tion. If we put $\nabla = \Phi(\nabla^0) + A$ then $\mathcal{U}(M, A)$ must be special, hence $A = \Omega A$. Conversely, if $\nabla = \Phi(\nabla^0) + \Omega(A)$ then $\Phi(\nabla) = \nabla$ and ∇ is special.

2.4. Proof of the proposition 3. We use the following.

Lemma 4. *The pair* (∇^1, ∇^2) *is special if and only if the pair* (∇^2, ∇^1) *is also special.*

Proof. If we suppose that the pair (∇^1, ∇^2) is special, then we have $\nabla^2 = 2\Phi(\nabla^1) - \nabla^1$. It follows now from $\Phi(\Phi(\nabla^1)) = \Phi(\nabla^1)$ that

$$\nabla^1 = 2\Phi(\nabla^2) - \nabla^2. \tag{2.3}$$

Let (∇^1, ∇^2) be a special pair of connections, let $\nabla = (\nabla^1 + \nabla^2)/2$ be the median connection and let $A = \nabla^2 - \nabla^1$ be the deformation tensor field. From (1.14) and (2.3) we find

$$\nabla = \Phi(\nabla), \tag{2.4}$$

and subtracting (2.3) from (1.14) we obtain $\Phi(\nabla^2) = \Phi(\nabla^1)$ hence we have

$$\Omega(A) = 0, \tag{2.5}$$

Conversely, (2.4) and (2.5) implies (1.14) and the proof is completed.

§ 3. Some remarks. If the manifold M is an almost product manifold of type (m_1, m_2), then the almost product structure of M is completely determined by the tensor field $J = \pi_1 - \pi_2$.

In this case, a linear connection ∇ is special if and only if $\nabla_X J = 0$ and a pair of two connections (∇^1, ∇^2) is special if and only if $\nabla^{1,2}_X J = \nabla^1_X \circ J - J \circ \nabla^2_X = 0$, for every vector field X.

More generally, in [4] Cruceanu studied the case in which J is an arbitrary tensor field of type (1.1) which has the property that rank $J = \dim M$. For an almost product manifold of type (m_1, m_2) with his general results ([4], the formulae (30), (34), (44), (46)) coincide with the relations (1.14), (2.1), (1.13), (1.15).

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EMBEDDINGS OF C^∞-SUBCARTESIAN SPACES

BY

DUMITRU MOTREANU

§ 1. Preliminaries. The C^∞-subcartesian spaces were introduced by N. Aronszajn [1] and their theory was developed by Ch. Marshall [6].

In this paper, we show that $J^k(S, S')$ is a C^∞-subcartesian space and study the Whitney C^k-topology on $C^\infty(S, S')$. Then we characterize the C^∞-subcartesian spaces which can be embedded into euclidean spaces and prove that $\mathrm{Im}(S, S')$ and $\mathrm{Emb}\,(S, S')$ are open subsets of $C^\infty(S, S')$.

We consider only locally compact and second countable C^∞-subcartesian spaces.

We review some basic definitions [6].

Definition 1.1. *A C^∞-subcartesian space is a Hausdorff space S together with a maximal C^∞-atlas $\mathcal{A}_S = \{\varphi : U_\varphi \to R^{n_\varphi}\}$ such that:* (A_1). *$\{U_\varphi\}$ is an open cover of S and each φ is a homeomorphism.* (A_2). If $\varphi, \psi \in \mathcal{A}_S$ and $p \in U_\varphi \cap U_\psi$ *there exists a C^∞-mapping s extending $\psi\varphi^{-1}$: $\varphi(U_\varphi \cap U_\psi) \to \psi(U_\varphi \cap U_\psi)$ in a neighborhood of $\varphi(p)$.*

Remark 1.2. Each topological subspace of S is a C^∞-subcartesian space.

Definition 1.3. *$f : S \to S'$ is a C^∞-mapping if for every $p \in S$, and every $\varphi \in \mathcal{A}_S$ and $\psi \in \mathcal{A}_{S'}$ with $p \in U_\varphi$ and $f(U_\varphi) \subset U_\psi$, there exists a C^∞-extension $\tilde{f}$ of $\psi f \varphi^{-1}$ in a neighborhood of $\varphi(p)$. The set of C^∞-mappings is denoted by $C^\infty(S, S')$.*

It is well-known that such a C^∞-subcartesian space admits C^∞-partition of unity [6].

The structural dimension of S at p is the number $n_{S,p} = \min\,\{n_\varphi / \varphi \in \mathcal{A}_S$ and $p \in U_\varphi\}$. $\varphi \in \mathcal{A}_S$ is called tangential at p if $p \in U_\varphi$ and $n_\varphi = n_{S,p}$.

In the set of triples (φ, p, v), where $p \in S$, $\varphi \in \mathcal{A}_S$ is tangential at p and $v \in T_{\varphi(p)}R^{n_\varphi}$, we define the equivalence relation $(\varphi, p, v) \sim (\psi, q, w) \Leftrightarrow p = q$ and $w = s_*(v)$, where s is as in (A_2), Definition 1.1. The tangent space of S at p, T_pS, is the set of all equivalence classes of $\sim$ with footpoint p. For each $\varphi \in \mathcal{A}_S$ tangential at p we have the bijection $\varphi_{*p} : T_pS \to T_{\varphi(p)} R^{n_\varphi}$, $\varphi_{*p}([\varphi, p, v]) = v$ (where $[\varphi, p, v]$ is the equivalence class defined by (φ, p, v)).

If $f \in C^\infty(S, S')$ and $p \in S$, we define $f_{*p} : T_pS \to T_{f(p)}S'$ by $f_{*p} = \psi_{*f(p)}^{-1} \tilde{f}_{*p}\, \varphi_{*p}$.

§ 2. **Jets of C^∞-mappings. Definition 2.1.** *Let S, S' be C^∞-subcar-tesian spaces, $p \in S$, V a neighborhood of p in S and $f, g \in C^\infty(V, S')$. We say that f and g have the same 1-jet at p, $j^1_p f = j^1_p g$, if $f(p) = g(p)$ and $f_{*p} = g_{*p}$.* Let $J^1(S, S')$ denote the set of 1-jets of C^∞-mappings. α: $J^1(S, S') \to S$, $\alpha(j^1_p f) = p$, is *the source map* and $\beta : J^1(S, S') \to S'$, $\beta(j^1_p f) = f(p)$, *is the target map.*

Lemma 2.2. a) *$J^1(S, S')$ is a C^∞-subcartesian space;*

b) *α and β are C^∞-mappings;*

c) *if $f \in C^\infty(V, S')$, then $j^1 f \in C^\infty(V, J^1(S, S'))$ $(j^1 f(p) = j^1_p f)$;*

d) *if $f \in C^\infty(S', S'')$, then $f^1_* \in C^\infty(J^1(S, S'), J^1(S, S''))$ $(f^1_*(j^1_p g) = j^1_p(f \circ g)$, $j^1_p g \in J^1(S, S'))$.*

Proof. We verify only a). Let $\varphi \in \mathcal{A}_S$ and $\psi \in \mathcal{A}_{S'}$. Define η: $J^1(U_\varphi, U_\psi) \to R^{n_\varphi} \times R^{n_\psi} \times L(R^{n_\varphi}, R^{n_\psi})$, $\eta(j^1_p f) = (\varphi(p), \psi(f(p)), \psi_{*f(p)} f_{*p} \varphi^{-1}_{*\varphi}$ … considering $L(R^{n_{S,p}}, R^{n_{S',f(p)}}) \subseteq L(R^{n_\varphi}, R^{n_\psi})$. Let $\eta' : J^1(U_{\varphi'}, U_{\psi'}) \to R^{n_{\varphi'}} \times R^{n_{\psi'}} \times L(R^{n_{\varphi'}}, R^{n_{\psi'}})$ corresponding to $\varphi' \in \mathcal{A}_S$ and $\psi' \in \mathcal{A}_{S'}$. The… $\eta' \circ \eta^{-1}(x, y, \lambda) = (\varphi' \varphi^{-1}(x), \psi' \psi^{-1}(y), \psi'_{*\psi^{-1}(y)} \psi_{*\psi^{-1}(y)} \lambda \varphi_{*\varphi^{-1}(x)} \varphi'^{-1}_{*\varphi^{-1}(x)})$. By … Proposition 3.2. [6] there exists a neighborhood V of $\varphi'\varphi^{-1}(x)$ and … C^∞-vector bundle map $F : TV \to TR^{n_\varphi}$ extending $\varphi_* \varphi'^{-1}_*$. Then $x \to F$… is a local C^∞-extension of $x \to \varphi_{*\varphi^{-1}(x)} \varphi'^{-1}_{*\varphi^{-1}(x)}$. With this remark the pro… of the lemma is obvious.

Assume inductively that we have defined $J^k(S, S')$.

Definition 2.3. *We say that $f, g \in C^\infty(V, S')$ (V neighborhood of … point $p \in S$) have the same $(k+1)$-jet at p if $j^1_p j^k f = j^1_p j^k g$.*

Remark 2.4. The standard arguments passing through coordinat… show that we can identify $j^1_p j^k f = j^{k+1}_p f$ for C^∞-manifolds [5].

Remark 2.5. We note that $J^0(S, S') = S \times S'$.

Theorem 2.6. 1) *$J^k(S, S')$, $k \geqslant 0$, is a C^∞-subcartesian space su… that the source map α and the target map β are C^∞-mappings;*

2) *for $f \in C^\infty(S, S')$, we have $j^k f \in C^\infty(S, J^k(S, S'))$;*

3) *if $f \in C^\infty(S', S'')$, then $f^k_* \in C^\infty(J^k(S, S'), J^k(S, S''))$, $f^k_*(j^k_p g) = j^k_p(f \circ g)$;*

4) *if $m < k$, then $\pi^k_m \in C^\infty(J^k(S, S'), J^m(S, S''))$ where $\pi^k_m(j^k_p g) = j^m_p g$, $j^k_p g \in J^k(S, S')$.*

Proof. For 1), 2), 3) we proceed by induction on k. If $k = 1$ w… apply Lemma 2.2. For $k > 1$ we have $J^k(S, S') \subseteq J^1(S, J^{k-1}(S, S'))$… Then by induction hypothesis and combining the arguments of Lemm… 2.2. and Remark 1.2. it follows 1) and 2). To see 3), note that $f^k_* = (f^{k-1}_*)^1_* | J^k(S, S')$. For 4), because $\pi^k_m = \pi^{m+1}_m \ldots \pi^k_{k-1}$, it is sufficient to sho… $\pi^i_{i-1} \in C^\infty(J^i(S, S'), J^{i-1}(S, S'))$ which is obvious by Definition 2.3. an… Lemma 2.2. b).

§ 3. **The Whitney C^k-topology.** The ideas and techniques of [4… p. 42—43, are basic for this section. For each $U \subset J^k(S, S')$ define

$$M(U) = \{f \in C^\infty(S, S') / j^k f(S) \subset U\}.$$

Definition 3.1. *The topology on $C^\infty(S, S')$ induced by the basis $M(U)$… is an open subset of $J^k(S, S')\}$ is called the Whitney C^k-topology. If W… is the set of open subsets of $C^\infty(S, S')$ in the Whitney C^k-topology, $\{W = \bigcup_{k=0}^{\infty}$ … is a basis for the Whitney C^∞-topology.*

Remark 3.2. If $f \in C^\infty(S', S'')$ there is an induced map $f_* : C^\infty(S, S') \to C^\infty(S, S'')$ defined as follows: $f^*(g) = f \circ g$. For each open subset D of $J^k(S, S'')$ we have $f_*^{-1}(M(D)) = M((f^k_*)^{-1}(D))$ and thus f_* is continuous in every C^k-topology.

Lemma 3.3. *Let d_k be a complete metric on $J^k(S, S')$. For each $f \in M(V)$ (V open subset of $J^k(S, S')$) the mapping $m : S \to \overline{R}_+$, $m(x) = \inf \{d_k(\tau, j^k f(x)) / \tau \in \alpha^{-1}(x) \cap (J^k(S, S') - V)\}$, is bounded below on any compact subset in S by a positive constant.*

The proof is elementary.

Theorem 3.4. *Let $f \in C^\infty(S, S')$ and $\delta \in C^\infty(S, R)$, $\delta > 0$. Define $B_\delta(f) = \{g \in C^\infty(S, S') / d_k(j^k f(x), j^k g(x)) < \delta(x),\ x \in S\}$. Then $\{B_\delta(f)\}$ is a neighborhood basis of f in the Whitney C^k-topology on $C^\infty(S, S')$.*

Proof. Given V, an open subset of $J^k(S, S')$, such that $f \in M(V)$, we must choose $\delta \in C^\infty(S, R)$, $\delta > 0$, so that $B_\delta(f) \subset M(V)$. Let $\mathcal{U} = \{U_i\}_{i \geqslant 1}$ be a countable open cover of S such that $\{U_i\}_{i \geqslant 1}$ is locally finite and each $\overline{U}_i$ is a compact set. Let $\{f_i\}_{i \geqslant 1}$ be a C^∞-partition of unity subordinated to $\mathcal{U}$. From Lemma 3.3 it follows that $m(x) \geqslant A_i > 0$, $x \in U_i$. Define

$$g_i(x) = \begin{cases} A_i f_i(x), & x \in U_i \\ 0, & x \notin U_i \end{cases}$$

Then $\delta = \sum_{i=1}^{\infty} g_i \in C^\infty(S, S')$ and $B_\delta(f) \subset M(V)$.

§4. **Immersions and embeddings. Definition 4.1.** *Let $f \in C^\infty(S, S')$. If* rank $f_{*p} = n_{S,p}$ *at each $p \in M$, f is said to be an immersion. If f is a homeomorphism (into) and is an immersion, it is called an embedding. The sets of C^∞-immersions and of C^∞-embeddings are denoted by* $\mathrm{Im}(S, S')$ *and* $\mathrm{Emb}(S, S')$ *respectively.*

Definition 4.2. *The number* $\dim S = \sup \{n_{S,p} / p \in S\}$ *is called the dimension of the C^∞-subcartesian space S.*

Theorem 4.3. *A necessary and sufficient condition for a C^∞-subcartesian space to be embedded in some euclidean space is that* $\dim S < \infty$(*).

Proof. Necessity is obvious. To show sufficiency let $\{(U_i, \varphi_i, R^{n_i})\}_{i \geqslant 1}$ be a countable C^∞-atlas of S such that $\{U_i\}_{i \geqslant 1}$ is an open locally finite cover of S by relatively compact sets $U_i = \varphi_i^{-1}(D_i)$, D_i open subset of R^{n_i} and for each $p \in S$ there exists φ_i with $n_{S,p} = n_i$. Let $n = \dim S < \infty$. Then as in [9], p. 328—329, we can prove that $\dim_L S = m < \infty$ ($\dim_L S$ represents the Lebesgue dimension of S). Apply Lemma 2.4 from [8] to obtain a locally finite open cover $\{G_{j\beta}\}$ $0 \leqslant j \leqslant m$ $\beta \in B_j$ refining $\{U_i\}_{i \geqslant 1}$ such that $G_{j\beta} \cap G_{j\beta'} = \emptyset$ if $\beta \neq \beta'$. Define $V_j = \bigcup_{\beta \in B_j} G_{j\beta}$. For each (j, β) choose $i(j, \beta)$ such that $G_{(j,\beta)} \subset U_{i(j,\beta)}$. Define $\psi_j : V_j \to R^n$, $\psi_j / G_j = i_{n, n_{i(j,\beta)}} \circ \varphi_{i(j,\beta)} / G_{j\beta}$, where $i_{n, n_{i(j,\beta)}} : R^{n_{i(j,\beta)}} \to R^n$ is the canonical injection. Then $(V_j, \psi_j, R^n)_{0 \leqslant j \leqslant m}$ is a C^∞-atlas on S.

*) After I submitted the paper to the redaction I noticed that this theorem was also proved, but in a different way, by M. Brener and Ch. D. Marshall in Math. Ann. 237 (1978), p. 112

Let $\{0_j\}_{0\leq j\leq m}$ be an open cover of S with $0_j \subseteq V_j$ and let $\{f_j\}_{0\leq j\leq m}$, $f_j \in C^\infty(S, R)$, with $0 \leqslant f_j \leqslant 1$, $f_j/0_j = 1$ and $\operatorname{supp} f_j \subseteq V_j$. We can now construct a C^∞-embedding $H = (f, h) : S \to R \times R^{(m+1)(n+1)}$ where $f : S \to R$ is an arbitrary proper C^∞-function (which can be constructed as in [2], p. 74) and $h : S \to R^{(m+1)(n+1)}$ is defined as in [3], p. 77, that is the first components of h are just $h_j^0 = f_j$, $0 \leqslant j \leqslant m$, and the other components are

$$h_j^k(p) = \begin{cases} \operatorname{pr}_k(\psi_j(p))\cdot f_j(p), & p \in V_j, \\ 0 & p \in S - V_j, \end{cases}$$

for $1 \leqslant k \leqslant n$, $0 \leqslant j \leqslant m$.
For $p \in S$, choose $(U_p, \varphi_p, R^{n_p}) \in \mathcal{A}_S$ tangential at p such that $U_p \subset \subset 0_j$, $0 \leq j \leq m$. Then rank $h_{*p} = \operatorname{rank}(h \circ \varphi_p^{-1})_{*\varphi_p(p)} = \operatorname{rank}(\psi_j\, \varphi_p^{-1})_{*\varphi_p(p)} = n_{S,p}$. The fact that H is a homeomorphism (into) is not difficult to prove.

Corollary 4.4. *If S is a compact C^∞-subcartesian space, there exists a C^∞-embedding of S into an euclidean space.*

Theorem 4.5. Im(S, S') *is an open subset of $C^\infty(S, S')$ in the Whitney C^k-topology, $k \geqslant 1$.*

Proof. It is sufficient to consider the case $k = 1$. Let $f \in \operatorname{Im}(S, S')$, $x_0 \in S$, K a compact neighborhood of x_0, $(U, \varphi, R^m) \in \mathcal{A}_S$ tangential at x_0, $(U', \psi, R^n) \in \mathcal{A}_{S'}$ tangential at $f(x_0)$ such that $K \supset U$ and $f(U) \subset U'$. Then there exists a finite family $\{(U_i, \varphi_i, R^{n_i})\}_{1\leq i\leq s} \subset \mathcal{A}_S$ so that $K \subset \bigcup_{i=1}^{s} U_i \subset U$ and for each $p \in K$ there is $1 \leqslant i \leqslant s$ with φ_i tangential at p. Let η be as in the proof of Lemma 2.2. Since $\eta(j^1 f(K))$ is compact and $\eta(j^1 f(K)) \subseteq \bigcup_{i=1}^{s} [\varphi(U_i) \times \psi(U') \times \{A \in L(R^{n_i}, R^n)/\operatorname{rank} A = n_i\}]$ which is open in $\bigcup_{i=1}^{s} [\varphi(U') \times \psi(U') \times L(R^{n_i}, R^n)]$, there exists a positive constant δ_K such that if $g \in C^\infty(S, S')$, $x \in K$, $y \in S'$, then $d_1(j^1 f(x), j^1 f(y)) < \delta_K$ implies that g is an immersion at p. Then we choose a locally finite countable family of compact sets $\{K_i\}_{i\geq 1}$ with $\bigcup_{i=1}^{\infty} \operatorname{Int} K_i = S$ and a corresponding family of positive constants $\{\delta_i\}_{i\geq 1}$ so that if $g \in C^\infty(S, S')$, $x \in K_i$, $y \in S$ with $d_1(j^1 f(x), j^1 g(y)) < \delta_i$, then g is an immersion at p. Now let $\{h_i\}_{i\geq 1}$ be a C^∞-partition of unity for S with $\operatorname{supp} h_i \subseteq \operatorname{Int} K_i$. Then put:

$$\bar{h}_i(p) = \begin{cases} \delta_i\, h_i(p), & p \in \operatorname{Int} K_i, \\ 0, & p \in \operatorname{Int} K_i, \end{cases}$$

and $\delta = \sum h_i \in C^\infty(S, S')$. Given $g \in B_\delta(f)$ and $p \in S$, denote $\delta_0 = \max \{\delta_i / p \in \operatorname{supp} h_i\}$. Then $d_1(j^1 f(p), j^1 g(p)) < \delta_0$ and so g is an immersion.

Lemma 4.7. *Let $\{K_i\}_{i\geq 1}$ be a locally finite countable family of compact sets of S with $\bigcup_{i=1}^{\infty} \operatorname{Int} K_i = S$. For each family $\{\varepsilon_i\}_{i\geq 1}$, $\varepsilon_i > 0$, there exists $\delta \in C^\infty(S, R)$, $\delta > 0$, such that $\delta(x) \leqslant \varepsilon_i$, $x \in K_i$.*

The proof follows by a straightforward use of the partition of unity.

Proposition 4.7. *Let $f \in \operatorname{Emb}(S, S')$ and $\{K_i\}_{i\geq 1}$ as in Lemma 4.6. Then there exists a neighborhood W of f in the C^k-topology, $k \geqslant 1$, such that g/K_i is injective, $i \geqslant 1$, for each $g \in W$.*

Proof. By Lemma 4.6, it is sufficient to prove that there exists a sequence $\{\varepsilon_i\}$, $i \geqslant 1$, $\varepsilon_i > 0$, such that g/K_i is one-to-one if $d_1(j^1 g(x), j^1 f(x)) < \varepsilon_i$, $x \in K_i$, $i \geqslant 1$, $g \in C^\infty(S, S')$. Assume there are $i \geqslant 1$ and the sequences $g_n \in C^\infty(S, S')$, x_n, $y_n \in K_i$ such that:

$$(\ast\ast) \qquad g_n(x_n) = g_n(y_n) \text{ and } d_1(j^1 f(p), j^1 g_n(p)) < 1/n,\ p \in K_i.$$

Since K_i is compact we may assume $x_n \to x$ and $y_n \to y$. From $(\ast\ast)$ it follows $f(x) = f(y)$ and thus $x = y$.

We choose a finite family $\{(U_\alpha, \varphi_\alpha, R^{n_\alpha})\}_{1\leq\alpha\leq s}$ of charts and a open cover $\{V_\alpha\}_{1\leq\alpha\leq s}$ of K_i such that each $\overline{V}_\alpha$ is a compact set included in U_α and if $p \in K_i$ there exists $1 \leqslant \alpha \leqslant s$ so that φ_α/V_α is tangential at p. Then we can find a subsequence of x_n and an integer $1 \leqslant \alpha \leqslant s$ such that φ_α/V_α is tangential at every x_n. It is routine to prove that φ_α is also tangential at x. Then using $(\ast\ast)$ we can continue as in the proof of Theorem 3.10 from [7], p. 30.

Theorem 4.8. Emb(S, S') *is an open subset of $C^\infty(S, S')$ in the Whitney C^k-topology, $k \geqslant 1$.*

Proof. It is sufficient to prove for $k = 1$. Let $f \in \operatorname{Emb}(S, S')$ and d' an admissible metrics for S'. Let $\{U_i\}_{i\geq 1}$ and $\{V_i\}_{i\geq 1}$ be countable covers of S such that $\{U_i\}_{i\geq 1}$ is locally finite and for every i, $\overline{U}_i$ is compact and $\overline{V}_i \subseteq U_i$. Since f is a homeomorphism, we have $\delta_i = d'(f(\overline{V}_i), f(S - U_i)) > 0$. Choose $\varepsilon_i > 0$ such that $d'(f(x), \beta(\sigma)) < \delta_i/2$, if $d_1(j^1 f(x), \sigma) < \varepsilon_i$, $x \in V_i$, $\sigma \in J^1(S, S')$.

By Lemma 4.6, Theorem 4.5 and Proposition 4.7 there exists $\delta \in C^\infty(S, R)$, $\delta > 0$, $\delta(x) \leqslant \varepsilon_i$, $x \in V_i$ and if $g \in B_\delta(f)$, then $g \in \operatorname{Im}(S, S')$ and g/U_i is injective, $i \geqslant 1$. Assume there would exist $g \in B_\delta(f)$, $x \in V_i$ and $y \in V_j$ such tht $g(x) = g(y)$. Then $d'(f(x), f(y)) \leqslant \max(\delta_i, \delta_j)$, and so x, $y \in U_i$ or x, $y \in U_j$, therefore $x = y$.

By a standard argument we can find a neighborhood of f in the Whitney C^0-topology whose intersection with $B_\delta(f)$ consists of homeomorphisms.

Remark 4.9. In the case of C^∞-manifolds the theorems 4.5. and 4.8. are well-known ([2], p. 67—69, or [7], p. 29—31).

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GLOBAL SLICES IN G-SPACES

BY

A. NEAGU

1. Let X be a Hausdorff space and G a compact Lie group. Suppose that G operates continuously on X, If H is a closed subgroup of G we denote:

$$X^H = \{x \in X / hx = x \text{ for all } h \in H\},$$

(the subspace of fixed points of H), X^H is a closed subspace of X[2].

For $x \in X$ let G_x denote the isotropy subgroup of x and $G(x)$ the orbit of x. Let A be a G_x-space and $G \times_{G_x} A$ the twist product of G and A. There is a G-tube about $G(x)$, more precisely, there is a map

$$\varphi : G \times_{G_x} A \to X,$$

such that $\varphi(G \times_{G_x} A)$ is an open neighbourhood of $G(x)$ and φ is an equivariant embeding. The image of $[e, A] \in G \times_{G_x} A$ is called the slice at x and we denote it by $\varphi[e, A] = S$.

Theorem 1. [2] *Let X be a G-space, let $x \in S \subset X$ and put $G_x = H$. The following statements are equivalent:*

1. There exists a G-tube about $G(x)$: $\varphi : G \times_{G_x} A \to X$;

2. S is a slice at x;

3. $G(S)$ is an open neighbourhood of $G(x)$ and there is an equivariant retraction $f : G(S) \to G(x)$ such that $f^{-1}(x) = S$.

Corollary 1. [2] *If S is a slice at x, then $g.S$ is a slice at $g.x$.*

Theorem 2. [2] *Let X be a G-space and let $x \in S$. Suppose that:*

1. S is closed in $G.S$;

2. $G \cdot S$ is an open neighbourhood of $g \cdot x$;

3. $G_x \cdot S = S$;

4. $g \cdot S \cap S \neq \varnothing$ then $g \in G_x$.

Then S is a slice at x. Conversely, every slice satisfies these conditions.

2. The next hypotesis is very important in what follows:

(A). If U_1 and U_2 are two G-tubes, there exists two slices S_1 and S_2 such that: $G \cdot S_1 = U_1$, $G \cdot S_2 = U_2$ and $U_1 \cap U_2 = G \cdot (S_1 \cap S.)$

Corollary 2. *Let S be a slice at x and let $y \in S$ such that* type $G(y) =$ type $G(x) =$ type G/H. *If $G_x = H$ then $G_y = H$.*