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ON STABILITY IN THE THEORY OF ELASTIC CONTINUA WITH DIRECTORS

BY

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1. Introduction. The study of stability of continuous systems is the subject of a large number of works. At the end of this paper a selective bibliography is given [1]—[5], in which a more comprehensive reference on this subject can be found.

In extending the ideas of Liapunov, Movchan [1] has given a useful definition of stability and has proved analogous theorems (his main results are also presented in [2]). In the present paper the stability of elastic directed bodies according to Movchan's definition, is investigated.

2. Basic equations of thermoelastic directed continuum. The motion of a continuous medium with directors is refered to a single fixed rectangular cartesian coordinate system ox^i. The material point X determines the position in ox^i of each particle in the medium at reference time $\tau = 0$. With each material point X a noncoplanar triad field of director vectors D_α (X) and a temperature θ_0 are associated. Then at time τ the spatial position x of a material point X, as well as the associated triad of directors d_α and the temperature θ are given by

$$x_i(\tau) = x_i(X_K, \tau), \; d_{\alpha i}(\tau) = d_{\alpha i}(X_K, \tau), \; \theta(\tau) = \theta(X_K, \tau). \tag{2.1}$$

For $\tau = 0$ we have

$$\mathbf{X} = \mathbf{x}(X_K, 0), \quad D_\alpha = d_\alpha(X_K, 0), \quad \theta_0 = \theta(X_K, 0). \tag{2.2}$$

The current position of the particle is denoted by $x_i = x_i(t)$, and as a consequence, Eq. (2.1) may be rewritten as

$$x_i(\tau) = x_i(x_j, t, \tau), \; d_{\alpha i}(\tau) = d_{\alpha i}(x_j, t, \tau), \; \theta(\tau) = \theta(x_j, t, \tau), \tag{2.3}$$

provided that

$$\det\left(\partial x_i(\tau)/\partial X_I\right) > 0.$$

The components of velocity at the point x_i are given by $\dot{x}_i$ and $\dot{d}_{\alpha i}$, the material derivative being taken with respect to τ and keeping x_j and t fixed in Eq. (2.3). The second order material time derivatives of x_i and $d_{\alpha i}$ are similarly defined.

Let the material volume Ω of the body be bounded by the surface Γ which is sufficiently regular to ensure the common laws of transformation of surface integrals and Friedrichs' inequality. We assume the energy balance at time t in the form [6]

$$\int_\Omega \rho \ddot{x}_i \dot{x}_i d\Omega + \int_\Omega \rho A^{\alpha\beta\gamma\delta} \dot{w}_{\alpha\beta} w_{\gamma\delta} \, d\Omega + \int_\Omega \rho \dot{e} d\Omega = \tag{2.4}$$

$$= \int_\Omega \rho(Q + F_i \dot{x}_i + B_i^\alpha \dot{d}_{\alpha i}) d\Omega + \int_\Gamma (t_i \dot{x}_i + p_i^\alpha \dot{d}_{\alpha i} - q) d\Gamma,$$

in which ρ is the density, e is the internal energy per unit mass, Q is the heat supply function per unit mass and unit time, F_l and B^{α}_l are the body forces per unit mass, t_l and p^{α}_l are the surface forces per unit area of Γ, $A^{\alpha\beta\gamma\delta}$ are defined by

$$I^{mnpq} = A^{\alpha\beta\gamma\delta} d^m_{\alpha} d^n_{\beta} d^p_{\gamma} d^q_{\delta},$$

where I^{mnpq} are the substructure inertia coefficients and $w_{\alpha\beta}$ are some deformation measures given by $w_{\alpha\beta} = d^n_{\alpha} d_{\beta n}$.

We admit the mass conservation in the form

$$\dot{\overline{\rho d\Omega}} = 0, \tag{2.5}$$

and we suppose that [6]

$$\dot{A}^{\alpha\beta\gamma\delta} = 0. \tag{2.6}$$

In order to ensure that director motions will contribute with a positive amount to kinetic energy it is required that $A^{\alpha\beta\gamma\delta} w_{\alpha\beta} w_{\gamma\delta} > 0$.

An entropy production inequality is also postulated, which at time t, has the form

$$\frac{d}{dt}\int_{\Omega} \rho\eta d\Omega - \int_{\Omega} \rho \frac{Q}{\theta} d\Omega + \int_{\Gamma} \frac{q}{\theta} d\Gamma \geqslant 0, \tag{2.7}$$

where η is the entropy per unit mass and $\theta (> 0)$ is local temperature.

Imposing that Eq. (2.4) be invariant under rigid body motions the basic equations are obtained [6]

$$t_{kl,k} + \rho F_l = \rho \ddot{x}_l, \quad \mu_{plk,p} + t_{lk} - \pi_{lk} + \rho B_{lk} = \rho \dot{\omega}_{lk}, \text{ in } \Omega, \tag{2.8}$$

$$t_l = t_{lj} n_j, \quad \mu_{lk} = \mu_{plk} n_p, \text{ on } \Gamma, \tag{2.9}$$

$$\pi_{[lj]} = 0, \tag{2.10}$$

$$q = q_l n_l, \tag{2.11}$$

where a comma denotes partial differentiation with respect to the coordinate x_l; B_{lk} and μ_{lk} are given by $B_{lk} = d_{\alpha l} B^{\alpha}_k$, $\mu_{lk} = d_{\alpha l} p^{\alpha}_k$ respectively; $\rho\dot{\omega}$ is the rate of change of substructure momentum, given by

$$\rho\dot{\omega}_{lj} = \rho(I_{(lj)pq} \dot{w}_{[pq]} + \frac{1}{2} \dot{I}_{(lj)pq} w_{[pq]}),$$

where $w_{pq} = d^{\alpha}_p d_{\alpha q}$; n_l is the unit outward normal to Γ and π_{lj} are the micro-stress average. The stress tensor t_{kl} and the first stress moments μ_{plk} are taken over the coordinate material surfaces, and the heat flux q_l is also taken over the coordinate material surfaces per unit area of this surface per unit time.

Let us introduce the Helmholtz free energy function

$$A = e - \theta\eta. \tag{2.12}$$

From the local form of the energy balance, it can easily be proved for a thermoelastic material, the following equations

$$\rho\theta\dot{\eta} - (\rho Q - q_{k,k}) = 0. \tag{2.13}$$

Using Eq. (2.13) in Eq. (2.4) we may writte

$$\int_{\Omega} \rho \ddot{x}_l \dot{x}_l d\Omega + \int_{\Omega} \rho A^{\alpha\beta\gamma\delta} \dot{w}_{\alpha\beta} w_{\gamma\delta} d\Omega + \int_{\Omega} \rho(\dot{A} + \dot{\theta}\eta) d\Omega =$$
$$= \int_{\Omega} \rho(F_l \dot{x}_l + B^{\alpha}_l \dot{d}_{\alpha l}) d\Omega + \int_{\Gamma} (t_l \dot{x}_l + p^{\alpha}_l \dot{d}_{\alpha l}) d\Gamma. \tag{2.14}$$

3. The energy criterion for stability of continuous system with directors. Let us establish sufficient conditions to ensure the stability of the motion given by Eq. (2.1) or Eq. (2.3). This is taken as unperturbed motion and we assume that at time t_0 it is perturbed by changes in the body and surface forces and also in the heat supply and heat flux. Let the perturbed body and surface forces be $\overline{F}_l(\tau)$, $\overline{B}^{\alpha}_l(\tau)$ per unit mass and $\overline{t}_l(\tau)$, $\overline{p}^{\alpha}_l(\tau)$ per unit area of Γ. Then for $\tau < t_0$ we have

$$\overline{F}_l(\tau) = F_l(\tau), \ \overline{B}^{\alpha}_l(\tau) = B^{\alpha}_l(\tau), \ \overline{t}_l(\tau) = t_l(\tau), \ \overline{p}^{\alpha}_l(\tau) = p^{\alpha}_l(\tau).$$

The particle X_K has coordinates $\overline{x}_l(\tau)$ and a temperature $\overline{\theta}(\tau)$ at time τ in the perturbed motion. Further we suppose that directors $\mathbf{D}_{\alpha}(\mathbf{X})$ will deform in vectors $\overline{\mathbf{d}}_{\alpha}(\tau)$ associated to position $\overline{x}_l(\tau)$. Hence we have

$$\overline{x}_l(\tau) = \overline{x}_l(X_K, \tau), \ \overline{d}_{\alpha l}(\tau) = \overline{d}_{\alpha l}(X_K, \tau), \ \overline{\theta}(\tau) = \overline{\theta}(X_K, \tau), \ t_0 \leqslant \tau \leqslant t. \tag{3.1}$$

By assuming that $\det(\partial x_i(\tau)/\partial X_J) > 0$, Eq. (3.1) may be written as:

$$\overline{x}_l(\tau) = \overline{x}_l(x_k, t, \tau), \ \overline{d}_{\alpha l}(\tau) = \overline{d}_{\alpha l}(x_k, t, \tau), \ \overline{\theta}(\tau) = \overline{\theta}(x_k, t, \tau), \tag{3.2}$$

so that the current position x_l of the particle becomes $\overline{x}_l = \overline{x}_l(x_k, t, t)$. The equation corresponding to Eq. (2.14) in the perturbed motion has the form

$$\int_{\Omega} \rho \ddot{\overline{x}}_l \dot{\overline{x}}_l d\Omega + \int_{\Omega} \rho A^{\alpha\beta\gamma\delta} \dot{\overline{w}}_{\alpha\beta} \overline{w}_{\gamma\delta} d\Omega + \int_{\Omega} \rho(\dot{\overline{A}} + \dot{\overline{\theta}}\overline{\eta}) d\Omega =$$
$$= \int_{\Omega} \rho(\overline{F}_l \dot{\overline{x}}_l + \overline{B}^{\alpha}_l \dot{\overline{d}}_{\alpha l}) d\Omega + \int_{\Gamma} (\overline{t}_l \dot{\overline{x}}_l + \overline{p}^{\alpha}_l \dot{\overline{d}}_{\alpha l}) d\Gamma, \tag{3.3}$$

where $\overline{A}$, $\overline{\eta}$, $\overline{\theta} (> 0)$ are the Helmholtz free energy function, the entropy per unit mass and local temperature of the body in its perturbed motion respectively. In (3.3) the integrals are extended over the unperturbed material volume and surface of the body.

Now let $u_l = \overline{x}_l - x_l$ and $\theta_{\alpha l} = \overline{d}_{\alpha l} - d_{\alpha l}$. Substraction of Eq. (2.14) from Eq. (3.3) then gives

$$\int_{\Omega} \rho(\dot{\overline{A}} - \dot{A}) d\Omega + \int_{\Omega} \rho(\ddot{\overline{x}}_l \dot{\overline{x}}_l - \ddot{x}_l \dot{x}_l + A^{\alpha\beta\gamma\delta}(\dot{\overline{w}}_{\alpha\beta} \overline{w}_{\gamma\delta} - \dot{w}_{\alpha\beta} w_{\gamma\delta})) d\Omega =$$
$$= \int_{\Omega} \rho(\dot{\theta}\eta - \dot{\overline{\theta}}\overline{\eta}) d\Omega + \int_{\Omega} (\overline{F}_l \dot{u}_l + (\overline{F}_l - F_l)\dot{x}_l + \overline{B}^{\alpha}_l \dot{\theta}_{\alpha l} + (\overline{B}^{\alpha}_l - B^{\alpha}_l)\dot{d}_{\alpha l}) d\Omega +$$
$$+ \int_{\Gamma} (\overline{t}_l \dot{u}_l + (\overline{t}_l - t_l)\dot{x}_l + \overline{p}^{\alpha}_l \dot{\theta}_{\alpha l} + (\overline{p}^{\alpha}_l - p^{\alpha}_l)\dot{d}_{\alpha l}) d\Gamma. \tag{3.4}$$

We take for the Movchan-Liapunov functional

$$
(3.5)\qquad \mathcal{J}_1(t) = \int_\Omega \rho(\bar{A} - A)d\Omega + \frac{1}{2}\int_\Omega \rho(\dot{\bar{x}}_i\dot{\bar{x}}_i - \dot{x}_i\dot{x}_i +
$$

$$
+ A^{\alpha\beta\gamma\delta}(\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta} - w_{\alpha\beta}w_{\gamma\delta}))d\Omega - \int_{t_0}^{t} H_1(p)dp,
$$

where $H_1(t)$ stands for the right side of Eq. (3.4). From Eq. (3.4) it follows that $\mathcal{J}_1(t) = 0$. Hence if $\mathcal{M}_0$ and $\mathcal{M}$ are the two metrics chosen for the stability investigation (under the assumption that conditions 3.3.1 and 3.3.2 of [2] are satisfied), the Theorem 3.3 [2] yields as sufficient conditions for stability:

(i) $\mathcal{J}_1(t)$ is continuous with respect to $\mathcal{M}_0$ for $t \geqslant t_0$,

(ii) $\mathcal{J}_1(t) \geqslant 0$,

(iii) $\mathcal{J}_1(t)$ satisfies the inequality $\mathcal{J}_1(t) \geqslant \delta(\varepsilon)$, whenever $\mathcal{M} \geqslant \varepsilon$ and $0 < \varepsilon < R$. The condition (iii) may be replaced by $\mathcal{J}_1(t) \geqslant c\mathcal{M}$ for positive constant c.

When the perturbation acts only for the finite time interval (t_0, t_{00}) condition (i) becomes

(i*) $\mathcal{J}_1(t)$ is continuous with respect to $\mathcal{M}_0$ for $t_0 \leqslant t \leqslant t_{00}$.

For some special cases of the unperturbed motion, other functionals may be taken instead of $\mathcal{J}_1(t)$ in the conditions (i)—(iii) or (i*)—(iii), to obtain sufficient conditions for stability. For example, we could take $\mathcal{J}_2(t)$ as given by

$$
(3.6)\qquad \mathcal{J}_2(t) = \int_\Omega \rho(\bar{A} - A)d\Omega + \frac{1}{2}\int_\Omega(\rho\dot{\bar{x}}_i\dot{\bar{x}}_i + A^{\alpha\beta\gamma\delta}\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta})d\Omega - \int_{t_0}^{t} H_2(p)dp,
$$

where

$$
(3.7)\qquad H_2(t) = \int_\Omega \rho(\bar{F}_i\dot{u}_i + \bar{B}_i^\alpha\dot{\theta}_{\alpha i})\,d\Omega + \int_\Gamma(\bar{t}_i\dot{u}_i + \bar{p}_i^\alpha\dot{\theta}_{\alpha i})d\Gamma,
$$

when isotermal conditions $(\dot{\bar{\theta}} = \dot{\theta} = 0)$ are assumed and the body is considered in equilibrium. Obviously

$$
(3.8)\qquad \mathcal{J}_2(t) \geqslant \int_\Omega \rho(\bar{A} - A)d\Omega - \int_{t_0}^{t_0} H_2(p)dp = \mathcal{J}_3(t)
$$

and thus $\mathcal{J}_3(t)$ may be used instead of $\mathcal{J}_2(t)$ in conditions (ii) and (iii).

When the perturbing loads $\bar{F}_i$, $\bar{B}_i^\alpha$, $\bar{t}_i$ and $\bar{p}_i^\alpha$ act only during a finite interval (t_0, t_{00}) and thereafter take the values F_i, B_i^α, t_i, p_i^α of the loads in equilibrium (the body is dead loaded for $\tau > t_{00}$), one may take the functional

$$
(3.9)\qquad \mathcal{J}_4(t) = \int_\Omega \rho(\bar{A} - A)d\Omega + \frac{1}{2}\int_\Omega \rho(\dot{\bar{x}}_i\dot{\bar{x}}_i + A^{\alpha\beta\gamma\delta}\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta})d\Omega -
$$

$$
- \int_{t_0}^{t} H_2(p)dp,\ t_0 \leqslant \tau \leqslant t_{00},
$$

and

$$
(3.10)\qquad \mathcal{J}_4(t) = \int_\Omega \rho(\bar{A} - A)d\Omega + \frac{1}{2}\int_\Omega \rho(\dot{\bar{x}}_i\dot{\bar{x}}_i + A^{\alpha\beta\gamma\delta}\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta})d\Omega -
$$

$$
- \int_\Omega \rho(F_iu_i + B_i^\alpha\theta_{\alpha i})d\Omega - \int_\Gamma(t_iu_i + p_i^\alpha\theta_{\alpha i})d\Gamma, \quad \tau > t_{00}.
$$

Again in conditions (ii) and (iii) we may use

$$
(3.11)\qquad \mathcal{J}_5(t) = \int_\Omega \rho(\bar{A} - A)d\Omega - \int_\Omega \rho(F_iu_i + B_i^\alpha\theta_{\alpha i})d\Omega - \int_\Gamma(t_iu + p_i^\alpha\theta_{\alpha i})d\Gamma, \quad \tau > t_{00}.
$$

If the body is further supposed in its undeformed and unloaded configuration, for which $A = F_i = B_i^\alpha = t_i = p_i^\alpha = 0$, the following functional may be taken as the Movchan-Liapunov functional

$$
(3.12)\qquad \mathcal{J}_6(t) = \int_\Omega \rho\bar{A}d\Omega + \frac{1}{2}\int_\Omega \rho(\dot{\bar{x}}_i\dot{\bar{x}}_i + A^{\alpha\beta\gamma\delta}\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta})d\Omega -
$$

$$
- \int_{t_0}^{t} H_2(p)dp,\ t_0 \leqslant \tau \leqslant t_{00},
$$

and

$$
(3.13)\qquad \mathcal{J}_6(t) = \int_\Omega \rho\bar{A}d\Omega + \frac{1}{2}\int_\Omega \rho(\dot{\bar{x}}_i\dot{\bar{x}}_i + A^{\alpha\beta\gamma\delta}\bar{w}_{\alpha\beta}\bar{w}_{\gamma\delta})d\Omega, \ \tau > t_{00}.
$$

For $\mathcal{J}_6(t)$ the condition (iii) may be replaced by:

$$
(3.14)\qquad \text{(iv)} \int_\Omega \rho\bar{A}d\Omega \geqslant \delta(\varepsilon), \text{ for } \mathcal{M} \geqslant \varepsilon,
$$

which represents a restriction on the free energy in order that stability occurs.

4. Stability and uniqueness in the linear theory of elastic Cosserat continuum. In this section sufficient conditions for the asymptotic stability of the equilibrium configuration under dissipative loadings are obtained (this condition with respect to forces has also been used in [7]) The equations of motions, the geometrical equations and the boundary conditions are [8]

$$
(4.1)\qquad t_{kl,k} + \rho F_l = \rho\ddot{u}_l, \quad m_{rk,r} + \varepsilon_{klm}t_{lm} + \rho L_k = \rho j\ddot{\varphi}_k,
$$

$$
(4.2)\qquad \varepsilon_{ij} = u_{i,j} - \varepsilon_{jik}\varphi_k, \quad \varkappa_{ij} = \varphi_{i,j}
$$

$$
(4.3)\qquad t_i = t_{ij}n_j, \quad m_i = m_{ij}n_j,
$$

where, u_k and φ_k are the components of displacement vectors and microrotation vectors respectively, while $\mathbf{t}$ and $\mathbf{m}$ are the stress tensor and the couple-stress tensor, ε_{klm} is the permutation symbol, L_k represents the mass density of the body couples, j is the mocro-inertia coefficient which satisfy $j = 0$ [8] (hence we are dealing with micro-isotropic solids only), and ε_{ij}, $\varkappa_{ij}$ are called the strain tensor and the curvature-twist tensor respectively.

In the case of linear isoterm theory the following constitutive equations are considered [9]

$$
(4.4)\qquad t_{ji} = C_{ijkl}\varepsilon_{kl} + D_{ijkl}\varkappa_{kl}, \quad m_{ji} = D_{ijkl}\varepsilon_{kl} + E_{ijkl}\varkappa_{kl},
$$

with

$$C_{ijkl} = C_{klij}, \quad E_{ijkl} = E_{klij}. \tag{4.5}$$

We assume that C_{ijkl}, E_{ijkl}, D_{ijkl} are bounded and measurable functions of x defined on $\bar{\Omega} = \Omega \cup \Gamma$.

The components of the displacement vector are denoted by u_i or v_i, the components of the micro-rotation vector by φ_i or ψ_i. We denote $\{u_i, \varphi_i\} = \mathbf{u}$ and $\{v_i, \psi_i\} = \mathbf{v}$ so that u or v represent a displacement and micro-rotation field. Let W be the space of vector - functions u with the norm

$$\|\mathbf{u}\|_W^2 = \int_\Omega (u_iu_i + \varphi_i\varphi_i + u_{i,j}u_{i,j} + \varphi_{i,j}\varphi_{i,j})d\Omega, \tag{4.6}$$

where $u_i, \varphi_i \in W_2^1(\Omega)$ ($W_2^1(\Omega)$ is the subspace of $L_2(\Omega)$ of functions whose first derivatives in the sense of distributions, are in $L_2(\Omega)$). Let I be the interval $I = [0, T]$ with $0 < T < \infty$ and let $W(\Omega, I)$ denote the linear space of mapping $\mathbf{u}(t)$ of I into $W(\Omega)$. We define the norm on $W(\Omega, I)$ by means of the equality

$$\|u\|_{W(\Omega,I)} = \sup_{t\in I} \|\mathbf{u}(t)\|_{W(\Omega)}. \tag{4.7}$$

The space ZT [2] may be taken as the subset of $W(\Omega, I)$ consisting of real valued vector-functions $\mathbf{u}(\mathbf{x}, t)$ satisfying the conditions

I) $\mathbf{u} \in C^2[\Omega]$ for $t \in I$,

II) $\mathbf{u} \in C^1[I]$ for $\mathbf{x} \in \Omega$,

III) $u_i = \varphi_i = 0$ when $\mathbf{x} \in \Gamma$ and $t \in I$.

Let us define the M o v c h a n - L i a p u n o v functional and the two metrics which will be used in the stability investigation (Theorem 3.3 [2])

$$\mathcal{S}(t) = \frac{1}{2}\int_\Omega (C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + 2D_{ijkl}\varepsilon_{ij}\varkappa_{kl} + E_{ijkl}\varkappa_{ij}\varkappa_{kl} + \rho\dot{u}_i\dot{u}_i + \rho j\dot{\varphi}_i\dot{\varphi}_i)d\Omega, \tag{4.8}$$

$$\mathcal{M}(t) = \int_\Omega (u_iu_i + \varphi_i\varphi_i + \rho\dot{u}_i\dot{u}_i + \rho j\dot{\varphi}_i\dot{\varphi}_i)d\Omega, \tag{4.9}$$

$$\mathcal{M}_0(t) = \mathcal{S}(t_0) = \text{const.} \tag{4.10}$$

We prove now the following stability theorem

Theorem. *If the following three conditions are satisfied:*
1) the body forces are dissipative, i. e. of the form

$$F_i = -c\dot{u}_i, \quad L_i = -\bar{c}\dot{\varphi}_i, \tag{4.11}$$

where $c(x)$ and $\bar{c}(x)$ are positive functions (we consider these forces as the effect of a surrounding dissipative medium in which the body is imbedded [7]),

2) the surface tractions are dead

$$\dot{t}_i = \dot{m}_i = 0, \tag{4.12}$$

3) the energy of deformation per unit volume is uniformly positive definite, i. e. there exists $c_1 > 0$ such that for each $\mathbf{x} \in \bar{\Omega}$

$$\mathcal{A}(\varepsilon_{ij}, \varkappa_{ij}) = \frac{1}{2} C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + D_{ijkl}\varepsilon_{ij}\varkappa_{kl} + \frac{1}{2}E_{ijkl}\varkappa_{ij}\varkappa_{kl} \geqslant \geqslant c_1\sum_{i,j=1}^{3}(\varepsilon_{ij}^2 + \varkappa_{ij}^2), \tag{4.13}$$

then the equilibrium solution is asymptotic stable with respect to the metrics given by Eq. (4.9) and Eq. (4.10) and the solution of the Eq. (4.1) is unique (in the sense of the Ddefinition 3.4.2 [2]).

Proof. Let us verify all the conditions of the Theorem 3.3 [2]. According to a theorem on integrals depending on a parameter, the conditions (I) and (II) imply the continuity of $\mathcal{M}(t)$ with respect to t and therefore the condition 3.3.1. is satisfied. In view of Eq. (4.1) and the conditions 1) and 2) we obtain

$$\dot{\mathcal{S}}(t) = -\int_\Omega (c\dot{u}_i\dot{u}_i + \bar{c}\dot{\varphi}_i\dot{\varphi}_i)\rho d\Omega \leqslant 0, \tag{4.14}$$

so that $\mathcal{S}(t)$ is a non-increasing functional ($3.3.3_{(i)}$). The condition $3.3.3_{(ii)}$ results immediately from Eq. (4.13) We will prove now the condition $3.3.3_{(iii)}$. The deformation operators ε_{ij} and $\varkappa_{ij}$ form a coercive system (Definition 1.2, [9]) in conformity with the Theorem 1.1 from [9]. Then, $\sum_{i,j=1}^{3}(\varepsilon_{ij}^2(\mathbf{u}) + \varkappa_{ij}^2(\mathbf{u})) = 0$ implies, using (III), that $\mathbf{u} = 0$ in Ω [9]. Therefore all the conditions of the Theorem 2.3 of [10] are satisfied and consequently we have

$$\int_\Omega \mathcal{A}(\varepsilon_{ij}(\mathbf{u}), \varkappa_{ij}(\mathbf{u}))d\Omega \geqslant c_2\|\mathbf{u}\|_W^2, \quad c_2 = \text{const.}, \ c_2 > 0, \tag{4.15}$$

for any $\mathbf{u} \in ZT$ and for any $t \in T$. We make use of the following Friedrichs' inequality ([2], Section 4)

$$\int_\Omega u^2 d\Omega \leqslant c_3\left(\int_\Gamma u^2 d\Gamma + \int_\Omega u_{,i}u_{,i}d\Omega\right), \quad u \in W_2^1(\Omega) \text{ and } c_3 > 0. \tag{4.16}$$

From (4.6), (4.7), (III) and 4.16) we obtain

$$\int_\Omega \mathcal{A}(\varepsilon_{ij}(\mathbf{u}), \varkappa_{ij}(\mathbf{u}))d\Omega \geqslant c_4\int_\Omega (u_iu_i + \varphi_i\varphi_i)d\Omega, \quad c_4 = \text{const.}, \ c_4 > 0 \tag{4.17}$$

and thus the following inequality may be written

$$\mathcal{S}(t) \geqslant c_5\int_\Omega (u_iu_i + \varphi_i\varphi_i + \rho\dot{u}_i\dot{u}_i + \rho j\dot{\varphi}_m\dot{\varphi}_m)d\Omega, \quad c_5 = \text{const.}, \ c_5 > 0, \tag{4.18}$$

i. e. $\mathcal{S}(t) \geqslant c_5\mathcal{M}(t)$ ($3.3.3_{(iii)}$). From (4.10), (4.14) and (4.18) it follows that

$$\mathcal{M}(t) \leqslant \frac{1}{c_5}\mathcal{S}(t) \leqslant \frac{1}{c_5}\mathcal{S}(t_0) = \frac{1}{c_5}\sqrt{\mathcal{M}_0}, \tag{4.19}$$

and now it becomes clear that conditions 3.3.2 and $3.3.3_{(iv)}$ are also satisfied. The proof of the asymptotic stability of the equilibrium solution and the proof of the uniqueness for the solution of (4.1), are similar to those given in [7] for the linear theory of elastic Cosserat rods.

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SYNTHESIS OF TWO-TERMINAL NETWORKS WITH RECTIFIERS AND A MINIMUM NUMBER OF RELAY-CONTACTS

BY

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1. Introduction. This paper considers a classical problem of realizing a given Boolean function, that describes the chains of relay-contacts in a contact network, into a two-terminal contact network N satisfying the contact demands of this function with a minimum number of relay-contacts. An abstract formulation of the problem is given in terms of graph theory and the base for constructing a desired network N is a directed graph. This directed graph is translated into N and chains of forbidden relay-contacts are excluded by using rectifiers.

There is a large number of roumanian works concerning the realization problem of this paper; a list of those papers is given at end of the monograph of Moisil [4]. As stated in [3], the problem is interesting but hard to solve. As expected, the abstract formulation of this paper does not give any practical solution method, but it offers a base for constructing reasonable approximative realization methods applicable to problems in practice. An approximation is considered in the last section.

The terminology of graph theory follows that of Harary in [2]. By a Boolean function $f(x_1^a,\dots,x_n^a)$ we shall always mean a function in its disjunctive normal form. The notation $\bar{x}$ stands for the negation of x, and x^a — means x, if $a=1$, and $\bar{x}$, if $a=0$.

2. A graphtheoretic formulation. Let $f(x_1^a\dots,x_n^a)$ be a given Boolean function. A two-terminal network N with terminal points α and β and with labels $x_1^a,\dots,x_n^a$ on lines, realizes $f(x_1^a,\dots,x_n^a)$, if there is a path, possibly directed, from α to β in N corresponding to any conjunction of $f(x_1^a,\dots,x_n^a)$ and containing only those lines with labels forming a conjunction in question; N contains no other paths from α to β. Let $x_{i_1}^a, x_{i_2}^a,\dots,x_{i_s}^a$ be a path from α to β in N with line labels $x_{i_r}^a$, $r=1,\dots,s$, given in that order in which they appear in the path. Clearly the paths define a set of orders, a set of chains, among the symbols $x_1^a,\dots,x_n^a$ and these chains form the subject of our interest here. The chains shall be detected by using the labels $x_1^a,\dots,x_n^a$ on points of graphs defined below. A graph with labels on points can be easily translated into a graph with labels on lines.

The determination process of N contains the following main steps. Let $f(x_1^a,\dots,x_n^a)$ contain m disjoint conjunctions. At first we construct an undirected graph $G_1=(V(G_1), E(G_1))$, $V(G_1)=\{x_{ij}^a \mid i=1,\dots,n,\ j=1,\dots,m, \text{ and } a=0, 1\}$. This graph will be decomposed into a set of specified induced subgraphs of G_1. The decomposition as the base, G_1 will be mapped onto an undirected graph G_2 satisfying a minimality condition. This graph determines the chains, or the order relations, under interest as follows: a line $(y, z)\in E(G_2)$ implies that $y>z$ or $z>y$. These order relations shall be translated into a directed graph $\vec{G}=(V(\vec{G}), E(\vec{G}))$, where $V(\vec{G})=V(G_2)$ and a directed path from a point y to a point z, $y, z\in V(\vec{G})$, implies that $y>z$. The graph $\vec{G}$ can be directly translated into a network N realizing $f(x_1^a,\dots,x_n^a)$ with a minimum number of relay-contacts.