et un nombre réel $M_0 \geqslant 1$ *tels que* $|f_t(x)|_{x,\alpha} \leqslant M_0$ *pour tout* $x \in U$, *et toute semi-norme* $\alpha \in A_0$. *Alors, pour tout* $x_0 \in U$, *il existe un voisinage raffiné ouvert* V *de* x_0, $V \subseteq U_0$, *un intervalle* $(-\varepsilon, \varepsilon) \subset R$ *et une application* $g: (-\varepsilon, \varepsilon) \times V \to U$ *telle que soient satisfaites les conditions* (a) *et* (b) *du théorème 3 et la condition* (c') $\dot{g}(t, x) = f(g(t, x'' \gg$ *pour tout* $(t, x) \in (-\varepsilon, \varepsilon) \times V$.

Remarque 4. Les définitions et les résultats de cette note seront utilisés dans [6] pour définir les LC et LCR-variétés de classe C^k_M modelées sur des espaces de Fréchet, le fibré tangent d'une LCR-variété et pour l'intégration locale des champs de vecteurs sur une LCR-variété de classe C^k_M.

BIBLIOGRAPHIE

1. C r a i o v e a n u M. — *Elemente de analiză locală*, partea I și II, Timișoara, 1976.
2. M a r i n s c u G. — *Tratat de analiză funcțională*, vol. I, Ed. Academiei, București, 1970.
3. M a r i n e s c u G. — *Tratat de analiză funcțională*, vol. II, Ed. Academiei, București, 1972.
4. P a p a g h i u c N. — *Un théorème d'existence pour les équations différentielles dans les espaces localement convexes*, An. șt. Univ. Iași, Tom. XXI, s. I a, 1975.
5. P a p a g h i u c N. — *Sur la topologie raffinée d'un espace localement convexe* (à parraitre dans Rev. Roum. Math. Pures et Appl., București).
6. P a p a g h i u c N. — *Variétés différentiables modelées sur des espaces de Fréchet; L'intégration des champs de vecteurs*, (à paraitre dans An. șt. Univ. Iași).
7. P e n o t J. P. — *Calcul différentiel dans les espaces vectoriels topologiques*, Studia Math, 47 (1973) pp. 1—23.
8. R o b b i n J. W. — *On the existence theorem for differential equations*, Proc. Amer. Math. Soc., 19 (1968), pp. 1 005—1 006.
9. Y a m a m u r o S. — *Differential Calculus in Topological Linear Spaces*, Lecture Notes in Math., 324, Springer-Verlag, Berlin.

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THE MANIFOLD OF SYMPLECTIC STRUCTURES ON A COMPACT MANIFOLD

BY

LILIANA MAXIM RĂILEANU

Introduction. In the present paper we consider the space **S** of symplectic structures which a given compact manifold may carry. The group **D** of diffeomorphisms of the manifold acts on **S** naturally by "pull-back„ and from this action we get the space **S**/**D** of equivalent symplectic structures. We shall analyse the action of **D** on **S**.

First we introduce some notations. M will be an even-dimensional, compact C^∞ orientable manifold without boundary, say $2m$-dimensional, which admits an almost complex structure and has a real cohomology class in dimension 2 which remains nonzero when raised to the power m, TM will be its tangent bundle, T^*M the cotangent bundle and $\Lambda^k T^*$ the bundle of k-forms over M. If E is a vector bundle on M, $C^k(E)$ $(0 \leqslant \leqslant k < \infty)$ will be the space of C^k-sections of E with the topology of uniform convergence in k derivatives. It has the topology of a Banach space. $C^\infty(E) = \bigcap_{k=0}^{\infty} C^k(E)$ is a Fréchet space whose topology is given by all C^k-norms. For notions and elementary results on infinite dimensional manifolds we refer to [5]. In particular, if $f: X \to Y$ is a smooth map between manifolds, $Tf: TX \to TY$ is the induced map on tangent bundles, $T_p f: T_p X \to T_{f(p)} Y$ is the map restricted to a fibre.

The set of all almost symplectic structures **AS** on M is, by definition, the set of all nondegenerate sections in $C^\infty(\Lambda^2 T^*)$. Denoting by $d: C^\infty(\Lambda^2 T^*) \to C^\infty(\Lambda^3 T^*)$ the exterior derivative, the set of symplectic structures **S** on M is, by definition, the open set of non-degenerate sections in the Fréchet subspace Ker d; hence **S** = **AS** $\cap$ Ker d. and, in the conditions imposed on M, it is a nonvoid set.

Let **D** be the group of diffeomorphisms of M with the C^∞-topology. **D** acts on $C^\infty(\Lambda^2 T^*)$ in the following way: if $f \in$ **D** and $\Omega \in C^\infty(\Lambda^2 T^*)$ $p \in M$ and $X_p, Y_p \in T_p M$, then $(f^*\Omega)_p$, considered as a bilinear form on $T_p M$, is defined by: $(f^*\Omega)_p (X_p, Y_p) = \Omega_{f(p)} (T_p f(X_p), T_p f(Y_p))$. We denote this right action by:

$$\rho: \mathbf{D} \times C^\infty(\Lambda^2 T^*) \to C^\infty(\Lambda^2 T^*).$$

It is clear that ρ is linear in its second variable and **AS**, **S** are invariant under the action ρ, so we can write:

$$\rho : \mathbf{D} \times \mathbf{AS} \to \mathbf{AS}, \quad \rho : \mathbf{D} \times \mathbf{S} \to \mathbf{S}.$$

If $\Omega \in \mathbf{S}$, we define the isotropy subgroup D_Ω at Ω, by:

$$\mathbf{D}_\Omega = \{f \in \mathbf{D}, \; f^*(\Omega) = \Omega\}.$$

In [6] H. Omori shows that the topological group $\mathbf{D}$ has a structure strong ILH-Lie group and $\mathbf{D}_\Omega$ is a closed, strong ILH-subgroup of

The main goal of this paper is to construct at each $\Omega \in \mathbf{S}$, a sli for the action ρ; that is a subset $\mathbf{S}_1$ of $\mathbf{S}$ such that:

1) if $f \in \mathbf{D}_\Omega$, $\rho(f, \mathbf{S}_1) = \mathbf{S}_1$.

2) if $f \in \mathbf{D}$, and $\rho(f, \mathbf{S}_1) \cap \mathbf{S}_1 \neq 0$, then $f \in \mathbf{D}_\Omega$.

3) there is a local cross-section $\mu : \mathbf{D}/\mathbf{D}_\Omega \to \mathbf{D}$ defined on a neig borhood U of the identity coset such that if the mapping $F : U \times \mathbf{S}_1 \to$ is given by $F(u,s) = \rho(\mu(u, s))$, then F is a homeomorphism onto a neig borhood of Ω.

Since for Fréchet spaces ($\mathbf{S}$ and $\mathbf{D}$ in our problem) the impli function theorem and the local existence theorem for ordinary differe tial equations are not true in general (see [5]), we do not know there exists an exponential map on $\mathbf{S}$ which is a diffeomorphism ne the zero section of the tangent bundle. The usual proof of the slice the rem for the action of a compact Lie group on a differential manif [1] uses specially the existence of the exponential map with the abo property. Because of these technical difficulties, we enlarge the sets and $\mathbf{D}$ somewhat, so that they are infinite dimensional manifolds, cally like neighborhoods in Hilbert spaces. The enlarged spaces, wh we call $\mathbf{D}^k$ and $H^k(\Lambda^2 T^*)$ are spaces of H^k-maps, maps which have p tial derivatives defined almost everywhere up to order k such t each of them is square integrable (see [3], [4], [7], [8]). In [6] (VIII p. 100), H. Omori proves that for $k \geq 2m + 5$, ρ can be exten to a smooth action of $\mathbf{D}^{k+1}$:

$$\rho : \mathbf{D}^{k+1} \times H^k(\Lambda^2 T^*) \to H^k(\Lambda^2 T^*).$$

For $k \geq 2m + 5$, let $\mathbf{S}^k = H^k(\Lambda^2 T^*) \cap C^0\,\mathbf{S}$. Since by the Sobo lemma ([8]) $H^k(\Lambda^2 T^*) \subset C^0(\Lambda^2 T^*)$ is a continous inclusion, $\mathbf{S}^k$ is clea an invariant subset of $H^k(\mathrm{Ker}\, d)$ under the action of $\mathbf{D}^{k+1}$, so by triction, we can define an action $\rho : \mathbf{D}^{k+1} \times \mathbf{S}^k \to \mathbf{S}^k$. First, we will c struct a slice for this action. The proof follows the ideas used by Ebin [2], [3] in the proof of the slice theorem for the manifold Riemannian metrics.

§1. A Riemannian Structure for the Space $\mathbf{S}^k$, $k \geq 2m+5$. shall endow $\mathbf{S}^k$ with a weak Riemannian structure and with a $\mathbf{D}^{k+1}$-in riant structure which is not weak. Since $\mathbf{S}^k$ is open in $H^k(\mathrm{Ker}\, d)$, is a Hilbert manifold whose tangent space at each point is canonic identified with $H^k(\mathrm{Ker}\, d)$. Therefore for each $\Omega \in \mathbf{S}^k$, we must def an inner product $<\,,\,>_\Omega$ on $H^k(\Lambda^2 T^*)$. To this aim we fix a smo Riemannian structure g on M; each $\Omega \in \mathbf{S}^k$ induces a Riemannian str ture g_Ω on $\Lambda^2 T^*$ as follows:

$$g_\Omega(p)\,(\Omega_p^1, \Omega_p^2) = \mathrm{tr}((\Omega(p))^{-1}\,(-\Omega_p^1)\,(\Omega(p))^{-1}\,(\Omega_p^2)),$$

where Ω_p^1, $\Omega_p^2 \in \Lambda^2 T_p$ are considered as the elements from $\mathrm{Hom}\,(T_p, T_p^*)$ and $(\Omega(p))^{-1} \in \mathrm{Hom}\,(T_p^*, T_p)$.

Let Ω^1, $\Omega^2 \in H^k(\Lambda^2 T^*)$; we define $g_\Omega(\Omega^1, \Omega^2) \in H^k(M, R)$ to be

$$g_\Omega(\Omega^1, \Omega^2)\,(p) = g_\Omega(p)\,(\Omega^1(p), \Omega^2(p)),$$

and the inner product on $H^k(\Lambda^2 T)$ to be:

$$<\Omega^1, \Omega^2>_\Omega = \int_M g_\Omega(\Omega^1, \Omega^2)\,\mu_0\,,$$

where μ_0 is the volume element induced on M by the fixed metric g.

Theorem 1. *With the previous hypotheses, we have:*

1) $<\,,\,>$ *defines a smooth weak Riemannian structure on* $\mathbf{S}^k$;

2) $<\,,\,>$ *has associated a unique affine connection* ∇, *such that for smooth vector fields* X, Y, Z *on* $\mathbf{S}^k$, *we have:*

(1) $$X<Y, Z> = <\nabla_X Y, Z> + <Y, \nabla_X Z>,$$

(2) $$\nabla_X Y - \nabla_Y X = [X, Y];$$

3) $<\,,\,>$ *is invariant under the action of* $\mathbf{D}^{k+1}$.

Proof. 1) Since $\Omega \in C^0(\Lambda^2 T^*)$, g_Ω is a C^0-Riemannian structure on $\Lambda^2 T^*$ and μ_0 is a C^∞ volume element on M. Hence $<\,,\,>$ is an inner product which induces the H^0-topology on $H^k(\Lambda^2 T^*)$ (see [3], [4], [8]) which is strictly weaker than the H^k-topology ($k > 0$). However, $<\,,\,>_\Omega$ is a positive definite bilinear form, so it gives a weak Riemannian structure on $\mathbf{S}^k$. Now we chek that the weak Riemannian structure on $\mathbf{S}^k$ is smooth. For this it suffices to show that for any Ω^1, $\Omega^2 \in H^k(\Lambda^2 T^*)$, the function $<\Omega^1, \Omega^2> : \mathbf{S}^k \to R$; $\Omega \to <\Omega^1, \Omega^2>_\Omega$ is smooth, uniformly in H^k-norms of Ω^1 and Ω^2. Define $f : \mathbf{S}^k \to H^k(M, R)$ by $f(\Omega) = g_\Omega(\Omega^1, \Omega^2)$. Let E be the subfiber bundle of $\Lambda^2 T^*$ consisting of closed nondegenerate forms on each $T_p M$. Then, it is easy to see that there exists a smooth function $\bar{f} : E \to M \times R$ such that $f = \bar{f} \circ \Omega$. Hence, by theorem 3.3 from [3], f is a smooth function from $\mathbf{S}^k (= H^k(E))$ to $H^k(M, R)$. Also the map $\int \mu_0 : H^k(M, R) \to R$ given by $f \to \int f \mu_0$ is a continuous linear map. It follows that $\Omega \to <\Omega^2, \Omega^1>_\Omega$ is smooth and the uniformity in H^k-norms holds at each step of the proof.

2) Let X, Y, Z be vector fields on $\mathbf{S}^k$ such that $[X, Y] = [X, Z] = [Y, Z] = 0$. Then from (1), (2) we get

(3) $$<\nabla_X Y, Z> = 1/2\,[X<Y, Z> - Z<X, Y> + Y<X, Z>].$$

In the case of a Riemannian metric on a Hilbert manifold there exists a unique vector field $\nabla_X Y$ satisfying (3). In the case of a weak metric, although we know there can be at most one solution, we do not know immediately that one exists. Because $<\,,\,>$ gives the topology of $H^0(\Lambda^2 T^*)$, we can say that there is a unique $\nabla_X Y \in H^0(\Lambda^2 T^*)$. We shall compute this $\nabla_X Y$. Let $\Omega \in \mathbf{S}^k$ be a fixed element and $\Omega_t = \Omega + tX(\Omega)$. Then

$$X<Y,Z>|_\Omega = \left[\frac{d}{dt} <Y,Z>_{\Omega_t}\right]_{t=0} =$$

$$= \int_M \frac{d}{dt} (\mathrm{tr}\,(\Omega_t(p))^{-1}(-Y(\Omega_t(p))(\Omega_t(p))^{-1}Z(\Omega_t(p))|_{t=0}\,\mu_0 =$$

$$= \int_M \mathrm{tr}((\Omega(p))^{-1}X(\Omega(p))\,(\Omega(p))^{-1}Y(\Omega(p))\,(\Omega(p))^{-1}Z(\Omega(p)))\,\mu_0 +$$

$$+\int_M \mathrm{tr}\,((\Omega(p))^{-1}Y(\Omega(p))\,(\Omega(p))^{-1}X(\Omega(p))\,(\Omega(p))^{-1}Z(\Omega(p)))\,\mu_0.$$

With a similar calculation for $Z<X,Y>$, and $Y<X,Z>$, we find th

$$(\nabla_X Y)_{\Omega'} = \frac12 \Omega[\Omega'^{-1}X(\Omega)\,\Omega'^{-1}Y(\Omega)+\Omega'^{-1}Y(\Omega)\,\Omega'^{-1}X(\Omega)]$$

satisfies (3). Since Ω', X, Y are H^k, $\nabla_X Y$ is H^k as well and $\nabla_X Y$ smooth as a function of Ω', so the weak Riemannian structure def a smooth connection on $\mathbf{S}^k$, $k \geq 2m+5$.

3) Fix $f \in \mathbf{D}^{k+1}$ and let $\rho_f : H^k(\Lambda^2 T^*) \to H^k(\Lambda^2 T^*)$ be the m given by $\rho_f(\Omega) = \rho(f,\Omega)$; obviously $\rho_f(\mathbf{S}^k) = \mathbf{S}^k$ and ρ_f is linear; hen $T_\Omega\, \rho_f = \rho_f$. Let $p \in M$, $\Omega^1, \Omega^2 \in T_\Omega(\mathbf{S}^k) = H^k(\Lambda^2 T^*)$.

$$g\, \rho_f(\Omega)(\rho_f(\Omega^1), \rho_f(\Omega^2))(p) = \mathrm{tr}((\rho_f(\Omega)(p))^{-1}(-\rho_f(\Omega^1)(p))(\rho_f(\Omega)(p))^{-1}(\rho_f(\Omega^2)(p))$$

$$= -\,\mathrm{tr}((Tf\circ\Omega_{f(p)}\circ(Tf)^t)^{-1}\circ(Tf\circ\Omega^1_{f(p)}\circ(Tf)^t)^{-1}\circ(Tf\circ\Omega^2_{f(p)}\circ(Tf)^t) =$$

$$= -\,\mathrm{tr}(\Omega^{-1}_{f(p)}\circ\Omega^1_{f(p)}\circ\Omega^{-1}_{f(p)}\circ\Omega^2_{f(p)}) = g_\Omega(\Omega^1,\Omega^2)\,(f(p)) = (g_\Omega(\Omega^1,\Omega^2)\circ f)($$

Then:

$$<\rho_f(\Omega^1),\ \rho_f(\Omega^2)>_{\rho_f(\Omega)} = \int_M g_{\rho_f(\Omega)}\,(\rho_f(\Omega^1),\ \rho_f(\Omega^2))\,\mu_0 = \int_M g_\Omega\,(\Omega^1,\ \Omega^2)\circ f\mu_0 =$$

$$= \int_M g_\Omega\,(\Omega^1,\ \Omega^2)\,\mu_0 = \ <\Omega^1,\ \Omega^2>_\Omega,$$

since $f^*\mu_0 = \mu_0$ and by the change of variables formula. This proves t theorem.

From this theorem it follows that the connection ∇ defines a smo exponential map exp, such that X, Y are vector fields on $\mathbf{S}^k$,

$$\rho_f(\nabla_X Y\,|_\Omega) = \nabla\rho_f(X_\Omega)\,\rho_f(Y_\Omega) \text{ and } \exp\circ T\rho_f = \rho_f\circ\exp.$$

Now we shall define a $\mathbf{D}^{k+1}$-invariant Riemannian structure $<,$ on $\mathbf{S}^k$ which is not weak. First we shall define $<\ ,\ >^k_\Omega$ for $\Omega \in \mathbf{S}^k$. know from [8] (Ch. IX §3) that to get an inner product $<\ ,\ >^k_\Omega$

$H^k(\Lambda^2 T^*)$ it suffices to define one on $H^0(J^k(\Lambda^2 T^*))$ where $J^k(\Lambda^2 T^*)$ is the k-jet bundle of $\Lambda^2 T^*$ (see [8] Ch. IV). Using the Riemannian structure g and the affine connection of M induced by g we shall construct the isomorphism:

$$J^k(\Lambda^2 T^*) \simeq \bigoplus_{m=0}^{k} L^m_s(TM, \Lambda^2 T^*) \simeq \bigoplus_{m=0}^{k} \Lambda^2 T^* \otimes S^m T^*$$

(see [8] Ch. IV). As in [3] (§4) it results that this isomorphism induces a continuous linear map from $H^k(\Lambda^2 T^*)$ to $H^0(\bigoplus_{m=0}^{k} \Lambda^2 T^* \otimes S^m T^*) = \bigoplus_{m=0}^{k} H^0(\Lambda^2 T^* \otimes S^m T^*)$. Now, we note that $\Omega \in \mathbf{S}^k$ induces a Riemannian structure on $S^m T^*$ as in [3]; hence it defines such a structure on $\bigoplus_{m=0}^{k} \Lambda^2 T^* \otimes S^m T^*$. Using this structure and the volume element of g on M, we get an inner product on $H^0(\bigoplus_{m=0}^{k} \Lambda^2 T^* \otimes S^m T^*)$, and by previous isomorphism we obtain a inner product on $H^0(J^k(\Lambda^2 T^*))$ which defines a strong inner product (i.e. gives the H^k-topology) on $H^k(\Lambda^2 T^*)$. The proof that $<\ ,\ >^k : \Omega \to\ <\ ,\ >^k_\Omega$ is a smooth, $\mathbf{D}^{k+1}$-invariant Riemannian structure on $\mathbf{S}^k$ is essentially the same as $<\ ,\ >^0$ case (theorem 1).

§2. **The slice theorem. Theorem 2.** *Let* $k \geq 2m+5$, $\rho : \mathbf{D}^{k+1} \times \mathbf{S}^k \to \mathbf{S}^k$ *be our usual action and* $\mathbf{D}^{k+1}_\Omega = \mathbf{D}^{k+1} \cap \mathbf{D}_\Omega$. *Then there exists a submanifold* $\mathbf{S}^k_1$ *of* $\mathbf{S}^k$ *containing* Ω, *which is diffeomorphic to a ball in a Hilbert space such that:*

1) *If* $f \in \mathbf{D}^{k+1}_\Omega$, $\rho(f, \mathbf{S}^k_1) = \mathbf{S}^k_1$;

2) *If* $f \in \mathbf{D}^{k+1}$ *and* $\rho\,(f, \mathbf{S}^k_1) \cap \mathbf{S}^k_1 \neq \varnothing$ *then* $f \in \mathbf{D}^{k+1}$;

3) *There exists a local cross section* $\chi : \mathbf{D}^{k+1}/\mathbf{D}^{k+1}_\Omega \to \mathbf{D}^{k+1}$ *defined in a neighborhood* U^k *of the identity coset such that the mapping* $F : U^k \times \mathbf{S}^k_1 \to \mathbf{S}^k$ *given by* $F(u, s) = \rho(\chi(u), s)$ *is a homeomorphism onto a neighborhood of* Ω.

Proof. Let $\rho_\Omega : \mathbf{D}^{k+1} \to \mathbf{S}^k$ be the smooth mapping defined by $\rho_\Omega(f) = f^*\Omega$. D. Ebin and J. Marsden in [4] show that $\mathbf{D}^{k+1}_\Omega$ is smooth Hilbert manifold of $\mathbf{D}^{k+1}$. Since $\rho_\Omega(\mathfrak{D}^{k+1}_\Omega) = \Omega$, ρ_Ω defines a map $\bar\rho_\Omega$: $\mathbf{D}^{k+1}/\mathbf{D}^{k+1}_\Omega \to \mathbf{S}^k$ by $\bar\rho_\Omega\ (\mathbf{D}^{k+1}_\Omega f) = f^*\Omega$. Using the implicit function theorem, H. Omori proves in [6] that $\bar\rho_\Omega$ is a homeomorphism of the factor set D^{k+1}/D^{k+1}_Ω with an open, closed subset O^k_Ω of $\mathbf{S}^k$, the orbit through Ω of the action ρ. We introduce a smooth structure on $\mathbf{D}^{k+1}/\mathbf{D}^{k+1}_\Omega$ by declaring $\bar\rho_\Omega$ to be a diffeomorphism. Then $\pi : \mathbf{D}^{k+1} \to \mathbf{D}^{k+1}/\mathbf{D}^{k+1}_\Omega$ is a smooth submersion because ρ_Ω is. Thus as any submersion admits smooth local sections, π becomes a principal bundle with fiber $\mathbf{D}^{k+1}_\Omega$. Our next goal is to show that $\bar\rho_\Omega$ is an injective immersion, hence an embedding. In order to prove this fact we must show firstly that the image of $T_{\pi(f)}\bar\rho_\Omega$ is closed in $T_{\rho_\Omega(f)}(\mathbf{S}^k)$. Since $\rho_\Omega = \bar\rho_\Omega \circ \pi$ and $T_f\pi$ is clearly onto $T_{\pi(f)}(\mathbf{D}^{k+1}/D^{k+1}_\Omega)$ it follows that the image of $T_f\,\rho_\Omega$ is the same as that of $T_{\pi(f)}\bar\rho_\Omega$, for all $f \in \mathbf{D}^{k+1}$. We recall that $T_{id}\,(\mathbf{D}^{k+1})$ is naturally identified with $H^{k+1}(TM)$ and $T_\Omega(\mathbf{S}^k)$ is identified with $H^k(\mathrm{Ker}\, d)$. As in [3]

(Lemma 6.2 and Corollary 6.4) it results that, with the above identi
fication, $T_{id}\,\rho_\Omega$ is the differential operator A_{k+1} defined by $A_{k+1}(X) =$
$= L_X\Omega = d(i_X\Omega)$ and the subspaces $T_f\,\rho_\Omega(T_f\,\mathbf{D}^{k+1}))$, $T_{id}\,\rho_\Omega\,T_{id}\,(\mathbf{D}^{k+1})$
are isomorphic subspaces of $H^k(\Lambda^2 T^*)$. So, to prove that the image
$T_{\pi(f)}\,\bar\rho_\Omega$ is closed in $T_{\rho_\Omega(f)}(\mathbf{S}^k)$, it is sufficient to do this for $T_f\,\rho_\Omega$ in th
case $f =$ id, i.e. to examine A_{k+1}. We consider the differential operato
$A = d(i\,\Omega) : C^\infty(T) \to C^\infty(\Lambda^2 T^*)$ and $B = d : C^\infty(\Lambda^2 T^*) \to C^\infty(\Lambda^3 T^*$
From [6] (§ VIII 2) it is known that $\square = A\,A^* + B^*B$ is elliptic becau
$d\delta + \delta d$ is elliptic, where $\delta = d^*$. Obviously we have $BA = 0$. Applyi
the Theorem 5.2.2 from [6], it results that the image of extended op
rator $A_{k+1} : H^{k+1}(TM) \to H^k(\Lambda^2 T^*)$ (see [8]) is closed and has clos
complement in $H^k(\Lambda^2 T^*)$ for $k \geqslant 2m + 5$.

The proof of the fact that $T\bar\rho_\Omega$ is injective on each tangent spa
follows the proof of the Proposition 6.11 from [3], so that $\bar\rho_\Omega$ is
embedding and $\mathbf{O}^k_\Omega$ is a smooth submanifold of $\mathbf{S}^k$.

Now we construct the normal bundle of $\mathbf{O}^k_\Omega$ in $\mathbf{S}^k$. Let $T(\mathbf{S}^k)/\mathbf{O}$
be the set of tangent vectors in $T(\mathbf{S}^k)$ whose base points are in $\mathbf{O}$
It is a vector bundle over $\mathbf{O}^k_\Omega$ and $T(\mathbf{O}^k_\Omega)$ is a subbundle of it. Defi
another subbundle $\nu = \nu(O^k_\Omega)$ to be:

$$\nu = \{\Omega' \in T(\mathbf{S}^k)/\mathbf{O}^k_\Omega \mid \forall \Omega'' \in T(\mathbf{O}^k_\Omega),\ <\Omega', \Omega''> = 0\},$$

where $< , >$ is the smooth weak Riemannian structure on $\mathbf{S}^k$ defin
in Th. 1. We show that ν is a smooth subbundle of $T(\mathbf{S}^k)/\mathbf{O}^k_\Omega$. To th
aim, as in [3], we shall find a smooth surjective vector bundle map P
$T(\mathbf{S}^k)/\mathbf{O}^k_\Omega \to T(\mathbf{O}^k_\Omega)$ whose kernel is ν. The fiber of ν at Ω is Ker A^*_{k}

Indeed $\Omega' \in \operatorname{Ker} A^*_{k+1}$ iff $< A_{k+1}(X), \Omega' >_\Omega = \int_M g(X, A^*_{k+1}(\Omega'))\,\mu_0 = 0$ Sin

the weak Riemannian structure of $\mathbf{S}^k$ is preserved by the action of $\mathbf{D}$
the fiber of ν at any point $f^*(\Omega)$ is $f^*(\operatorname{Ker} A^*_{k+1})$. Define P on the fib
at Ω to be $A_{k+1} \circ (A^*_{k+1} A_{k+1})^{-1} \circ A^*_{k+1}$ and on the fiber at $f^*(\Omega)$ to
$f^* \circ A_{k+1}(A^*_{k+1} A_{k+1})^{-1} \circ A^*_{k+1} \circ f^{*-1}$. The proof that $\nu = \operatorname{Ker} P$, that P
onto and P is a smooth bundle map is essentially the same as the pr
of the slice theorem for the action of $\mathbf{D}^{k+1}$ on the manifold of Riem
nian metrics (see [3]).

Let $\exp : T(\mathbf{S}^k) \to \mathbf{S}^k$ be the exponential map of the weak Ri
mannian structure on $\mathbf{S}^k$. Then $\exp/_\nu$ is a diffeomorphism from a neig
borhood of the zero section of ν to a neighborhood of $\mathbf{O}^k_\Omega$ in $\mathbf{S}^k$. He
if we consider $H^k(\operatorname{Ker} d)$ as $T_\Omega(\mathbf{S}^k)$ and the strong inner product $<,$
on $H^k(\operatorname{Ker} d)$ defined in §1, there exists a neighborhood U^k of Ω in
(which is identified with $\mathbf{D}^{k+1}/\mathbf{D}^{k+1}_\Omega$), a cross section $\mu : U^k \to \mathbf{D}^{k+1}$ o
U^k of the bundle $\pi : \mathbf{D}^{k+1} \to \mathbf{O}^k_\Omega$ and an $\varepsilon > 0$ such that the neighb
hood of zero in ν_Ω, $V^\varepsilon_\Omega = \{\Omega' \in T_\Omega(\mathbf{S}^k) \mid \Omega' \in \nu_\Omega$ and $<\Omega', \Omega'>^k <$
has the property: if $W_{\mu, V^\varepsilon_\Omega} = \{f^*(\Omega') \mid \Omega' \in V^\varepsilon_\Omega$ and $f \in \mu(U^k)\} \subseteq$
$\exp|W_{\mu, V^\varepsilon_\Omega}$ is a diffeomorphism onto a neighborhood of Ω in $\mathbf{S}^k$ and
$(W_{\mu, V^\varepsilon_\Omega}) \cap \mathbf{O}^k_\Omega = U^k$. Let B^r_Ω the ball about Ω of radius r with respect to
metric defined on $\mathbf{S}^k$ by the Riemannian structure $< , >^k$. Then for so

positive δ, $\exp(W_{\mu, V^\varepsilon_\Omega}) \supseteq B^{2\delta}_\Omega$. Pick $U^k_1 \subseteq U^k$ and $\varepsilon_1 < \varepsilon$ so that if $W_1 = \{f^*(\Omega') \mid \Omega' \in V^{\varepsilon_1}_\Omega, f \in \mu(U^k_1)\}$, $\exp(W_1) \subseteq B^\delta_\Omega$. Let $\mathbf{S}^k_1 = \exp(V^{\varepsilon_1}_\Omega)$. As in [13] it results that $\mathbf{S}^k_1$ satisfies the conditions 1), 2), 3).

To obtain the proof of the slice theorem related to the action ρ, which is stated in introduction, we define $\mathbf{S}_1 = \mathbf{S}^k_1 \cap \mathbf{S}$, $U = U^k \cap \mathbf{D}/\mathbf{D}_\Omega$, where $\mathbf{S}^1_k$ and U^k are constructed in Th. 2.

Now we denote by $\mathbf{S}_0$ the set of symplectic structures on M with trivial isotropy group.

Theorem 3. $\mathbf{S}_0$ *is an open set in* $\mathbf{S}$.

Proof. We consider $\Omega \in \mathbf{S}_0$ and V a neighborhood of id in $\mathbf{D}$. Using notations from the slice theorem, let $\pi : \mathbf{D} \to \mathbf{D}/\mathbf{D}_\Omega$, $U' = U \cap \pi(V)$ and $V_\Omega = F(U', \mathbf{S}_1)$. If $\Omega_1 = F(u, s) \in V_\Omega$ then $\chi = (\mu(u)^{-1})^*(\Omega_1) \in \mathbf{S}_1$ hence by 2) of the slice theorem it results that $\mathbf{D}_\chi \subseteq \mathbf{D}_\Omega$; but $f = \mu(u)$ has the property that $f^{-1}\mathbf{D}_{\Omega_1} f \subseteq \mathbf{D}_\Omega$ and, since $\pi(f) = u \in U$, $f \in V$. From these considerations it follows that $\mathbf{D}_{\Omega_1} = \{\text{id}\}$, $\forall \Omega_1 \in V_\Omega$, hence $V_\Omega \subset \mathbf{S}_0$

REFERENCES

1. Borel A. — *Seminar on transformation groups*, Princeton Univ. Press, Princeton, N. J. 1960.
2. Ebin D. G. — *On the space of Riemannian metrics*, Bull. Amer. Math. Soc. 4, (1968), 1001—1003.
3. Ebin D. G. — *On the space of Riemannian metrics*, Proceedings of Symposia in Pure Math., Global Analysis, XV, Amer. Math. Soc., 1970.
4. Ebin D. G. and Marsden J. — *Groups of diffeomorphisms and the motion of an incompresible fluid*, Ann. of Math. 92 (1970), 102—163.
5. Lang S. — *Introduction to differential manifolds*, Interscience, New York, 1962.
6. Omori H. — *Infinite Dimensional Lie Transformation Groups*, Lecture Notes in Math., 427, Springer-Verlag, 1974.
7. Palais R. — *Foundations of non-linear functional Analysis*, Benjamin, New York, 1968.
8. Palais R. — *Seminar on the Atiyah-Singer Index Theorem*, Princeton Univ. Press, 1965.

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