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CURGEREA NESTAȚIONARĂ A UNUI FLUID DE ORDINUL AL DOILEA ÎNTRE DOI CILINDRI COAXIALI POROȘI

Rezumat

Se studiază curgerea elicoidală nestaționară a unui fluid de ordinul al doilea între doi cilindri coaxiali poroși. Presupunind că numărul lui Reynold, *R*, este mic, se determină viteza în forma unei serii după puterile lui *R*. Citeva cazuri speciale sint considerate.

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ON BENDING OF MICROPOLAR PLATES

BY

AUREL IDELOVICI

1. Statement of the problem. Consider a thin plate of constant thickness $2h$ with its middle surface in the x_1Ox_2 — plane. Let Ω be the region occupied by the plate, Σ — its boundary surface, S — the edge surface, C — its boundary curve given by $x_\alpha = x^*_\alpha(s)$ ($\alpha = 1, 2$), where s is arc length. The plate is assumed to consist of isotropic, homogeneous elastic material.

In what follows, we use the notation of [2], [6]. Greek indices take the values 1, 2, and Latin indices take the values 1, 2, 3. Subscripts following commas denote partial differentiation.

The basic equations of linear micropolar elasticity with independent local rotations, in the absence of body forces and couples, are:

— the equilibrium equations

$$\sigma_{ji,j} = 0, \quad \mu_{ji,j} + \varepsilon_{ikl}\sigma_{kl} = 0; \tag{1}$$

— the kinematical relations

$$\beta_{ij} = U_{j,i} - \varepsilon_{ijk}\Phi_k, \quad \varphi_{ij} = \Phi_{j,i}; \tag{2}$$

— the constitutive equations

$$\sigma_{ij} = \lambda\beta_{hh}\delta_{ij} + 2\mu\beta_{(ij)} + 2\alpha\beta_{[ij]}, \qquad \mu_{ij} = \beta\varphi_{hh}\delta_{ij} + 2\gamma\varphi_{(ij)} + 2\varepsilon\varphi_{[ij]}, \tag{3}$$

where σ_{ij} are stresses, μ_{ij} — couple stresses, β_{ij} and φ_{ij} — strain tensors, U_i — displacements, Φ_i — microrotations, $\lambda, \mu, \alpha, \beta, \gamma, \varepsilon$ — constants, ε_{ikl} — the permutation symbol, δ_{ij} — Kronecker's delta; the symbols ( ) and [ ] denote the symmetric, respectively the skew-symmetric part of a tensor.

The boundary conditions are

$$\sigma_{ji}n_j = \overline{P}_i, \quad \mu_{ji}n_j = \overline{Q}_i, \tag{4}$$

where $\overline{P}_i$, $\overline{Q}_i$ are exterior loads (surface tractions and couples, respectively), and n_i are the components of the outward unit normal to Σ.

In what follows we consider only the bending of the plate, assuming that U_3, Φ_α, $\sigma_{\alpha\beta}$, $\sigma_{3\alpha}$, μ_{33} are even functions of x_3, and U_α, Φ_3, $\sigma_{\alpha 3}$, $\mu_{\alpha 3}$, $\mu_{3\alpha}$, are odd functions of x_3. Accordingly we impose the following boundary conditions:

$$\sigma_{3\alpha} = 0, \quad \sigma_{33} = \mp \theta\sigma_0\frac{p}{2}, \quad \mu_{3\alpha} = \mp \theta\mu_0\frac{q_\alpha}{2}, \quad \mu_{33} = 0 \quad \text{— on faces } (x_3 = \pm h). \tag{5}$$

$$\left.\begin{aligned} P_\alpha &\equiv \sigma_{\lambda\alpha}n_\lambda = \theta\sigma_0\overline{p}_\alpha, \quad P_3 \equiv \sigma_{\lambda 3}n_\lambda = \sigma_0\overline{p}_3 \\ Q_\alpha &\equiv \mu_{\lambda\alpha}n_\lambda = \mu_0\overline{q}_\alpha, \quad Q_3 \equiv \mu_{\lambda 3}n_\lambda = \theta\mu_0\overline{q}_3 \end{aligned}\right\} \quad \text{— on } S\ (x_\alpha = x^*_\alpha(s), |x_3| \leq h). \tag{6}$$

where p and q_α are fonctions of x_1 and x_2, $\bar{p}_i$ and $\bar{q}_i$ are functions of s and x_3($\bar{p}_\alpha$, $\bar{q}_3$ — odd in x_3, $\bar{p}_3$, $\bar{q}_\alpha$ — even in x_3). σ_0 and μ_0 are a reference stress and a reference couple stress, θ is the small parameter h/L, where L is the smallest characteristic lenght of the plate; here p, q_α, $\bar{p}_i$, $\bar{q}_i$ are dimensionless.

2. Non-dimensionalization. We introduce dimensionless quantities by setting (cf. [1], [5]):

$$
\begin{aligned}
&x_\alpha = L\xi_\alpha, \quad x_3=\theta\zeta, \quad \sigma_{\alpha\beta}=\sigma_0\theta s_{\alpha\beta}, \quad \sigma_{\alpha 3}=\sigma_0 s_{\alpha 3}, \quad \sigma_{3\alpha}=\sigma_0\theta^2 s_{3\alpha}, \quad \sigma_{33}=\sigma_0\theta s_{33},\\
&\mu_{\alpha\beta}=\mu_0 m_{\alpha\beta}, \quad \mu_{\alpha 3}=\mu_0\theta m_{\alpha 3}, \quad \mu_{3\alpha}=\mu_0\theta m_{3\alpha}, \quad \mu_{33}=\mu_0\theta^2 m_{33},\\
&EU_\alpha=\sigma_0\theta L u_\alpha, \quad EU_3=\sigma_0 L u_3, \quad E^*\Phi_\alpha=\mu_0 L\varphi_\alpha, \quad E^*\Phi_3=\mu_0\theta L\varphi_3,
\end{aligned}
\tag{7}
$$

where

$$
E=\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}, \quad \nu=\frac{\lambda}{2(\lambda+\mu)}, \quad E^*=\frac{\gamma(3\beta+2\gamma)}{\beta+\gamma}, \quad \nu^*=\frac{\beta}{2(\beta+\gamma)}. \tag{8}
$$

We can write the equations (3) in a more convenient form, by using (2) and (8), and then setting $E^*=L^2\bar{E}$, $\nu^*=\bar{\nu}$, $\varepsilon=L^2\bar{\varepsilon}$ (cf. [1], [2]), so that $\bar{E}$ and $\bar{\varepsilon}$ are of the same dimension as E and α, and ν, $\bar{\nu}$ are dimensionless. We suppose $\mu_0=L\sigma_0$. Introduction of (7) into the modified equations (1) and (3) yields

$$
\begin{aligned}
&s_{\lambda\alpha,\lambda}+s_{3\alpha,3}=0, \quad s_{\lambda 3,\lambda}+s_{33,3}=0, \quad u_{3,3}=\theta^2(s_{33}-\nu s_{\lambda\lambda}), \quad \varphi_{3,3}=\theta^2 m_{33}-\nu m_{\lambda\lambda},\\
&m_{\lambda\alpha,\lambda}+m_{3\alpha,3}+\varepsilon_{\alpha\lambda}(s_{\lambda 3}-\theta^2 s_{3\lambda})=0, \quad m_{\lambda 3,\lambda}+m_{33,3}+s_{12}-s_{21}=0,\\
&s_{\alpha\beta}=\left(\frac{\nu}{1-\nu}s_{33}+\frac{\nu}{1-\nu^2}u_{\lambda,\lambda}\right)\delta_{\alpha\beta}+\frac{1}{1+\nu}u_{(\alpha,\beta)}-2\frac{\alpha}{E}\left(u_{[\alpha,\beta]}+\frac{E}{\bar{E}}\varepsilon_{\alpha\beta}\varphi_3\right),\\
&\left(\frac{E}{1+\nu}+2\alpha\right)s_{\alpha 3}-\theta^2\left(\frac{E}{1+\nu}-2\alpha\right)s_{3\alpha}=\frac{4\alpha}{1+\nu}\left(u_{3,\alpha}+\frac{E}{\bar{E}}\varepsilon_{\alpha\lambda}\varphi_\lambda\right),\\
&\theta^2 s_{3\alpha}=\frac{1}{1+\nu}u_{(3,\alpha)}-2\frac{\alpha}{E}\left(u_{[3,\alpha]}+\frac{E}{\bar{E}}\varepsilon_{\alpha\lambda}\varphi_\lambda\right), \quad \theta^2(m_{\alpha 3}+m_{3\alpha})=\frac{1}{1+\bar{\nu}}(\theta^2\varphi_{3,\alpha}+\varphi_{\alpha,3}),\\
&m_{\alpha\beta}=\left(\theta^2\frac{\bar{\nu}}{1-\bar{\nu}}m_{33}+\frac{\bar{\nu}}{1-\bar{\nu}^2}\varphi_{\lambda,\lambda}\right)\delta_{\alpha\beta}+\frac{1}{1+\bar{\nu}}\varphi_{(\alpha,\beta)}-2\frac{\bar{\varepsilon}}{\bar{E}}\varphi_{[\alpha,\beta]},\\
&\left(\frac{\bar{E}}{1+\bar{\nu}}+2\bar{\varepsilon}\right)m_{\alpha 3}-\left(\frac{\bar{E}}{1+\bar{\nu}}-2\bar{\varepsilon}\right)m_{3\alpha}=\frac{4\bar{\varepsilon}}{1+\bar{\nu}}\varphi_{3,\alpha},
\end{aligned}
\tag{9}
$$

where $\varepsilon_{\alpha\beta}$ is the alternating symbol, and now $(\ldots)_{,\alpha}=\partial(\ldots)/\partial\xi_\alpha$, $(\ldots)_{,3}=\partial(\ldots)/\partial\zeta$. (5) and (6) become

$$
s_{3\alpha}=0, \quad s_{33}=\mp\frac{p^*}{2}, \quad m_{3\alpha}=\mp\frac{\overset{*}{q}_\alpha}{2}, \quad m_{33}=0, \quad (\zeta=\pm 1). \tag{10}
$$

$$
P_i^*\equiv s_{\lambda i}n_\lambda=\bar{p}_i^*, \quad Q_i^*\equiv m_{\lambda i}n_\lambda=\bar{q}_i^*, \quad (\xi_\alpha=\xi_\alpha^*(\sigma), \ |\zeta|\leqslant 1), \tag{11}
$$

where $p^*=p^*(\xi_1, \xi_2)$, $\overset{*}{q}_\alpha=q_\alpha^*(\xi_1, \xi_2)$, $ds=Ld\sigma$, $\bar{p}_i^*=\bar{p}_i^*(\sigma, \zeta)$, $\bar{q}_i^*=\bar{q}_i^*(\sigma, \zeta)$, $n_\alpha=\varepsilon_{\alpha\lambda}\xi^*_{\lambda,s}$, $p^*(\xi_1, \xi_2)=p(L\xi_1, L\xi_2)$, etc.; p^*, q_α^*, $\bar{p}_i^*$, $\bar{q}_i^*$, as well as their derivatives are of order unity.

3. Asymptotic expansion. We assume expansions of the form

$$
u_i=u_i^{(0)}+\theta^2 u_i^{(2)}+\ldots, \quad \varphi_i=\varphi_i^{(0)}+\theta^2\varphi_i^{(2)}+\ldots, \quad s_{ij}=s_{ij}^{(0)}+\theta^2 s_{ij}^{(2)}+\ldots, \quad m_{ij}=m_{ij}^{(0)}+\theta^2 m_{ij}^{(2)}+\ldots \tag{12}
$$

Inserting (12) into (9), (10) and (11), and equating the coefficients of the same powers of θ^2, we obtain successive systems of differential equations and boundary conditions. For instance, the conditions (11) on the lateral surface become

$$
P_i^{*(0)}\equiv s_{\lambda i}n_\lambda=\bar{p}_i^*, \quad Q_i^{*(0)}\equiv m_{\lambda i}^{(0)}n_\lambda=\bar{q}_i^*; \quad P_i^{*(k)}=Q_i^{*(k)}=0, \ k=2, 4, \ldots \quad (\xi_\alpha=\xi_\alpha^*(\sigma), \ |\zeta|\leqslant 1). \tag{13}
$$

By using only the face-boundary conditions, these systems can be integrated explicitly with respect to ζ. We give here only the firstorder approximation:

$$
\begin{aligned}
&u_3^{(0)}=w^{(0)}(\xi_1, \xi_2), \quad \varphi_\alpha^{(0)}=\psi_\alpha^{(0)}(\xi_1, \xi_2), \quad s_{\alpha 3}^{(0)}=\frac{4\alpha}{E+2\alpha(1+\nu)}\left(w^{(0)}_{,\alpha}+\frac{E}{\bar{E}}\varepsilon_{\alpha\lambda}\psi_\lambda^{(0)}\right),\\
&m_{\alpha\beta}^{(0)}=\frac{\bar{\nu}}{1-\bar{\nu}^2}\psi^{(0)}_{\lambda,\lambda}\delta_{\alpha\beta}+\frac{1}{1+\bar{\nu}}\psi^{(0)}_{(\alpha,\beta)}-2\frac{\bar{\varepsilon}}{\bar{E}}\psi^{(0)}_{[\alpha,\beta]}, \quad s_{33}^{(0)}=-\frac{p^*}{2}\zeta, \quad m_{3\alpha}^{(0)}=-\frac{\overset{*}{q}_\alpha}{2}\zeta,\\
&\varphi_3^{(0)}=-\frac{\bar{\nu}}{1-\bar{\nu}}\psi^{(0)}_{\lambda,\lambda}\zeta, \quad u_\alpha^{(0)}=\left[-\frac{\mu-\alpha}{\mu+\alpha}w^{(0)}_{,\alpha}+\frac{2E\alpha}{\bar{E}(\mu+\alpha)}\varepsilon_{\alpha\lambda}\psi_\lambda^{(0)}\right]\zeta,\\
&s_{\alpha\beta}^{(0)}=-\left\{-\frac{\nu}{1-\nu^2}w^{(0)}_{,\lambda\lambda}\delta_{\alpha\beta}+\frac{1}{1+\nu}\left[-\frac{\mu-\alpha}{\mu+\alpha}w^{(0)}_{,\alpha\beta}+\frac{E\alpha}{\bar{E}(\mu+\alpha)}(\varepsilon_{\alpha\lambda}\psi^{(0)}_{\lambda,\beta}+\varepsilon_{\beta\lambda}\psi^{(0)}_{\lambda,\alpha})\right]+\right.\\
&\quad\left.+2\varepsilon_{\alpha\beta}\frac{\alpha}{\bar{E}}\left(-\frac{\alpha}{\mu+\alpha}+\frac{\bar{\nu}}{1-\bar{\nu}}\right)\psi^{(0)}_{\lambda,\lambda}\right\}\zeta, \quad m_{\alpha 3}^{(0)}=-\left[\frac{\bar{\gamma}-\bar{\varepsilon}}{\bar{\gamma}+\bar{\varepsilon}}\frac{\overset{*}{q}_\alpha}{2}+\frac{2\bar{\nu}\bar{\varepsilon}}{(1-\bar{\nu}^2)(\bar{\gamma}+\bar{\varepsilon})}\psi^{(0)}_{\lambda,\lambda\alpha}\right]\zeta,\\
&s_{3\alpha}^{(0)}=S_{3\alpha}^{(0)}(w, \psi_\lambda)\frac{\zeta^2-1}{2}, \quad m_{33}^{(0)}=M_{33}^{(0)}(w, \psi_\lambda)\frac{\zeta^2-1}{2},
\end{aligned}
\tag{14}
$$

(we do not write $S_{3\alpha}^{(0)}$ and $M_{33}^{(0)}$ here explicitly) where the functions $w^{(0)}$ and $\psi_\alpha^{(0)}$ satisfy the following equations:

$$
\begin{aligned}
&\frac{4\alpha}{E+2\alpha(1+\nu)}\left(w^{(0)}_{,\lambda\lambda}+\frac{E}{\bar{E}}\varepsilon_{\lambda\mu}\psi^{(0)}_{\mu,\lambda}\right)=\frac{p^*}{2},\\
&\frac{2}{1}\left(\frac{1}{1+\bar{\nu}}+2\frac{\bar{\varepsilon}}{\bar{E}}\right)\psi^{(0)}_{\alpha,\lambda\lambda}+\frac{1}{2}\left(\frac{1}{1-\bar{\nu}}-2\frac{\bar{\varepsilon}}{\bar{E}}\right)\psi^{(0)}_{\lambda,\lambda\alpha}+\frac{4\alpha}{E+2\alpha(1+\nu)}\left(\varepsilon_{\alpha\lambda}w^{(0)}_{,\lambda}-\frac{E}{\bar{E}}\psi_\alpha^0\right)=\frac{\overset{*}{q}_\alpha}{2}.
\end{aligned}
\tag{15}
$$

One can also obtain easily the higher-order approximations depending on $w^{(k)}(\xi_1, \xi_2)$, $\psi_\alpha^{(k)}(\xi_1, \xi_2)$ and lower-order approximations.

4. Boundary conditions. To derive the boundary conditions for the edge of the plate, we use a variational theorem, similar to that of [4], [5]. Let be:

$$
\begin{aligned}
I=\iiint_\Omega\Big\{&\sigma_{ij}\beta_{ij}+\mu_{ij}\varphi_{ij}-\frac{1}{2}\Big[-\frac{\nu}{E}(\sigma_{hh})^2+\frac{1+\nu}{E}\sigma_{(ij)}\sigma_{(ij)}+\frac{1}{2\alpha}\sigma_{[ij]}\sigma_{[ij]}-\\
&-\frac{\nu^*}{E^*}(\mu_{hh})^2+\frac{1+\nu^*}{E^*}\mu_{(ij)}\mu_{(ij)}+\frac{1}{2\varepsilon}\mu_{[ij]}\mu_{[ij]}\Big]\Big\}dV-\iint_\Sigma(\bar{P}_i U_i+\bar{Q}_i\Phi_i)d\sigma.
\end{aligned}
\tag{16}
$$

$\delta I=0$ leads to:

$$
\begin{aligned}
(17)\quad &\iiint\limits_{\Omega}\Big\{\Big[\beta_{ij}+\frac{\nu}{E}\sigma_{hh}\,\delta_{ij}-\frac{1+\nu}{E}\sigma_{(ij)}-\frac{1}{2\alpha}\sigma_{[ij]}\Big]\delta\sigma_{ij}+\Big[\frac{\nu^*}{E^*}\mu_{hh}\sigma_{ij}-\frac{1+\nu^*}{E^*}\mu_{(ij)}-\\
&\frac{1}{2\varepsilon}\mu_{[ij]}\Big]\delta\mu_{ij}-\sigma_{ij,i}\,\delta U_j-(\mu_{ij,i}+\varepsilon_{ijk}\sigma_{ik})\delta\Phi_j\Big\}dx_1\,dx_2\,dx_3+\iint\limits_{S}(\sigma_{\lambda j}n_\lambda\,\delta U_j+\\
&+\mu_{\lambda j}n_\lambda\delta\Phi_j)dx_3\,ds+2\iint\limits_{D}(\sigma_{3j}\delta U_j+\mu_{3j}\delta\Phi_j)_{x_3=h}dx_1\,dx_2-\iint\limits_{S}(\theta\sigma_0\bar p_\alpha\delta U_\alpha+\sigma_0\bar p_3\delta U_3+\\
&\mu_0\bar q_\alpha\delta\Phi_\alpha+\theta\mu_0\bar q_3\delta\Phi_3)dx_3\,ds+2\iint\limits_{D}\Big(\theta\sigma_0\frac{p}{2}\,\delta U_3+\theta\mu_0\frac{q_\lambda}{2}\,\delta\Phi_\lambda\Big)_{x_3=h}dx_1\,dx_2=0.
\end{aligned}
$$

Taking into account (7) to (11), and using (12), the equation (17) reduces to

$$
(18)\qquad \iint\limits_{S^*}\Big\{\theta^2(P_\lambda^*-\bar p_\lambda^*)\delta u_\lambda+(P_3^*-\bar p_3^*)\delta u_3+\frac{E}{E}[(Q_\lambda^*-\bar q_\lambda^*)\,\delta\varphi_\lambda+\theta^2(Q_3^*-\bar q_3^*)\,\delta\varphi_3]\Big\}d\zeta\,d\sigma=0.
$$

Introduction of the expansions $P_\lambda^*=P_\lambda^{*(0)}+\theta^2P_\lambda^{*(2)}+\ldots$, etc., into (18) yields

$$
(19)\qquad \iint\limits_{S^*}\Big[(P_3^{*(0)}-\bar p_3^{*(0)})\delta u_3+\frac{E}{E}(Q_\lambda^{*(0)}-\bar q_\lambda^*)\,\delta\varphi_\lambda\Big]d\zeta\,d\sigma=0,\ \text{etc.}
$$

If we take in (19) $\delta u_3=\delta u_3^{(0)}$, $\delta\varphi_\alpha=\delta\varphi_\alpha^{(0)}$, where $u_3^{(0)}$, $\varphi_\alpha^{(0)}$ are given by (14), we obtain for the first approximation

$$
(20)\qquad \int_{-1}^{1}P_3^{*(0)}d\zeta=\int_{-1}^{1}\bar p_3^*\,d\zeta,\quad \int_{-1}^{1}Q_\alpha^{*(0)}d\zeta=\int_{-1}^{1}\bar q_\alpha^*\,d\zeta,
$$

or

$$
(21)\qquad s_{\lambda 3}^{(0)}n_\lambda=\frac{1}{2}\int_{-1}^{1}\bar p_3^*d\zeta,\quad m_{\lambda\alpha}^{(0)}n_\lambda=\frac{1}{2}\int_{-1}^{1}\bar q_\alpha^*\,d\zeta,\quad (\xi_\alpha=\xi_\alpha^*(\sigma),\ |\zeta|\leqslant 1).
$$

The boundary value problem (15), (21) is similar to that of [3], (for which an existence and uniquenes theorem has been proved).

5. Validity of iterative procedure. Assuming ν and ν^* of order unity, the conditions for the validity of the proposed iterative procedure are

$$
(22)\qquad (\beta,\ \gamma,\ \varepsilon)=O(L^2(\lambda,\ \mu,\ \alpha)).
$$

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GENERALIZED LINEAR AUTOMATA

BY

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1. In this paper we have studied a notion of generalized linear automaton (G. L. A.), whose particular case is the notion of linear automaton. We have investigated the properties of the G. L. A. s regarding the equivalence of automata, minimal automata, a. s. o. Then we have obtained an application of the G. L. A. s to the generation of the numerical pseudo-random sequences.

1.1. Definition. *A G. L. A. is a 5-uple (I, S, O, f, g), where :*

a) *I, S, O are nonempty sets called respectively the set of inputs, the set of states and the set of outputs, $f : S\times I\to S$ and $g : S\times I\to S$ are the next state function, and respectively the output fonction ;*

b) *S and O are organized as semigroups with the internal operation denoted by $\ast$ and the unit elements O_s, respectively O_0 ;*

c) *there exists the semigroup homomorphisms $A : S\to S$, $C : S\to O$ and the mappings $B : I\to S$, $D : I\to O$ so that $f(s, i)=As\ast Bi$ and $g(s, i)=Cs\ast Di$, for every $(s, i)\in S\times I$;*

d) *there exists the element $O_I\in I$, so that $BO_I=O_s$ and $DO_I=O_0$. When $|I|=1$, the G.L.A. is called autonomous.*

1.2. If I, S, O are linear spaces and A, B, C, D are linear mappings, we obtain the linear automata ([2], p. 26).

1.3. The mappings f and g can be extended to $S\times I^*$ by taking

$$
f(s,\ px)=f(f(s,\ p),\ x),
$$

$$
g(s,\ px)=g(f(s,\ p),\ x),
$$

where $s\in S$, $x\in I$ and $p\in I^*$.

We get by induction the formulae :

$$
f(s,\ i_1\ldots i_n)=A^n s\ast\sum_{j=1}^{n}A^{n-j}\,Bi_j\ \text{(the general next state equation)},
$$

$$
g(s,\ i_1\ldots i_n)=CA^{n-1}s\ast\sum_{j=1}^{n-1}CA^{n-j-1}\,Bi_j\ast Di_n\ \text{(the general output equation)},
$$

where $i_j\in I$ for $j=1,\ 2,\ldots,\ n$ and $\sum$ is referring to the operation $\ast$.

1.4. Since the mappings A, B, C and D determine the next state and output functions, we shall also denote a G. L. A. by $\mathfrak{a}=(I, S, O ;\ A, B, C, D)$.

1.5. Definition. *Let $\mathfrak{a}=(I, S, O ;\ A, B, C, D)$ and $\mathfrak{a}'=(I', S', O' ;\ A', B', C', D')$; a G.L.A. epimorphism is a triple $h=(\alpha, \beta, \gamma)$ of surjections $\alpha : I\to I'$, $\beta : S\to S'$, $\gamma: O\to O'$ so that*

$$
\beta(As\ast Bi)=A'\beta(s)\ast B'\alpha(i),\qquad \gamma(Cs\ast Di)=C'\beta(s)\ast D'\alpha(i),\qquad \forall(s,\ i)\in S\times I.
$$

If α, β and γ are bijections, then h will be called G. L. A. isomorphism.

2. A connection between states which give the same answer when the same sequence is introduced in the automaton is given by :