$$

$$\left|\arg\frac{\alpha}{x^2}\right|<\frac{T\pi}{2},$$

$$T'=\sum_{\tau=1}^{m_1}\alpha_\tau+\sum_{\tau=1}^{m_2}\varepsilon_\tau+\sum_{\tau=1}^{n_2}\tau_\tau-\sum_{\tau=m_1+1}^{p_1}\alpha_\tau-\sum_{\tau=1}^{q_1}\beta_\tau-\sum_{\tau=m_3+1}^{p_3}\varepsilon_\tau-\sum_{\tau=n_3+1}^{q_3}\tau_\tau>0 ;$$

$$\left|\arg\frac{\beta}{y^2}\right|<\frac{T'\pi}{2},$$

$$Q=\sum_{1}^{m_1}\alpha_\tau+\sum_{1}^{p_2}\gamma_\tau-\sum_{1}^{q_2}\delta_\tau-\sum_{m_1+1}^{p_1}\alpha_\tau-\sum_{1}^{q_1}\beta_\tau\leqslant 0,$$

$$Q'=\sum_{1}^{m_1}\alpha_\tau+\sum_{1}^{p_3}\varepsilon_\tau-\sum_{1}^{q_3}\tau_\tau-\sum_{m_1+1}^{p_1}\alpha_\tau-\sum_{1}^{q_1}\beta_\tau\leqslant 0 .$$

Series 2 :

$$\pi P\left[\begin{array}{c|c|c} \begin{vmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{vmatrix} & \{(ap_1,\ \alpha p_1)\}\ ;\quad \{(bq_1,\ \beta q_1)\} & \dfrac{x}{\sin^2\left(\frac{1}{2}\theta\right)} \\ \begin{pmatrix} m_2,\ n_2 \\ p_2-m_2,\ q_2-n_2 \end{pmatrix} & \{(cp_2,\ \gamma p_2)\}\ ;\quad \{(dq_2,\ \delta q_2)\} & \\ \begin{pmatrix} m_3,\ n_3 \\ p_3-m_3,\ q_3-n_3 \end{pmatrix} & \{(cp_3,\ \varepsilon p_3)\}\ ;\quad \{(fq_3,\ \tau q_3)\} & \dfrac{y}{\sin^2\left(\frac{1}{2}\Phi\right)} \end{array}\right]=$$

$$=P\left[\begin{array}{c|c|c} \begin{vmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{vmatrix} & \{(ap_1,\ \alpha p_1)\}\ ;\qquad \{(bq_1,\ \beta q_1)\} & x \\ \begin{pmatrix} m_2,\ n_2+1 \\ p_2+1-m_2,\ q_2-n_2 \end{pmatrix} & \{(cp_2,\ \gamma p_2)\}\ (1,1)\ ;\ \left(\frac{1}{2},\ 1\right)\{(dq_2,\ \delta q_2)\} & \\ \begin{pmatrix} m_3,\ n_3+1 \\ p_3+1-m_3,\ q_3-n_3 \end{pmatrix} & \{(cp_3,\ \varepsilon p_3)\}\ (1,1)\ ;\ \left(\frac{1}{2},\ 1\right)\{(fq_3,\ \tau q_3)\} & y \end{array}\right]+$$

$$+4\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}P\left[\begin{array}{c|c|c} \begin{pmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{pmatrix} & \{(ap_1,\ \alpha p_1)\};\ \{(bq_1,\ \beta q_1)\} & x \\ & (1-r,1)\{(cp_2,\gamma p_2)\}(1+r,1):\left(\frac{1}{2},\ 1\right) & \\ \begin{pmatrix} m_2+1,\ n_2+1 \\ p_2+1-m_2,\ q_2+1-n_2 \end{pmatrix} & \{(dq_2,\delta q_2)\}(1,1) & \\ & (1-s,1)\{(cp_3,\varepsilon p_3)\}(1+s,1);\ \left(\frac{1}{2},1\right) & \\ \begin{pmatrix} m_3+1,\ n_3+1 \\ p_3+1-m_3,\ q_3+1-n_3 \end{pmatrix} & \{(fq_3,\ \tau q_3)\}(1,1) & y \end{array}\right]$$

$$\cos(r\theta)\ .\ \cos(s\Phi)\ +$$

$$+\ 2\sum_{r=1}^{\infty}P\left[\begin{array}{c|c|c} \begin{pmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{pmatrix} & \{(ap_1,\ \alpha p_1)\}:\quad \{(bq_1,\beta q_1)\} & x \\ & (1-r,\ 1)\{(cp_2,\ \gamma p_2)\};\ (1+r,\ 1)\ ;\left(\frac{1}{2},1\right) & \\ \begin{pmatrix} m_2+1,\ n_2+1 \\ p_2+1-m_2,\ q_2+1-n_2 \end{pmatrix} & \{(dq_2,\ \delta q_2)\}\ (1,\ 1) & \\ \begin{pmatrix} m_3,\ n_3+1 \\ p_3+1-m_3,\ q_3-n_3 \end{pmatrix} & \{(cp_3,\ \varepsilon p_3)\}\ (1,1);\ \left(\frac{1}{2},1\right)\{(fq_3,\ \tau q_3)\} & y \end{array}\right]$$

$$\cos r\theta\ + \tag{2.2}$$

$$+\ 2\sum_{S=1}^{\infty}P\left[\begin{array}{c|c|c} \begin{pmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{pmatrix} & \{(ap_1.\ \alpha p_1)\}\ ;\ \{(bq_1.\ \beta q_1)\} & x \\ & \{(cp_2,\gamma p_2)\}\ (1,\ 1)\ ;\left(\frac{1}{2}.\ 1\right)\{(dq_2,\ \delta q_2)\} & \\ \begin{pmatrix} m_2,\ n_2+1 \\ p_2+1-m_2,\ q_2-n_2 \end{pmatrix} & & \\ & (1-s,1)\{(cp_3,\ \varepsilon p_3)\}(1+s,1);\left(\frac{1}{2},1\right), & \\ \begin{pmatrix} m_3+1,\ n_3+1 \\ p_3+1-m_3.q_3+1-n^3 \end{pmatrix} & \{(fq_3,\ \tau q_3)\}\ (1,\ 1) & y \end{array}\right]\cos s\Phi.$$

provided,

$0<\theta<\pi$; $0\leqslant\Phi\leqslant\pi$, $|\operatorname{amp} x|<\pi$; $|\operatorname{amp} y|<\pi$,

$$T>0\ ;\ |\arg x|<\frac{T\pi}{2},\quad T'>0\ ;\ |\arg y|<\frac{T'\pi}{2},$$

$$Q\leqslant 0\ ;\ Q'\leqslant 0$$

T, T' and Q, Q' are same as in previous result (2.1).

Series 3 :

$$\sum_{n=0}^{\infty} \frac{(l+2n)\,\Gamma(l+n)\,(l+a_1;\ n)\,(k:n)\,\Gamma(1-a_1-k)}{n!\ \Gamma(1-a_1+n)\,\Gamma(1+l-k+n)\,x^n y^n}\cdot \pi^2 \operatorname{cosec} \pi(2a_1)\times$$

$$\times \operatorname{cosec} \pi(k+a_1-l)\times$$

$$P\left[\begin{array}{c|l} \begin{matrix} m_1,\ 0 \\ p_1+2-m_1,\ q_1+2 \end{matrix} & \begin{matrix}(a_1+n-2n\alpha_1,\ \alpha_1)\{(ap_1'-2n\alpha p_1',\ \alpha p_1')\},\\ (a_1-l-n-2n\alpha_1,\ \alpha_1),(2a_1+k-2n\alpha_1,\ \alpha_1);\end{matrix} \\ \begin{pmatrix} m_2,\ n_2 \\ p_2-m_2,\ q_2-n_2 \end{pmatrix} & \{(cp_2,\ \gamma p_2)\}; \\ \begin{pmatrix} m_3,\ n_3 \\ p_3-m_3,\ q_3-n_3 \end{pmatrix} & \{(ep_3,\ \varepsilon p_3)\}; \end{array}\right.$$

$$\left.\begin{array}{l|c} \begin{matrix}; \{(bq_1-2n\beta q_1,\ \beta q_1)\}\ (k+a_1-l-2n\alpha_1,\\ ;\qquad\qquad ,\alpha_1),\ (2a_1-2n\alpha_1,\ \alpha_1);\end{matrix} & x \\ ;\{(dq_2,\ \delta q_2)\} & \\ ;\{(fq_3,\ \tau q_3)\} & y \\ p_1'=2,\ 3,\ldots\ldots, p_1. & \end{array}\right] =$$

$$(2.3)\qquad = P\left[\begin{array}{c|ll|c} \begin{bmatrix} m_1,\ 0 \\ p_1-m_1,\ q_1 \end{bmatrix} & \{(ap_1,\ \alpha p_1)\}; & \{(bq_1,\ \beta q_1)\} & x \\ \begin{pmatrix} m_2,\ n_2 \\ p_2-m_2,\ q_2-n_2 \end{pmatrix} & \{(cp_2,\ \gamma p_2)\}; & \{(dq_2,\ \delta q_2)\} & \\ \begin{pmatrix} m_3,\ n_3 \\ p_3-m_3,\ q_3-n_3 \end{pmatrix} & \{(ep_3,\ \varepsilon p_3)\}; & \{(fq_3,\ \tau q_3)\} & y \end{array}\right]$$

provided,

$$Re(1-a_1-k)>0,\ |\operatorname{amp} x|<\pi;\ |\operatorname{amp} y|<\pi,$$

$$T>0;\ |\arg x|<\frac{T\pi}{2},\ \ T'>0;\ |\arg y|<\frac{T'\pi}{2},$$

$$Q\leqslant 0,\ Q'\leqslant 0$$

T, T' and Q, Q' are same as given in (2.1).

We shall give a brief proof of series 1 (2.1) similarly series 2 and series 3 can be obtained with suitable adjustment by using the Fourier-series due to [2, p. 143] and Dougall's formula [3, p. 372] respectively.

Proof of Series — 1:

Expanding the H-function of two variables in the left of (2.1) as its Mellin-Barne's type integral, we get

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{\alpha^m\beta^n}\cdot\frac{(xt)^m}{m!}\cdot\frac{(yz)^n}{n!}\cdot\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}\frac{\prod_{\tau=1}^{m_1}\Gamma(a_\tau-(m+n)\alpha_\tau+\alpha_\tau(\xi+\eta))}{\prod_{\tau=m_1+1}^{p_1}\Gamma(1-a_\tau+(m+n)\alpha_\tau+\alpha_\tau(\xi+\eta))}\times$$

$$\times\frac{\Gamma(\xi)\prod_{\tau=1}^{p_1}\Gamma(1-c_\tau-m\gamma_\tau+\gamma_\tau\xi)\prod_{\tau=1}^{n_2}\Gamma(d_\tau+m\delta_\tau-\delta_\tau\xi)\Gamma(\eta)}{\prod_{\tau=1}^{q_1}\Gamma(b_\tau-(m+n)\beta_\tau+\beta_\tau(\xi+\eta))\prod_{\tau=m_2+1}^{p_1}\Gamma(c_\tau+m\gamma_\tau-\gamma_\tau\xi)\prod_{\tau=n_2+1}^{q_2}\Gamma(1_\tau-d_\tau-m\delta_\tau+\delta_\tau\xi)}$$

$$\frac{\prod_{\tau=1}^{m_3}\Gamma(1-e_\tau-n\varepsilon_\tau+\varepsilon_\tau\eta)\prod_{\tau=1}^{n_3}\Gamma(f_\tau+n\tau_\tau-\tau_\tau\eta)}{\prod_{\tau=m_3+1}^{p_3}\Gamma(e_\tau+n\varepsilon_\tau-\varepsilon_\tau\eta)\prod_{\tau=n_3+1}^{q_3}\Gamma(1-f_\tau-n\tau_\tau+\tau_\tau\eta)}\cdot\ \alpha^\xi\beta^\eta x^{-2\xi}y^{-2\eta}d\xi\,d\eta\,.$$

replacing ξ by $\xi+m$ and η by $\eta+n$, changing the order of summation and integration which is valid due to absolute convergence. Proceeding on the lines of Macrobert [2] and using the definition of H-function of two variables, we obtain series 1.

I am thankful to Dr. H. M. Srivastava for kind guidance during the preparation of this paper.

Department of Mathematics
University of Lucknow
Lucknow — India

R E F E R E N C E S

1. Pathak R. S. — Bull. Cal. Math. Soc. 2, 62 (1970), 95—106.
2. Macrobert T. M. — Math. Zeitsch 71 (1959), 143—145.
3. Macrobert T. M. — *Functions of Complex Variables.*

CÎTEVA SERII PENTRU FUNCȚIA H DE DOUĂ VARIABILE

Rezumat

Autorul stabilește trei dezvoltări în serie pentru funcția H de două variabile definită de R. S. Pathak [1] relativ la care alte rezultate au fost date și de T. M. Macrobert [2].