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THE EXISTENCE AND THE UNIQUENESS OF THE SOLUTION OF THE EQUILIBRIUM EQUATIONS FOR THE LINEAR THEORY OF COSSERAT RODS

BY

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Introduction. The main paper where the rods and shells are treated as oriented bodies is that of J. L. Ericsen and C. Truesdell ([1]). Recently some other authors have dealt with such bodies, for example A. B. Whitman and C. N. DeSilva ([2], [3], [4] and [5]).

In this paper we have started from the linear theory presented in [4] using also the results of J. Hlavacek, M. Hlavacek and J. Nečas ([6], [7], [8] and [9]) to obtain an existence and uniqueness theorem for the weak solution of the equilibrium equations of the elastic Cosserat rods. The rod is considered initial straight and unstressed.

In § 1 some basic theorems about elliptic systems of partial differential equations are stated.

In § 2 the basic equations of the linear theory of Cosserat rod are presented and the boundary-value problem is formulated.

In § 3 we define the weak solution of this boundary-value problem and we establish the existence and uniqueness theorem.

§1. Boundary-value problems for elliptic system of partial differential equations. In this section we present some basic theorems whose proofs can be found in [6] and [8].

Definition 1.1. *A region $\Omega \subset E_n$ (the euclidian N-dimensional space) is called Lipschitz if it is bounded and its frontier Γ has the following properties: a) to each point $P \in \Gamma$ there exists a sphere B_p with the centre in P so that $B_p \cap \Gamma$ can be described by means of a Lipschitz function and b) $B_p \cap \Gamma$ divides B_p into internal and external parts with respect to Ω.*
We consider such a region Ω.

Let $L_2(\Omega)$ be the space of real functions which are integrable in Ω (in the sense of Lebesgue) and let $W_2^k(\Omega)$ be the subspace of $L_2(\Omega)$ of all functions whose derivatives up to the order k (in the sense of distributions) are in $L_2(\Omega)$. We introduce the scalar product on $W_2^k(\Omega)$ by:

$$(v, u) = \sum_{|\alpha| \leq k} \int_\Omega D^\alpha v \, D^\alpha u \, dx, \tag{1.1}$$

where $D^\alpha = \partial^{|\alpha|}/\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \ldots \partial x_m^{\alpha_m}$. In the following $\mathbf{W}$ denotes the Cartezian product $\prod_{s=1}^{m} W_2^{k_s}(\Omega)$.

Let $\mathbf{u} = \{u_1, u_2, \ldots u_m\}$ and $\mathbf{v} = \{v_1, v_1, \ldots v_m\}$ be two vectors of $\mathbf{W}$. We now define the bilinear form $A(\mathbf{v},\mathbf{u})$ on $\mathbf{W} \times \mathbf{W}$ by:

$$A(\mathbf{v},\mathbf{u}) = \int_\Omega \sum_{r,s=1}^{m} a_{ij}^{rs} D^i v_r D^j u_s dx, \tag{1.2}$$

where a_{ij}^{rs} are real measurable and bounded functions on Ω. Let $\mathcal{D}(\Omega)$ be the space of real functions with compact support in Ω which are infinitely differentiable, $\mathring{W}_2^k(\Omega)$ the closure of $\mathcal{D}(\Omega)$ in $W_2^k(\Omega)$ and $\mathring{\mathbf{W}} = \prod_{s=1}^{m} \mathring{W}_2^{k_s}(\Omega)$. Denote by $\mathbf{V}$ a closed subspace of $\mathbf{W}$ so that $\mathring{\mathbf{W}} \subset \mathbf{V} \subset \mathbf{W}$. We define the functionals:

$$\left\{\begin{aligned} & f(\mathbf{v}) = \int_\Omega \sum_{s=1}^{m} f_s v_s \, dx, \quad f_s \in L_2(\Omega), \ \mathbf{v} \in \mathbf{W} \\ & g(\mathbf{v}) = \int_{\Gamma'} \sum_{s=1}^{m} g_s v_s \, d\Gamma + \int_B \sum_{s=1}^{m} G_s v_s dB, \quad g_s \in L_2(\Gamma'), \quad G_s \in L_2(B), \\ & \qquad \mathbf{v} \in \mathbf{W} \text{ and if } k_s < 2 \text{ then } G_s = 0. \end{aligned}\right. \tag{1.3}$$

Here $L_2(\Gamma')$ and $L_2(B)$ denote the spaces of real functions, square-integrable on $\Gamma' \subset \Gamma$ and $B \subset \Gamma$, respectively. B is a one-dimensional set of finit measure. The embedding theorems imply that $f(\mathbf{v})$ and $g(\mathbf{v})$ are continuous on $\mathbf{W}$.

Definition 1.2. *Let be* $\bar{\mathbf{u}} \in \mathbf{W}$. *We say that* $\mathbf{u} \in \mathbf{W}$ *is a weak solution of the boundary-value problem if*

$$\mathbf{u} - \bar{\mathbf{u}} \in \mathbf{V}$$

and

$$A(\mathbf{v}, \mathbf{u}) = f(\mathbf{v}) + g(\mathbf{v}) \tag{1.4}$$

hold for each $\mathbf{v} \in \mathbf{V}$.

Let N_e $(e = 1, 2 \ldots h)$, be the operators given by:

$$N_e \mathbf{v} = \sum_{r=1}^{m} \sum_{|\alpha| \leq k_r} n_{er\alpha} D^\alpha v_r, \tag{1.5}$$

where $n_{er\alpha}$ are bounded and measurable functions on Ω. This operators map $\mathbf{W}$ into $L_2(\Omega)$.

Definition 1.3. *We say that the operators* N_e *form a coercive system on* $\mathbf{W}$ *if for each* $\mathbf{v} \in \mathbf{W}$ *we have:*

$$\sum_{e=1}^{h} |N_e \mathbf{v}| + \sum_{s=1}^{m} |v_s|_{L_2}^2 \geq c_1 |\mathbf{v}|_{\mathbf{W}}^2, \quad c_1 > 0, \tag{1.6}$$

where c_1 does not depend of $\mathbf{v}$, $|\ |_{L_2}$ and $|\ |_{\mathbf{W}}$ denote the usual norm on L_2 and $\mathbf{W}$, respectively.

We suppose that the system (1.5) is coercive and that for each $\mathbf{v} \in \mathbf{W}$ we have:

$$A(\mathbf{v}, \mathbf{v}) \geq c_2 |N_e \mathbf{v}|_{L_2}^2, \quad c_2 > 0, \tag{1.7}$$

where c_2 does not depend on $\mathbf{v}$. We now state the following theorem (theorem 2.3 of [6])

Theorem 1.1 *Let us suppose that (1.6) and (1.7) hold and that we have:*

$$\mathbf{v} \in \mathbf{V}, \quad \sum_{e=1}^{h} |N_e \mathbf{v}|_{L_2}^2 = 0 \Rightarrow \mathbf{v} = 0. \tag{1.8}$$

Then the inequality

$$\sum_{e=1}^{h} |N_e \mathbf{v}|_{L_2}^2 \geq c_3 |\mathbf{v}|_{\mathbf{W}}^2 \quad c_3 > 0 \tag{1.9}$$

holds.

Definition 1.4. *A bilinear form* $[\mathbf{v}, \mathbf{u}] = A(\mathbf{v}, \mathbf{u})$ *is called* $\mathbf{V}$*-elliptic if for each* $\mathbf{v} \in \mathbf{V}$ *we have:*

$$[\mathbf{v}, \mathbf{v}] \geq c_4 |\mathbf{v}|_{\mathbf{W}}^2. \quad c_4 > 0 \tag{1.10}$$

Theorem 1.2. (Theorem 7.2 of [8]). *Let us suppose that* $A(\mathbf{v}, \mathbf{u}) = [\mathbf{v}, \mathbf{u}]$ *defines a bilinear form for* $\mathbf{v}, \mathbf{u} \in \mathbf{W}$ *and that it is* $\mathbf{V}$*-elliptic. Then there is a unique solution of the boundary-value problem which satisfies the inequality:*

$$|\mathbf{u}|_{\mathbf{W}} \leq c_5 \left[|f|_{(L_2)^m} + |\bar{\mathbf{u}}|_{\mathbf{W}} + |g|_{\mathbf{V}'} \right]. \quad c_5 > 0 \tag{1.11}$$

We now state a simple conditions for coercivity of the system (1.5).

Theorem 1.3. *Let* $n_{es\alpha}$ *be constants for* $|\alpha| = k_s$. *Then the system* N_e *is coercive on* $\mathbf{W}$ *if and only if the rank of the matrix*

$$N_{es}\xi = \sum_{|\alpha|=k_s} n_{es\alpha} \xi_\alpha$$

equals m for each $\xi \neq 0$, $\xi \in C_3$ *where* C_3 *denotes the complex three-dimensional space and* $\xi_\alpha = \xi_1^{\alpha_1} \xi_2^{\alpha_2} \xi_3^{\alpha_3}$.

§2. **Basic equations. Formulation of the boundary-value problem.** A Cosserat rod is defined as a curve with a non-coplanar triad of deformable vectors called directors, which are associated to every point of the curve. By deformation of a rod we mean the displacement of every point of the rod and the independent rotations and stretches of the directors. A deformed configuration c of a Cosserat rod at every time is defined by:

$$\mathbf{r} = \mathbf{r}(s, t), \quad \mathbf{d}_\alpha = \mathbf{d}_\alpha(s, t), \qquad \alpha = 1, 2, 3, \tag{2.1}$$

where $\mathbf{r}$ is the position vector of a point of the deformed rod, d are the deformed directors and s is the arc length of c. We take as reference configuration the configuration at time $t = 0$. Let $\mathcal{C}$ denote the undeformed rod. We have now:

$$\mathbf{R}(S) = \mathbf{r}|_{t=0}, \ \mathbf{D}_\alpha(S) = \mathbf{d}_\alpha|_{t=0} \tag{2.2}$$

where $\mathbf{R}, \mathbf{D}_\alpha$, and S are respectively the position vector, the undeformed directors and are lenght in the reference configuration. The deformation of the rod is given by the equation $s = s(S, t)$. We assume the stretch $\lambda = \dfrac{ds}{dS}$ to be bounded and positive. We shall use the following clasical notation for derivatives:

$$(\wedge) = \frac{d}{dS}, \ (\wedge) = \frac{d}{ds}. \tag{2.3}$$

Since we consider Cosserat rods, the directors form a rigid triad with three rotational degrees of liberty. We suppose the directors form an orthonormal triad right oriented at every time. Likewise we suppose that $\mathbf{D}_3$ is tangent to the rod $\mathcal{C}$. Hence we have:

$$\mathbf{D}_3 = \mathbf{T}, \quad \mathbf{T}_\alpha = \mathbf{T} \cdot \mathbf{D}_\alpha = (0, 0, 1). \tag{2.4}$$

We make now the hypothesis that the rod is initially straight and unstressed. In this case, the following basic equations for the linear theory are obtained [4]:

the equilibrium equations:

$$\begin{cases} \hat{\tau}_\alpha + \rho_R f_\alpha = 0, \\ \hat{m}_\alpha - e_{\alpha\beta\gamma} \tau_\beta T_\gamma + \rho_R l_\alpha = 0; \end{cases} \tag{2.5}$$

the deformations measures:

$$\begin{cases} z_\alpha = \hat{u}_\alpha + e_{\alpha\beta\gamma} T_\beta \varphi_\gamma, \\ \hat{k_\alpha} = \varphi_\alpha, \end{cases} \tag{2.6}$$

where

$\mathbf{f}$ and $\mathbf{l}$ are respectively the body force vector and the body couple vector,
ρ_R is the mass density on the unit length of $\mathcal{C}$,
τ and $\mathbf{m}$ are respectively the stress vector and the couple stress vector,
z_α and k_α are the Ericsen-Truesdell measures of deformation which are liniarized and vanish in the reference configuration,
u_α and φ_α are respectively the components of displacement vector and micro-rotation vector, φ_α $e_{\alpha\beta\gamma}$ is the permutation symbol.
The greek indices denote the anholonomic components of a tensor with respect to d_α and take the values 1, 2, 3.

Assume the energy of deformation per unit volume given by the quadratic form:

$$2\,\mathcal{A}(z_\alpha, k_\alpha) = E_{\alpha\beta} z_\alpha z_\beta + 2F_{\alpha\beta} z_\alpha k_\beta + G_{\alpha\beta} k_\alpha k_\beta = 2\rho_R\, \varepsilon. \tag{2.7}$$

The constitutive equations for the nonlinear theory are given in the form ([2])

$$\tau_\alpha = \rho_R \frac{\partial \varepsilon}{\partial z_\alpha}, \qquad m_\alpha = \rho_R \frac{\partial \varepsilon}{\partial k_\alpha}.$$

From (2.7) these equations can be written

$$\tau_\alpha = E_{\alpha\beta} z_\beta + F_{\alpha\beta} k_\beta \qquad m_\alpha = F_{\alpha\beta} z_\beta + G_{\alpha\beta} k_\beta, \tag{2.8}$$

where E, F and G are bounded and measurable functions of S on $[0, L]$. Without loss of generality, we can take E, F and G to be symmetric.

We make now the basic hypothesis that the form (2.7) is uniformly positive definite, i. e. there exists $c > 0$ so that for each $S \in [0, L]$ we have:

$$\mathcal{A}(z_\alpha, k_\alpha) \geqslant c \sum_{\alpha=1}^{3} (z_\alpha^2 + k_\alpha^2). \tag{2.9}$$

We shall consider the first boundary-value problem when u_α and φ_α vanish at the ends of the rod:

$$\begin{cases} u_\alpha = 0, & S = 0 \text{ or } L \\ \varphi_\alpha = 0, & S = 0 \text{ or } L \end{cases} \qquad \alpha = 1, 2, 3. \tag{2.10}$$

(in the sense of traces).

§3. **The theorem of existence and uniqueness.** Using the results of §1 we put $m = 6$ and $k_s = 1$. The components of two displacement vectors are denoted by u_α and v_α respectively, and the components of two micro-rotation vectors by φ_α and ψ_α. We denote

$$\{u_1, u_2, u_3, \varphi_1, \varphi_2, \varphi_3\} = \{u_\alpha, \varphi_\alpha\} = \mathbf{u}, \quad u_\alpha, \varphi_\alpha \in W_2^1(0, L)$$

$$\{v_1, v_2, v_3, \psi_1, \psi_2, \psi_3\} = \{v_\alpha, \psi_\alpha\} = \mathbf{v}, \quad v_\alpha, \psi_\alpha \in W_2^1(0, L)$$

so that $\mathbf{u}$ and $\mathbf{v}$ represent displacement and micro-rotation fields.

We define the space $\mathbf{W}$ and the space $\mathring{\mathbf{W}}$ as follows:

$$\mathbf{W} = [W_2^1(0, L)]^6, \ \mathring{\mathbf{W}} = [\mathring{W}_2^1(0, L)]^6 .$$

The norm in $\mathbf{W}$ is given by the formula:

$$\| u \|_W^2 = \sum_{\alpha=1}^{3} (\| u_\alpha \|^2_{W_2^1(0,L)} + \| \varphi_\alpha \|^2_{W_2^1(0,L)}). \tag{3.1}$$

Let $\mathbf{V}$ be that subspace of $\mathbf{W}$ containing all elements $\mathbf{v} \in \mathbf{W}$ which fulfil the homogenous boundary conditions (2.10). It is well known that $\mathbf{V} = \mathring{\mathbf{W}}$.

The bilinear form $A(\mathbf{v}, \mathbf{u})$ on $\mathbf{W} \times \mathbf{W}$ is the following:

$$A(\mathbf{v}, \mathbf{u}) = \int_0^L [E_{\alpha\beta}\, z_\alpha(\mathbf{v})\, z_\beta(\mathbf{u}) + F_{\alpha\beta}\, (z_\alpha(\mathbf{v})\, k_\beta(\mathbf{u}) + z_\beta(\mathbf{u})\, k_\alpha(\mathbf{v})) + G_{\alpha\beta}\, k_\alpha(\mathbf{u})\, k_\alpha(\mathbf{v})]\, dS \ , \tag{3.2}$$

where

$$z_\alpha(\mathbf{v}) = \hat{v}_\alpha + e_{\alpha\beta\gamma} T_\beta \psi_\gamma ; \quad k_\alpha(\mathbf{v}) = \hat{\psi}_\alpha,$$

$$z_\alpha(\hat{\mathbf{u}}) = \hat{u} + e_{\alpha\beta\gamma} T_\beta\, \varphi_\gamma ; \quad k_\alpha(\mathbf{u}) = \hat{\varphi}_\alpha,$$

and where E, F and G satisfy the conditions of § 2.

We consider now the functional:

$$f(\mathbf{u}) = \int_0^L \rho_R (f_\alpha v_\alpha + l_\alpha \psi_\alpha)\, dS ; \quad \mathbf{v} \in \mathbf{V}, f_\alpha, l_\alpha \in L_2(0, L). \tag{3.3}$$

Definition 3.1. *We say that* $\mathbf{u} \in \mathbf{W}$ *is a weak solution of the boundary-value problem if* $\mathbf{u} \in \mathbf{V}$ *and if for each* $\mathbf{v} \in \mathbf{V}$ *the following relation:*

$$\int_0^L [E_{\alpha\beta}\, z_\alpha(\mathbf{u})\, z_\beta(\mathbf{v}) + F_{\alpha\beta}\, (z_\alpha(\mathbf{v})\, k_\beta(\mathbf{u}) + z_\beta(\mathbf{u})\, k_\alpha(\mathbf{v})) + G_{\alpha\beta}\, k_\alpha(\mathbf{u})\, k_\beta(\mathbf{v})]\, dS = \int_0^L \rho_R (f_\alpha v_\alpha + l_\alpha \psi_\alpha)\, dS . \tag{3.4}$$

holds.

Remark. The definition of the weak solution expresses the variational principle of minimum potential energy: the quadratic functional

$$\mathcal{E}(\mathbf{u}) = \int_0^L \left(\frac{1}{2} E_{\alpha\beta}\, z_\alpha(\mathbf{u})\, z_\beta(\mathbf{u}) + F_{\alpha\beta}\, z_\alpha(\mathbf{u})\, k_\beta(\mathbf{u}) + \frac{1}{2} G_{\alpha\beta}\, k_\alpha(\mathbf{u})\, k_\beta(\mathbf{u}) \right) dS - \int_0^L \rho_R (f_\alpha v_\alpha + l_\alpha \psi_\alpha)\, dS$$

is minimized on $\mathbf{V}$ if and only if $\mathbf{u}$ is the weak solution.

If $\mathbf{u}$ is a weak solution then $\tau_\alpha, m_\alpha \in L_2(0, L)$ but they need not belong to $W_2^1(0, L)$. We choose the operators N_e from §1 in the form:

$$N_1(\mathbf{v}) = z_1(\mathbf{v}),\ N_2(\mathbf{v}) = z_2(\mathbf{v}),\ N_3(\mathbf{v}) = z_3(\mathbf{v}).$$
$$N_4(\mathbf{v}) = k_1(\mathbf{v}),\ N_5(\mathbf{v}) = k_2(\mathbf{v}),\ N_6(\mathbf{v}) = k_3(\mathbf{u}). \tag{3.5}$$

We give now the following theorem of existence and uniqueness:

Theorem 3.1 *If the condition* (2.9) *holds, then there exists one and only one weak solution* $\mathbf{u} \in \mathbf{W}$ *and the estimate*

$$\| \mathbf{u} \|_W \leqslant C \left(\sum_{j=1}^{3} \| f_\alpha \|^2_{L_2(0,L)} + \| l_\alpha \|^2_{L_2(0,L)} \right)^{\frac{1}{2}} \tag{3.6}$$

holds.

Proof. Concerning the operators N_e, we have from (3.5)

$$\int_0^L \sum_{\alpha=1}^{3} (z_\alpha^2 + k_\alpha^2)\, dS = \sum_{e=1}^{6} \| N_e \mathbf{v} \|^2_{L_2(0,L)}$$

and from (2.9) and (3.2) we obtain:

$$A(\mathbf{v}, \mathbf{u}) \geqslant c_2 \sum_{e=1}^{6} \| N_e \mathbf{v} \|^2_{L_2(0,L)}, \ c_2 > 0 . \tag{3.7}$$

From (1.5) and (3.5) it is easy to see that $n_{(i)i} = 1$ ($i = 1, 2, \ldots 6$ non summed) and $n_{(i)j} = 0$ for $i \neq j$. Hence the matrix $N_{rs}\xi$ has the rank 6 and from Theorem 1.3 the system (3.5) is coercive. The set $\mathcal{P}$ defined by

$$\mathcal{P} = \left\{ \mathbf{v} \in \mathbf{V}, \sum_{e=1}^{6} \| N_e \mathbf{v} \|^2_{L_2} = 0 \right\}.$$

consists of those $\mathbf{v} \in \mathbf{V}$ for which $z_\alpha(\mathbf{v}) = k_\alpha(\mathbf{v}) = 0$. Applying the Fundamenthal Lemma of the Calculus of Variations $k_\alpha(\mathbf{v}) = 0$ implies $\varphi_\alpha = b_\alpha = \text{const}$. Similarly with the case of the classical elasticity (cf. Lemma II.1 of [2]) we can prove that:

$$v_\alpha = a_\alpha - e_{\alpha\beta\gamma} T_\beta\, b_\gamma\, S, \qquad a_\alpha = \text{const.}$$

Hence we have:

$$v \in P \Rightarrow \begin{cases} v_\alpha = a_\alpha - e_{\alpha\beta\gamma}\, T_\beta\, b_\gamma\, S, \\ \psi_\alpha = b_\alpha . \end{cases}$$

Since we consider zero displacements on the boundary, from $v_\alpha = \psi_\alpha = 0$ it results:

$$\mathcal{D} = \{0\}. \tag{3.8}$$

We can now use the Theorem 1.1 to obtain the V-ellipticity of the bilinear form $A(\mathbf{v}, \mathbf{u})$. Theorem 3.1 follows immediately from Theorem 1.2, whose hypotheses are satisfied. From the inequality (1.11), taking $\bar{\mathbf{u}} = g_s = G_s = 0$, it follows obviously (3.6).

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EXISTENŢA ŞI UNICITATEA SOLUŢIEI ECUAŢIILOR DE ECHILIBRU PENTRU TEORIA LINIARĂ A BARELOR SUBŢIRI ELASTICE COSSERAT

Rezumat

În această Notă se porneşte de la teoria liniară a barelor subţiri elastice Cosserat dată recent de A. B. Whitman şi C. N. DeSilva. Pentru această teorie aplicăm unele teoreme referitoare la probleme la limită pentru sisteme eliptice de ecuaţii cu derivate parţiale şi obţinem o teoremă de existenţă şi unicitate a soluţiei slabe în cazul echilibrului. Ipoteza de bază necesară în demonstraţie este uniform pozitiva definire a funcţiei energie de deformare. S-a considerat bara iniţial dreaptă şi lipsită de tensiuni.

LE SCIENZE MATEMATICHE. Raccolta di saggi. Unione Matematica Italiana, Nicola Zanichelli. Editore, Bologna, 1973. XII, 312 p.

Culegerea de articole apărută sub titlul de mai sus este traducerea cărţii *The mathematical sciences*, apărută în 1969 la Massachussets Institute of Technology prin grija „Comitetului pentru promovarea cercetării în ştiinţele matematice", al Consiliului Naţional pentru Cercetare din S.U.A., cu colaborarea lui George A. W. Boehm.

Pornind de la necesitatea de a evalua situaţia actuală a matematicienilor, de a prevedea viitorul ei, Comitetul pentru promovarea cercetării în ştiinţele matematice a înaintat un raport C.N.C. din S.U.A. Cu această ocazie, pentru a da o idee oamenilor de ştiinţă — aş adăuga eu — şi oamenilor de cultură în general, despre rolul şi natura cercetării matematice, s-a alcătuit această culegere de 22 articole scrise de persoane de mare renume în diverse ramuri de matematică, care expun — la un nivel foarte elementar — obiectul, rolul şi perspectiva disciplinei cu care se ocupă; S. Ulam, L. Bers, J. Kemeny, J. Lederberg, H. S. M. Coxeter, J. Kiefer, J. T. Schwartz, E. J. Mc Shane, F. Dyson, A. S. Wightman, P. Davis, G. Forsythe, S. Eilenberg, L. R. Klein, A. M. Gleason, L. Harris, G. C. Rota, R. H. Bing, H. Cohen, M. Kac, R. Smullyan acoperă aproape toată aria matematicii şi a aplicaţiilor ei; de la Topologia algebrică, Funcţii complexe, la Ştiinţe biomedicale, Lingvistică, Ştiinţe sociale şi Topologie moleculară.

Este remarcabil faptul că, deşi lucrînd în domenii atît de diverse, cartea a putut să apară la un nivel unitar de accesibilitate, încît să poată fi considerată ca un model pentru modul cum trebuie făcută popularizarea unei discipline ştiinţifice.

Adolf Haimovici

M. MILLER: *Gelöste und ungelöste mathematische Probleme*. Teubner Verlagsgesellschaft, Leipzig, 1973, 96 p.

Autorul a reunit un număr de 25 de probleme care şi-au dobîndit celebritatea atît prin numele matematicienilor care s-au ocupat de ele, cît şi prin interesul lor mai aparte. Întîlnim astfel probleme privitoare la numere prime (Goldbach, Fermat, Euler, Wilson ş.a.), probleme de împărţire a unui poligon, a planului, a spaţiului, a unei sfere, prin drepte, respectiv plane, cercuri (Euler, Catalan, Steiner ş.a.), probleme privitoare la ecuaţii sau sisteme de ecuaţii (Fermat, Leibniz, Newton, Abel ş.a.). Interesul cărţii constă atît în notiţele istorice, cît şi în indicaţiile cheie privind soluţiile.

David Rimer