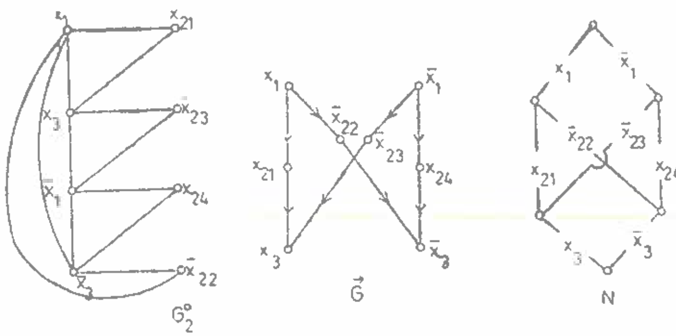
Now, $f(X)=x_1(x_2x_3\vee\bar{x}_2\bar{x}_3)\vee\bar{x}_1\bar{x}_2x_3\vee\bar{x}_1x_2\bar{x}_3$ and $f_{11}(X^{11})=x_2x_3\vee\bar{x}_2\bar{x}_3\,f_{11}(X^{11})$ is the function of the type reported in (I) and we choose x_2 from $f_{11}(X^{11})$; the corresponding residual is $f_{22}(X^{22})=x_3\vee\bar{x}_2\bar{x}_3$. Thus $V(S_1)=\{\{x_{11}, x_{12}\}, \{x_{21}\}\}$ and the new function is $x_3\vee\bar{x}_2x_3\vee\bar{x}_1\bar{x}_2x_3\vee\vee\bar{x}_1x_2\bar{x}_3=x_3(\varnothing\vee\bar{x}_1\bar{x}_2)\vee\bar{x}_2x_3\vee\bar{x}_1x_2\bar{x}_3$. According to the case (II) we choose from $\bar{x}_1\bar{x}_2$ the variable $\bar{x}_2$. Hence the new function is $\bar{x}_1\vee\bar{x}_2x_3\vee\bar{x}_1x_2\bar{x}_3$ and $V(S_2)=\{\{x_{31}, x_{33}\}, \{\bar{x}_{23}\}\}$. $\bar{x}_1\vee\vee\bar{x}_2\bar{x}_3\vee\bar{x}_1x_2\bar{x}_3=\bar{x}_1(\varnothing\vee x_2\bar{x}_3)\vee\bar{x}_2x_3$ and from $x_2\bar{x}_3$ we choose x_2. $V(S_3)=\{\{\bar{x}_{13}, \bar{x}_{14}\}, \{x_{24}\}\}$ and the new function is $\bar{x}_2x_3\vee\bar{x}_3=\bar{x}_3(\varnothing\vee\bar{x}_2)$. After choosing $\bar{x}_2$ there are no variables and hence we have decomposed G_1; $V(S_4)=\{\{\bar{x}_{32}, \bar{x}_{34}\}, \{\bar{x}_{22}\}\}$. By denoting the sets $\{x_{11}, x_{12}\}$, $\{x_{31}, x_{33}\}$, $\{\bar{x}_{13}, \bar{x}_{14}\}$, $\{\bar{x}_{32}, \bar{x}_{34}\}$ briefly by x_1, x_3, $\bar{x}_1$, $\bar{x}_3$, respectively, we can directly draw the graph G_2^0 of Fig. 4 (*a*). Fig. 4 (*b*) shows the corresponding graph $\vec{G}$ and Fig. 4 (*c*) illustrates the network N realizing the function (2).



(*a*) (*b*) (*c*)

Fig. 4

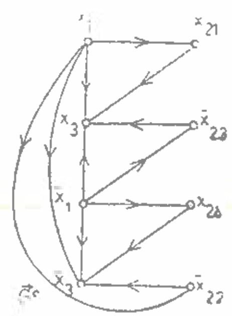


Fig. 5

We have no efficient algorithm for determining a graph $\vec{G}$ from the graph G_2 (or from G_2^0) with a minimum number of lines. A labelling algorithm giving a graph $\vec{G}$ such that $|E(\vec{G})|=|E(G_2)|$ (or $|E(\vec{G})|=|E(G_2^0)|$), can be constructed, but such a graph is clearly not a desired graph $\vec{G}$. Figure 5 shows such a graph $\vec{G}$ derived from G_2^0 of Figure 4(*a*). For further information on directing the lines of G_2 and G_2^0 and for constructing suitable algorithms the reader is referred to the papers [6] and [7] of Jung.

Received 5.IX.1974

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Analele ştiinţifice ale Universităţii „AL. I. CUZA" din Iaşi
Tomul XXII, s. I a, 1976, f. 1

ON THE THEORY OF REDUCTION OF SEMILATTICIAL AUTOMATA

BY

DAN A. SIMOVICI

I. Introduction. We intend to study in this paper several aspects of the reduction theory of semilatticial automata. This new class of automata is an extension of the class of linear Boolean automata [3], which includes as special cases Leticevskij automata [2] and macroautomata related to nondeterministic automata (see [1], for instance).

A semilatticial automaton (SLA) is a 5-uple $\mathbf{A}=(I, L, K, \{f_i\mid i\in I\}, \{g_i\mid i\in I\})$ where I is the input alphabet, $L=(S, \cup)$ is the state semilattice of te automaton, $K=(0, \cup)$ is the semilattice of output signals, $\{f_i\mid i\in I\}$ is the set of transition mappings which are endomorphisms of the lattice L and $\{g_i\mid i\in I\}$ is the set of output mapping which consists of lattice-homomorphisms from L to K.

When an input signal is presented at the input wire of the automaton, its interior state being $s\in S$ we obtain $g_i(s)$ at the output wire of the semiautomaton, the next state being $f_i(s)$. The transition mappings and the output mappings can be extended for words belonging to I^* by induction on the lenght of the words. Namely, for every p from I^* we shall take

$$f_{pi}=f_i\circ f_p \text{ and } g_{pi}=g_i\circ f_p.$$

It is clear that $f_p\in\mathrm{Hom}(L, L)$ and $g_p\in\mathrm{Hom}(L, K)$. In the sequel we shall present some classes of automata which appear to be special cases of semilatticial automata.

Let $\mathbf{A}=(B_2^{(m,1)}, B_2^{(n,1)}, B_2^{(l,1)}, f, g)=\langle A, B, C, D\rangle$ be a linear Boolean automaton [3]. It is clear that $B_2^{(m,1)}$, $B_2^{(n,1)}$, $B_2^{(l,1)}$ are disjunctive semilattices. The mappings f_i and g_i are given by $f_i(s)=As\cup Bi$ and $g_i(s)=Cs\cup Di$. We remark that they are homomorphisms since

$$f_i(\cup(s_j\mid j\in J))=A(\cup(s_j\mid j\in J))\cup Bi=$$
$$=\cup(As_j\mid j\in J)\cup Bi=\cup(As_j\cup Bi\mid j\in J)=\cup(f_i(s_j)\mid j\in J)$$

and

$$g_i(\cup(s_j\mid j\in J))=C(\cup(s_j\mid j\in J))\cup Di=$$
$$=\cup(Cs_j\mid j\in J)\cup Di=\cup(Cs_j\cup Di\mid j\in J)=\cup(g_i(s_j)\mid j\in J).$$

Let $\mathbf{A}=(I, S, O, f, g)$ be a nondeterministic automaton. The *macroautomaton* related to $\mathbf{A}$ is defined as $\mathbf{M(A)}=(I, P(S), P(O), \bar{F}, \bar{G})$, where $\bar{F}=\{\bar{f}_i\mid\bar{f}_i: P(S)\to P(S),\ i\in I,\ \bar{f}_i(S_1)=\cup(f_i(s)\mid s\in S_1),\ \forall S_1\in P(S)\}$ and $\bar{G}=\{\bar{g}_i\mid\bar{g}_i: P(S)\to P(O),\ i\in I,\ \bar{g}_i(S_1)=\cup(g_i(s)\mid s\in S_1),\ \forall S_1\in P(S)\}$. It is immediat that $\mathbf{M(A)}$ is a SLA for, $\bar{f}_i(\cup(S_j\mid j\in J))=\cup(\bar{f}_i(S_j\mid j\in J))$ and $\bar{g}_i(\cup(S_j\mid j\in J))=\cup(\bar{g}_i(S_j)\mid j\in J)$.

A Leticevskij - like automaton is an infinite semiautomaton $\mathbf{A}=(I, P_f(Z^*), F, E_0)$ where I is the input alphabet, $P_f(Z^*)$ the set of finite subsets of the free semigroup Z^* generated by Z, F is the state set, $E_0\in P_f(Z^*)$ is the subset from Z which is the initial state for $\mathbf{A}$ and the family F satisfies the following conditions:

i) $f_i(E_1 \cup E_2) = f_i(E_1) \cup f_i(E_2),\ \forall E_1, E_2 \subseteq I^*$,

ii) $f_i(zp) = f_i(z)p,\ \forall z \in Z,\ p \in Z^*$,

iii) $f_i(e) = \emptyset$,

iv) $f_1(z_1)$ consists only of words having the length less then 2.

A Leticevskij-like automaton is a SLA since $(P_f(Z^*), \cup)$ is a semilattice and $f_i \in \mathrm{Hom}(P_f(Z^*), P_f(Z^*))$.

II. **Equivalences and coverings of SLA** **Definition 1.** *A basis of SLA* $\mathbf{A} = (I, L, K, \{f_i \mid i \in I\}, \{g_i \mid i \in I\})$ *is a set of generators of the semilattice L. An atomic basis is a basis for the semilattice L having a first element O, for which* $0 \leqslant s_1 \leqslant s_2 \in G$ *implies* $s_1 = s_2$ *or* $s_1 = 0$. *A strict basis is a basis G for which* $f_i(G) \subseteq G$ *for every* $i \in I$.

Every SLA has at least the trivial basis which consists of the whole set S. If G is a strict basis then $\mathbf{A}' = (I, (G, \cup), K, \{f_i \mid i \in I\}, \{g_i \mid i \in I\})$ is a subautomaton of the automaton $\mathbf{A}$.

Let $\mathbf{A}_j = (I, L_j, K, F^j, G^j)$, $j = 1, 2$, two SLA having the same input alphabet and the same output semilattice.

Definition 2. *The automaton* $\mathbf{A}_2$ *covers the automaton* $\mathbf{A}_1$, $\mathbf{A}_2 > A_1$ *if there exists* $h \in \mathrm{Hom}(L_1, L_2)$ *so that for every* $p \in I^*$ *it holds:* $g_p^1 = g_p \circ h$ *and* $h(s_1) > 0$ *(if there exists* $0 \in S_2$).

The automaton $\mathbf{A}_2$ *covers* $\mathbf{A}_1$ *with respect to* G_2 *and* G_1, $\mathbf{A}_2 > (G_2, G_1)\mathbf{A}_1$ if there exists a mapping $h \in G_2^{G_1}$ so that for every $s_1 \in G_1$ we have $g_p^1(s_1) = g_p^2(h(s_1))$, $\forall p \in I^*$ and $h(s_1) > 0$ (if there exists $0 \in S_2$). Here G_j is a basis for $\mathbf{A}_j$, $j = 1, 2$.

A_1 is equivalent with $\mathbf{A}_1$, $\mathbf{A}_2 \sim \mathbf{A}_2$ *if* $\mathbf{A}_1 > \mathbf{A}_2$ *and* $\mathbf{A}_2 > \mathbf{A}_1$; $\mathbf{A}_1$ *is equivalent with* $\mathbf{A}_2$ *with* $\mathbf{A}_2$ *with respect to the bases* G_1 *and* G_2, $\mathbf{A}_1 \sim (G_1, G_2)\mathbf{A}_2$, *if* $\mathbf{A}_1 > (G_1, G_2)\mathbf{A}_2$ *and* $\mathbf{A}_2 > (G_2, G_1)\mathbf{A}_1$.

Theorem 1. *If* $\mathbf{A}_1 \sim \mathbf{A}_2$, *then* $\mathbf{A}_1 \sim (S_1, S_2)\mathbf{A}_2$ *and* $\mathbf{A}_1 > (G_1, G_2)\mathbf{A}_2$ *implies* $\mathbf{A}_1 > \mathbf{A}_2$.

Proof. In view of the previous definition the first part is obvious. Let us suppose that $\mathbf{A}_1 > (G_1, G_2)\mathbf{A}_2$. There exists $h : G_2 \to G_1$ so that for every $s_2 \in G_2$, $g_p(s_2) = g_p(h(s_2))$, $\forall p \in I^*$. Let $s \in S_2$. Since G_2 is a basis there exists a set J for which $s = \cup(s_{2j} \mid s_{2j} \in G_2, j \in J)$, $|J| < \infty$. Then the state $s_1 = \cup(h(s_{2j}) \mid j \in J) \in S_1$ is equivalent to s since, for every $p \in I^*$ we have:

$$g_p^1(\cup (h(s_{2j}) \mid j \in J) = \cup(g_p^1(h(s_{2j}) \mid j \in J)) =$$
$$= \cup(g_p^2(s_{2j}) \mid j \in J) = g_p^2(\cup(s_{2j} \mid j \in J)) = g_2^p(s).$$

Therefore we can extend $h : G_2 \to G_1$ to a mapping $h : S_2 \to S_1$ by taking $h(s) = s_1$. We have

$$g_p^2(s_2) = g_p(h(s_2)), \quad \forall s_2 \in S_2$$

hence $\mathbf{A}_1 > \mathbf{A}_2$.

Definition 3. *A is reduced with respect to the basis G if* $(s_1, s_2) \in \rho \cap (G \times G)$ *implies* $s_1 = s_2$. *A is strongly reduced with respect to the basis G if* $(s_1, s_2) \in \rho \cap (G \times S)$ *implies* $s_1 = s_2$. *A is reduced if it is reduced with respect to the trivial basis S.*

Theorem 2. *For every SLA there exists an equivalent reduced automaton which is also a SLA.*

Proof. Let $\mathbf{A} = (I, L, K, \{f_i \mid i \in I\}, \{g_i \mid i \in I\})$ be a SLA and let us consider the automaton $\mathbf{A}/\rho = (I, L/\rho, K, \{f_{\rho i} \mid i \in I\}, \{g_{\rho i} \mid i \in I\})$, where $f_{\rho i}([s]) = [f_i(s)]$ and $g_{\rho i}([s]) = [g_i(s)]$, where $[s]$ is the equivalence class of the state s in the quotient semilattice L/ρ and $\rho = \{(s_1, s_2) \mid g_p(s_1) = g_p(s_2),\ \forall p \in I^*\}$.

The automaton $\mathbf{A}/\rho$ is reduced for, if $([s_1], [s_2]) \in \rho$ it follows $g_p(s_1) = g_{\rho p}([s_1]) = g_{\rho p}([s_2]) = g_p(s_2)$, $\forall p \in I^*$ hence $(s_1, s_2) \in \rho$ and $[s_1] = [s_2]$.

We have $\mathbf{A}/\rho \sim \mathbf{A}$ since $(s, [s]) \in \rho$.

Theorem 3. *Let* $\mathbf{A}$ *be a SLA which is reduced with respect to the basis G and* $\mathbf{A}'$ *a SLA which is strongly reduced with respect to* G'. *Suppose* $\mathbf{A}' > \mathbf{A}$. *If* $\hat{G} = \{s \mid s \in G, |[s]| \geqslant 2\}$ *then for every* $s \in \hat{G}$, $s' \in G'$ *we have* $(s, s') \notin \rho$.

Suppose that in would exist $s' \in G'$ and $s \in \hat{G}$ so that $(s, s') \in \rho$. Since $s \in \hat{G}$ there exists $s_1 \in S$ with $(s_1, s) \in \rho$, $s_1 \neq s$. Taking into account that G is a basis for $\mathbf{A}$ we have $s_1 = \cup(s_j \mid j \in J)$.

From $\mathbf{A}' > \mathbf{A}$ it follows the existence of a mapping $h : S \to S'$ so that for every $s_1 \in S_1$ it holds $(s_1, h(s_1)) \in \rho$, or $(\cup(s_j \mid j \in J), \cup(h(s_j) \mid j \in J)) \in \rho$. Taking $r' = \cup(h(s_j) \mid j \in J)$ we have $(s_1, r') \in \rho$. Therefore, from $(s, s') \in \rho$, $(s, s_1) \in \rho$ and $(s_1, r') \in \rho$ we obtain $(s', r') \in \rho$ hence $r' = s' = \cup(h(s_j) \mid j \in J)$ since $\mathbf{A}'$ is strongly reduced with respect to the basis G. Due to the fact that G' is an atomic basis, we have $h(s_j) = s'$, $\forall j \in J$ and, since $(h(s_j), s_j) \in \rho$ it follows $(s_j, s') \in \rho$, $\forall j \in J$ or $(s_j, s) \in \rho$. Since $\mathbf{A}$ is a reduced automaton with respect to G and $s \in \hat{G} \subseteq G$ we obtain $s_j = s$, $\forall j \in J$ hence $s_1 = \cup(s_j \mid j \in J) = s$, which contradicts the hypothesis.

Theorem 4. *Let* $\mathbf{A}$ *be a SLA reduced with respect to G and* $\mathbf{A}'$ *a SLA strongly reduced with respect to the atomic basis* G'. *Suppose* $\mathbf{A} \sim \mathbf{A}'$. *Then we have* $\mathbf{A} > (G, G')\mathbf{A}'$.

Proof. It is obvious that $\mathbf{A} > \mathbf{A}'$. Let us consider $s' \in G$. There exists $h(s') \in S$ with $(s', h(s')) \in \rho$. Since G is a basis for $\mathbf{A}$ we have $h(s') = \cup(s_j \mid j \in J, s_j \in G)$ and taking into account that $\mathbf{A}' > \mathbf{A}$ there exists $k(s_j) \in S'$ so that $(s_j, k(s_j)) \in \rho$ which implies:

$$g'_p(s') = g_p(h(s)) = \cup(g_p(s_j) \mid j \in J) = \cup(g_p(k(s_j)) \mid j \in J).$$

From the last equality we obtain $g'_p(s') \geqslant g'_p(k(s_j))$, $\forall j \in J$ hence $g'_p(s' \cup k(s_j)) = g'_p(s') \cup g'_p(k(s_j)) = g'_p(s')$ for every $p \in I^*$, $j \in J$. Therefore $s' \cup k(s_j) = s'$, since $\mathbf{A}'$ is strongly reduced with respect to G', hence $s' \geqslant k(s_j) > 0$, which implies $k(s_j) = s'$, $\forall j \in J$, G' being an atomic basis.

For every pair of indices $j_1, j_2 \in J$ we obtain $k(s_{j_1}) = k(s_{j_2}) = s'$, hence $(s_{j_1}, s_{j_2}) \in \rho$. Since $\mathbf{A}$ is strongly reduced it follows $s_{j_1} = s_{j_2}$, $\forall j_1, j_2 \in J$, which means $h(s') \in G$.

Theorem 5. *Let* $\mathbf{A}$ *be a SLA reduced with respect to the basis G. If* $\mathbf{A}'$ *is a strongly reduced SLA with respect to the atomic basis* G' *and* $\mathbf{A} \sim \mathbf{A}'$ *then there exists a mapping* $k : \hat{G} \to S$ *so that:*

$$(1) \qquad \forall \hat{s} \in \hat{G}, / (\hat{s}, k(\hat{s})) \in \rho \text{ and } k(\hat{s}) \in [G \setminus \hat{G}].$$

where $[G \setminus \hat{G}]$ *is the semilattice spanned by* $G \setminus \hat{G}$ *and* $\hat{G}$ *is the set introduced in Theorem 3.*

Proof. Since $A' > A$ there exists a mapping $h : S \to S'$ so that for every $\hat{s} \in \hat{G}$ we have $(\hat{s}, h(\hat{s})) \in \rho$. We have $h(\hat{s}) = \cup(s'_j \mid j \in J, s'_j \in G')$. Let us consider the mapping $k : \hat{G} \to S$ defined by:

$$k(\hat{s}) = \cup(s \mid s \in [s'_j] \cap G,\ j \in J).$$

Due to the Th. 4 we have $\mathbf{A} > (G, G')\mathbf{A}'$ and $[s'_j] \cap G \neq \emptyset$. We have also $(h(\hat{s}), k(\hat{s})) \in \rho$ since

$g_p(k(\hat{s})) = g_p(\cup(s \mid s \in [s_j] \cap G, j \in J)) = \cup(g_p(s) \mid s \in [s'_j] \cap G, j \in J) = \cup(g'_p(s'_j) \mid j \in J) = g'_p(\cup(s'_j \mid j \in j)) = g'_p(h(\hat{s}))$, $\forall p \in I^*$. Since $(\hat{s}, h(\hat{s})) \in \rho$ we find $(\hat{s}, k(\hat{s})) \in \rho$. We can write $k(\hat{s}) = \cup(s_m \mid m \in M, s_m \in G)$. Let us suppose that for a certain $m_1 \in M$ we have $s_{m_1} \in \hat{G}$. Then, there exists s'_j so that $(s_{m_1}, s'_j) \in \rho$ which contradicts Th. 3. Indeed, if this is the case $s_{m_1} \in [s'_j]$ which implies $(s_{m_1}, s_j) \in \rho$. We have $k(\hat{s}) \in [G \setminus \hat{G}]$ since in the development of $k(\hat{s})$, which was earlier considered, we have not any element of $\hat{G}$.

The next result allows a characterization of SLA which are reduced with respect to atomic bases and admit equivalent automata strongly reduced with respect to atomic bases.

Theorem 6. *If* $\mathbf{A}$ *is an ASL reduced with respect to the atomic basis G and there exists the mapping* $k : \hat{G} \to S$ *which satisfies the condition (1) there exists a strongly reduced SLA with respect to its atomic basis* G' *with* $\mathbf{A} \sim \mathbf{A}'$.

Proof. We shall prove that the SLA, $A' = (I, [G \setminus \hat{G}], K, \{f_i' \mid i \in I\}, \{g_i' \mid i \in I\})$, where $f_i' = f_i \mid [G \setminus \hat{G}]$, $g_i' = g_i \mid [G \setminus \hat{G}]$ satisfies the required conditions. Here $h \mid M_1$ is the restriction of the mapping h to the subset M_1 of its domain.

The automaton A′ is equivalent to A. Let us define the mapping $h: S \to S'$ by

$$h(s) = \begin{cases} s, & \text{if } s \in [G \setminus \hat{G}] \\ k(s), & \text{if } s \in \hat{G} \end{cases}$$

where $k: \hat{G} \to S$ (which exists due to Th. 5) and $(s, k(s)) \in \rho$.

We have $(s, h(s)) \in \rho$ hence $A' > A$. Since $A > A'$ (A' being a subautomaton of A) we have $A \sim A'$.

Let us prove that A' is strongly reduced with respect to the basis $G \setminus \hat{G}$. If $(s, s_1) \in \rho'$, $s \in G \setminus \hat{G}$, $s_1 \in [G \setminus \hat{G}]$, where ρ' is the equivalence in the automaton A', we have $(s, s_1) \in \rho$. If $s_1 \neq s$ we have $s \in \hat{G}$, which contradicts the hypothesis. Hence A′ is strongly reduced with respect to the set $G \setminus \hat{G}$, which is composed only by atoms.

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Received 8. VI. 1974

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Analele științifice ale Universității „AL. I. CUZA" din Iași
Tomul XXII, s. I a, 1976, f. 1

RECENZII

JAQUES HADAMARD: *Essai sur la psychologie de l'invention dans le domaine mathématique.* Gauthier-Villars, Paris, 1975, 136 p.

Cartea apare în colecția „Discours de la méthode", colecție care grupează texte, mărturii, ale cercetătorilor și inventatorilor contemporani despre activitatea lor. Matematicianul Jacques Hadamard este cunoscut pentru lucrările sale din domeniul funcțiilor analitice și al ecuațiilor cu derivate parțiale și a avut o mare influență asupra școlii matematice franceze de la începutul secolului.

Studiul despre invenție în matematică este o dezvoltare a cursurilor ținute de autor la New York în 1943. Hadamard cercetează mecanismele creației matematice degajînd rolul „imaginilor vagi" care sînt apoi formalizate prin analiză și raționament. Se încearcă astfel o descifrare a gîndirii creatoare revelînd partea de muncă, meditație, hazard, subconștient, inspirație sau influență care însoțește acest act.

Conținînd reflecții ale unuia din marii creatori ai timpului nostru despre propria disciplină, cartea de dovedește utilă pentru cunoașterea mecanismului invenției științifice.

Corneliu Ursescu

SOLOMON MARCUS (sub redacția): *Semiotica folclorului: abordare lingvistico-matematică.* București, Editura Academiei R. S. R., 1975, 267 pp.

Volumul reunește un număr de treisprezece contribuții elaborate de unsprezece autori de formație diversă (matematicieni, lingviști, filologi folcloriști etc.) și subsumate ideii de a aplica metodele teoriei limbajelor formale în studiul unor probleme fundamenttale ale folclorului; această manieră de abordare constituie — cum se afirmă și în prefață — o prioritate pe plan mondial. Conturarea unei viziuni unitare asupra volumului este facilitată cititorului prin lectura primului capitol, *Modelarea lingvistico-matematică și aspectele semiotice ale folclorului*, în care S. Marcus aduce importante clarificări de obiect și de metodă privind cercetările prezentate, în contextul preocupărilor (atît în perspectivă istorică, cît și în cea a actualității pe plan mondial) pentru cercetarea structurală a folclorului, cercetarea cu ajutorul calculatorului, modelarea probabilistică informațională a structurilor prozodice etc.

În ce privește problematica studiilor incluse în celelalte șase capitole, chiar și simpla enumerare a autorilor și titlurilor acestor studii ar ocupa un spațiu depășind limitele unei astfel de recenzii. Vom menționa deci numai titlurile capitolelor și autorii contribuțiilor incluse în fiecare din ele: *Paradigmatica baladelor populare* (Tudor Bălănescu, Ion Rădoi), *Rima în poezia populară* (Mihai Dinu, Ion Rădoi), *Structurile repetitive ale folclorului în lumina algebrei omologice* (Irina Gorun), *Mecanismele generative ale basmelor populare* (Stanca Fotino & Solomon Marcus, Mihaela Dumitru, Adriana Polith, Gheorghe Păun), *Invarianțe în structura cimiliturilor* (Adrian Rogoz, Sorin Ciobotaru), *Genealogia eroilor mitologici în lumina teoriei grafurilor* (Mihai Dinu).

Alexandru Cărăușu

JOE ROSEN: *Symmetry Discovered. Concepts and Applications in Nature and Science.* Cambridge University Press, 1975, xi + 138 p.

Parcurgînd această carte, cititorul descoperă lumea minunată și plină de frumusețe a simetriei. Autorul introduce conceptele și terminologia uzuală ale acesteia (incluzînd și teoria elementară a grupurilor). Discuția nu este însă restrînsă la construcții geometrice, ci este extinsă la aspectele simetriei, caracteristice unor evenimente temporale și spațiale, precum și unor fenomene mai generale și mai abstracte. Ca exemple, autorul prezintă numeroase obiecte din lumea vie, microcosmos și macrocosmos. Este scos în evidență rolul simetriei aproximative, precum și raportul dintre simetriile cauzei și efectului unui fenomen natural. Lucrarea se încheie cu o discuție a rolului simetriei în cercetarea științifică. Simbolismul matematic este evitat pe cît posibil, fapt care face cartea accesibilă unui cerc foarte larg de cititori.

Jack Weinstein

J. B. MORGAN and K. S. SNELL: *Unified mathematics. Teacher's Guide.* Book 1–3. Cambridge University Press, Cambridge, London, New York, Melbourne 1976. VIII + 164 p.

Așa cum o arată și subtitlul, cartea se adresează profesorilor și constituie o anexă utilă la manualele elaborate de aceiași autori pentru elevi. Urmînd tabla de materii din cărțile 1–3, se dau indicații, la fiecare capitol asupra scopului și a obiectivelor ce trebuie atinse, asupra dificultăților probabile, se fac sugestii privind modul de abordare a unor exerciții și se dau răspunsurile la toate exercițiile.

D. Rimer

J.B. MORGAN and K. S. SNELL: *Unified mathematics.* Book 3. Cambridge University Press, Cambridge, London, New York, Melbourne 1975. X + 388 p.

Cartea face parte dintr-un ciclu de manuale de liceu, elaborate în concordanță cu spiritul de modernizare a programelor și a tehnologiei didactice. Se tratează chestiuni de algebră (fracții, ecuații