10. Mignot, F. — *Contrôle dans les inéquations variationnelles elliptiques*, J. Functional Analysis 22 (1976), 130—185.

11. Mignot F. and Puel J. P. — *Inéquations d'évolution paraboliques avec convexes dépendent du temps. Applications aux inéquations quasi-variationnelles d'évolution*, Arch. Rat. Mech. Anal. 61 (1977), 59—91.

12. Moreno, C. Saguez, Ch. — *Dependence par rapport aux données de la frontière libre associée à certaines inequations variationnelles d'evolution*, Rapport Laboria, Mai 1978.

13. Rockafellar, R.T. — *Integral functional, normal integrands and measurable selections*, in *Nonlinear Operators and the Calculus of Variations Bruxelles 1975*, Lecture Notes in Mathematics, J. P. Gossez ed. Springer-Verlag 1976.

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OSCILLATION THEOREMS FOR SECOND ORDER ORDINARY DIFFERENTIAL INEQUALITIES AND EQUATIONS WITH ALTERNATING COEFFICIENTS

BY

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1. The oscillation problem for differential equations with alternating coefficients is of special importance in applications. Therefore, there has been an increassing interest in the study of the oscillatory behavior of solutions of such differential equations. Among numerous papers dealing with this problem, we choose to refer to [1]—[12]. The equations considered in these papers are the following ones:

$$x''(t) + p(t)\,x(t) = 0,\ t \geq t_0, \tag{1}$$

$$x''(t) + p(t)\,f[x(t)] = 0,\ t \geq t_0, \tag{2}$$

$$[r(t)x'(t)]' + p(t)\,f[x(t)] = 0,\ t \geq t_0, \tag{3}$$

where the real-valued functions p, f and r are subject to appropriate assumptions.

The results obtained there establish sufficient conditions in order that all solutions be oscillatory, and they can be classified in two categories. The first category (cf. [1], [4], [7]—[10] and [12]) concerns sufficient conditions of the form

$$\int^{\infty} \varphi(t)\,p(t)\,dt = \infty,$$

while the second one (cf. [2], [3], [5], [6] and [11]) concerns conditions involving the averaging of the function

$$\int_{T_0}^{t} \varphi(s)\,p(s)\,ds,\ T_0 \geq \max\{t_0, 0\},$$

where, in both cases, φ is an appropriate function.

For the equation (1), Wintner (cf. [11]) introduced the averaging condition

$$\lim_{t\to\infty} \frac{1}{t}\int_{T_0}^{t} ds \int_{T_0}^{s} p(\tau)\,d\tau = \infty.$$

Some more general averages for (1) were introduced by Coles (cf. [2]) and by Coles and Willett (cf. [3]). Later, the concept of averaging was extended by Kamenev (cf. [5] and [6]) to the equation (2).

Our purpose here is to study the equations (1), (2), (3) and more general ones under weaker assumptions. The results obtained below (Sections 3 and 4) concern both of the above categories and generalize and extend previous ones due to Kiguradze,, Kamenev and others (cf. [1]—[12]).

2. Consider the differential equation of the form

$$(*) \qquad [r(t)x'(t)]' + p(t)f[x(t)] + H[t, x'(t), x(t)] = 0, \quad t \geqq t_0.$$

where for the real-valued functions r, p, f and H the following assumptions are made:

(i) r is continuous and positive on $[t_0, \infty)$.

(ii) p is locally summable on $[t_0, \infty)$.

(iii) f is increasing and continuously differentiable on $\mathbf{R}-\{0\}$ and has the sign property

$$yf(y) > 0 \text{ for every } y \in \mathbf{R}-\{0\}.$$

(iv) H is continuous on the set $E = [t_0, \infty) \times \mathbf{R}^2$ and such that for every $(t, y, z) \in E$ with $y \neq 0$

$$yH(t, y, z) > 0.$$

Note. Although the function H is not involved in the process of the proofs, its significance lies in the fact that it broaden the class of the equations (*) so that it responds to a wider area of applications.

In what follows, we consider only such solutions of (*) which are defined for all large t. The oscillatory character is considered in the usual sence, i.e. a continuous real-valued function u on $[t_u, \infty)$ is called *oscillatory* if it has no last zero, ortherwise it is called *nonoscillatory*.

In order to obtain general results and for technical reasons, it is more convenient to work with the corresponding to equation (*) differential inequality

$$(**) \qquad [r(t)x'(t)]' + p(t)f[x(t)] \leqq 0, \quad t \geqq t_0.$$

The results for the equation (*) follow then as immediate corollaries of corresponding theorems obtained for the inequality (**).

3. We start with results concerning the first category of conditions.

Theorem 1. *Consider the differential inequality (**) subject to the conditions* (i)—(iii) *and*

$$(\mathrm{v}) \qquad \int^{\infty} \frac{dy}{f(y)} < \infty.$$

Let Φ be a positive, increasing and continuously differentiable function on $[t_0, \infty)$ such that

$$(\mathrm{C}_1) \qquad \int^{\infty} \frac{dt}{\Phi(t)\, r(t)} = \infty.$$

If either $\Phi'r$ is decreasing on $[t_0, \infty)$ and

$$(\mathrm{C}_2) \qquad \int^{\infty} \Phi(t)\, p(t)\, dt = \infty.$$

or $\Phi'r$ is positive and increasing on $[t_0, \infty)$ and

$$(\mathrm{C}_3) \qquad \lim_{t\to\infty} \frac{1}{|\Phi'(t)|\, r(t)} \int_{t_0}^{t} \Phi(s)\, p(s)\, ds = \infty,$$

then the inequality (**) has no positive solutions.

Proof. We assume the existence of a positive solution x of the differential inequality (**) with domain $[t_0, \infty)$. From (**), because of (iii), we obtain that

$$\frac{[r(t)\, x'(t)]'}{f[x(t)]} + p(t) \leqq 0 \text{ for every } t \geqq t_0.$$

Multiplying this inequality by $\Phi(t)$, integrating the result obtained from t_0 to t and taking into account the fact that $f' \geqq 0$ on $\mathbf{R}-\{0\}$, we deduce that for every $t \geqq t_0$

$$(4) \qquad \frac{\Phi(t)\, r(t)\, x'(t)}{f[x(t)]} + \int_{t_0}^{t} \Phi(s)\, r(s) f'[x(s)] \left[\frac{x'(s)}{f[x(s)]}\right]^2 ds$$

$$\leqq L + \int_{t_0}^{t} \Phi'(s)\, r(s) \frac{x'(s)}{f[x(s)]}\, ds - \int_{t_0}^{t} \Phi(s)\, p(s)\, ds, \text{ where } L = \frac{\Phi(t_0)\, r(t_0) x'(t_0)}{f[x(t_0)]}.$$

We shall prove that for some $T \geqq t_0$ and for every $t \geqq T$.

$$(5) \qquad \frac{\Phi(t)\, r(t)\, x'(t)}{f[x(t)]} + \int_{t_0}^{t} \Phi(s)\, r(s) f'[x(s)] \left[\frac{x'(s)}{f[x(s)]}\right]^2 ds \leqq -1.$$

In fact, if $\Phi'r$ is decreasing on $[t_0, \infty)$, then, by (v) and the Bonnet second mean-value theorem, we easily derive that the first integral on the right-side of (4) is bounded above by a positive constant. Thus, by (C_2), there exists a $T \geqq t_0$ such that for every $t \geqq T$, (5) holds. If now $\Phi'r$ is increasing on $[t_0, \infty)$, then, by using again the Bonnet theorem, for some positive constant c and for every $t \geqq t_0$ we have

$$\int_{t_0}^{t} \Phi'(s)\, r(s) \frac{x'(s)}{f[x(s)]}\, ds \leqq c\, \Phi'(t)\, r(t),$$

and consequently, due to the positivity of $\Phi'r$, from (4) we obtain:

$$\text{(6)}\qquad \frac{1}{\Phi'(t)\, r(t)}\left[\frac{\Phi(t)\, r(t)\, x'(t)}{f[x(t)]} + \int_{t_0}^{t} \Phi(s)\, r(s)\, f'[x(s)]\left(\frac{x'(s)}{f[x(s)]}\right)^2 ds\right] \leqq$$

$$\leqq L_1 + c - \frac{1}{\Phi'(t)\, r(t)}\int_{t_0}^{t} \Phi(s)\, p(s)\, ds, \text{ where } L_1 = \frac{L}{\Phi'(t_0)\, r(t_0)}.$$

From (6), using (C_3), it follows that for some $T \geqq t_0$ and for every $t \geqq T$ (5) holds. Since from (5) we have $x'(t) < 0$ for every $t \geqq T$, (5) can be rewritten as follows

$$\text{(7)}\qquad \frac{\Phi(t)\, r(t)\, |\, x'(t)\, |}{f[x(t)]} \geqq 1 + \int_{t_0}^{t} \Phi(s)\, r(s)\, f'[x(s)]\left(\frac{x'(s)}{f[x(s)]}\right)^2 ds, \quad t \geqq T.$$

Multiplying (7) by

$$\frac{f'[x(t)]\, |\, x'(t)\, |}{f[x(t)]}\left[1 + \int_{t_0}^{t} \Phi(s)\, r(s)\, f'[x(s)]\left(\frac{x'(s)}{f[x(s)]}\right)^2 ds\right]^{-1}, \quad t \geqq T,$$

and integrating the result obtained from T to t, we obtain that for every $t \geqq T$

$$\text{(8)}\qquad \log\left[1 + \int_{t_0}^{t} \Phi(s)\, r(s)\, f'[x(s)]\left(\frac{x'(s)}{f[x(s)]}\right)^2 ds\right] \geqq \log\frac{f[x(T)]}{f[x(t)]}.$$

By (7) and (8), we conclude that

$$x'(t) \leqq -\frac{f[x(T)]}{\Phi(t)\, r(t)} \quad \text{for every } t \geqq T,$$

which, by (C_1), leads to the contradiction $\lim_{t\to\infty} x(t) = -\infty$.

In a similar way one can prove the following theorem.

Theorem 2. *Consider the differential inequality (**) subject to the conditions* (i)—(iii) *and*

$$\text{(vi)}\qquad \int_{0+} \frac{dy}{f(y)} < \infty.$$

Let Φ be a positive, decreasing and continuously differentiable function on $[t_0, \infty]$ satisfying (C_1). If either $\Phi' r$ is increasing on $[t_0, \infty)$ and (C_2) holds or $\Phi' r$ is negative and decreasing on $[t_0, \infty)$ and (C_3) holds, then the inequality (**) has no positive solutions.

From Theorems 1 and 2 we have the following results:

Corollary 1. *Let the conditions* (i)—(iv) *and*

$$\text{(v)}\qquad \int^{\infty} \frac{dy}{f(y)} < \infty \text{ and } \int^{-\infty} \frac{dy}{f(y)} < \infty$$

be satisfied. Then under the assumptions of Theorem 1 all the solutions of the differential equations (*) are oscillatory.

Proof. Let x be a nonoscillatory solution of the equation (*) with domain $[t_0, \infty)$. This solution can be supposed positive on $[t_0, \infty)$, since the substitution $y = -x$ transforms the equation (*) into an equation of the same form satisfying the assumptions of the corollary. Indeed, the transformed equation is

$$[r(t)\, y'(t)]' + p(t)\, \hat{f}[y(t)] + \hat{H}[t\,;\, y(t),\, y'(t)] = 0,$$

where

$$\hat{f}(y) = -f(-y) \text{ and } \hat{H}(t\,;\, y,\, y') = -H(t\,;\, -y,\, -y').$$

Then, due to the positivity of the solution x and because of (iv), x is obviously a solution of the inequality (**), which contradicts Theorem 1.

Corollary 2. *Let the conditions* (i)—(iv) *and*

$$\text{(vi)}\qquad \int_{0+} \frac{dy}{f(y)} < \infty \text{ and } \int_{0-} \frac{dy}{f(y)} < \infty$$

be satisfied. Then under the assumptions of Theorem 2, all the solutions of the differential equation (*) are oscillatory.

Proof. The proof of this corollary is similar to that of Corollary 1.

Theorem 3. *Consider the differential inequality (**) subject to the conditions* (i)—(iii), (v) *and*

$$\text{(vii)}\qquad \int^{\infty} \frac{dt}{r(t)} < \infty.$$

If

$$(C_4)\qquad \limsup_{t\to\infty} \int_{t_0}^{t} \frac{1}{r(\tau)} \int_{t_0}^{\tau} p(s)\, ds\, d\tau = \infty,$$

then for every positive solution x of the inequality (**) we have $\liminf_{t\to\infty} x(t) = 0$.

Proof. Let x be a positive solution of the inequality (**) with domain $[t_0, \infty)$. Then, by (iii), from (**) we have

$$\frac{[r(t)\, x'(t)]'}{f[x(t)]} \leqq -p(t) \quad \text{for every } t \geqq t_0.$$

Integrating this inequality from t_0 to t we obtain that for every $t \geqq t_0$

$$\text{(9)}\qquad \frac{r(t)\, x'(t)}{f[x(t)]} + \int_{t_0}^{t} r(s)\, f'[x(s)]\left(\frac{x'(s)}{f[x(s)]}\right)^2 ds \leqq L - \int_{t_0}^{t} p(s)\, ds, \quad L = \frac{r(t_0)\, x'(t_0)}{f[x(t_0)]}.$$

Taking into account the fact that $f' \geqq 0$ on $\mathbf{R} - \{0\}$, from (9) we get

$$\frac{r(t)\, x'(t)}{f[x(t)]} \leqq L - \int_{t_0}^{t} p(s)\, ds \quad \text{for every } t \geqq t_0.$$

Dividing this inequality by $r(t)$ and integrating the result obtained from t_0 to t we deduce that for every $t \geq t_0$

(10) $$\int_{x(t_0)}^{x(t)} \frac{dy}{f(y)} \leq L \int_{t_0}^{t} \frac{ds}{r(s)} - \int_{t_0}^{t} \frac{1}{r(\tau)} \int_{t_0}^{\tau} p(s)\, ds\, d\tau.$$

From (10), using (vii) and (C_4), we easily get:

(11) $$\liminf_{t\to\infty} \int_{x(t_0)}^{x(t)} \frac{dy}{f(y)} = -\infty.$$

If $\liminf_{t\to\infty} x(t) > 0$, then there exists a $K > 0$ such that

$$x(t) \geq K \text{ for every } t \geq t_0,$$

and consequently, by (v), for every $t \geq t_0$

$$\left| \int_{x(t_0)}^{x(t)} \frac{dy}{f(y)} \right| \leq \int_{K}^{\infty} \frac{dy}{f(y)} < \infty,$$

which contradicts (11). Thus, we must have $\liminf_{t\to\infty} x(t) = 0$.

Corollary 3. *Let the conditions* (i) — (iv), (v) *and* (vii) *be satisfied. Then, under the assumptions of Theorem 3, every solution x of the differential equation* (*) *is such that* $\liminf_{t\to\infty} |x(t)| = 0$.

Proof. The proof of this corollary can be carried out as in Corollary 1.

4. We turn to the second category of conditions involving averages. Following [6] we consider the real-valued functions ψ_i, ω_i, Φ, α_i and Θ, which we suppose to satisfy the following:

The functions ψ_i $(i = 1, 2, \ldots, n)$ are positive and locally summable on $[t_0, \infty)$.

For every $t > t_0$, we define $\omega_0(t) = \psi_1(t)$, and

$$\omega_i(t) = \left(\int_{t_0}^{t} \psi_i(s)\, ds \right)^{-1} \psi_{i+1}(t) \quad (i = 1, 2, \ldots, n).$$

The function Φ is positive and continuously differentiable on $[t_0, \infty)$ and such that either

(I) Φ is monotone and $|\Phi'|\, r$ is decreasing on $[t_0, \infty)$,

or

(II) Φ is strictly monotone and $|\Phi'|\, r$ is increasing on $[t_0, \infty)$.

For every $t \geq t_0$ we define

$$\alpha_1(t) = \frac{1}{\psi_1(t)} \int_{t_0}^{t} \Phi(s)\, \psi_1^2(s)\, ds$$

and

$$\alpha_{i+1}(t) = \frac{1}{\omega_i(t)} \int_{t_0}^{t} \alpha_i(s)\, \omega_i^2(s)\, ds, \quad (i = 1, 2, \ldots, n-1).$$

The function Θ is continuous, nonnegative and increasing on $[0, \infty)$ and such that

(C_5) $$\int^{\infty} \frac{\Theta(v)}{v^2}\, dv < \infty.$$

Moreover, we suppose that the function f in addition to (iii) satisfies

(ε) $$f'(y) \geq \varepsilon > 0 \text{ for every } y \in \mathbf{R} - \{0\}.$$

Now, for every $t > t_0$ we put

$$A_n(t) = \left(\int_{t_0}^{t} \psi_n(\tau)\, d\tau \right)^{-1} \int_{t_0}^{t} \omega_{n-1}(s_{n-1}) \int_{t_0}^{s_{n-1}} \omega_{n-2}(s_{n-2}) \cdots$$

$$\cdots \int_{t_0}^{s_2} \omega_1(s_1) \int_{t_0}^{s_1} \psi_1(s) \int_{t_0}^{s} \Phi(\tau)\, p(\tau)\, d\tau\, ds\, ds_1 \ldots ds_{n-1},$$

and

$$B_n(t) = \left(\int_{t_0}^{t} \psi_1(\tau)\, d\tau \right)^{-1} \int_{t_0}^{t} \omega_{n-1}(s)_{n-1} \int_{t_0}^{s_{n-1}} \omega_{n-2}(s_{n-2}) \cdots$$

$$\cdots \int_{t_0}^{s_2} \omega_1(s_1) \int_{t_0}^{s_1} \frac{\psi_1(s)}{|\Phi'(s)|\, r(s)} \int_{t_0}^{s} \Phi(\tau)\, p(\tau)\, d\tau\, ds\, ds_1 \ldots ds_{n-1},$$

and we give the following two theorems.

Theorem 4. *Let the conditions* (i) — (iii, ε), (v), (I) *and* (C_5) *be satisfied. If, in addition,*

(C_6) $$\int^{\infty} \frac{1}{\alpha_n(t)}\, \Theta\left(\int_{T}^{t} \omega_{n-1}(s)\, ds \right) dt = \infty, \quad T > t_0,$$

and

(C_7) $$\lim_{t\to\infty} A_n(t) = \infty$$

hold, then the differential inequality (**) *has no positive solutions.*

Proof. Let x be a positive solution of the inequality (**) with domain $[t_0, \infty)$. Then, we consider the function w defined by the formula

(12) $$w(t) = \frac{r(t)\, x'(t)}{f[x(t)]}, \quad \text{for every } t \geq t_0.$$

From (12), using (i), (iii, ε) and (**), we get :

$$w'(t) + \frac{w^2(t) f'[x(t)]}{r(t)} \leq -p(t) \text{ for every } t \geq t_0. \tag{13}$$

Multiplying (13) by $\Phi(t)$ and integrating the result obtained from t_0 to t we derive that for every $t \geq t_0$

$$\Phi(t)\, w(t) + \int_{t_0}^{t} \frac{\Phi(\tau)\, w^2(\tau) f'[x(\tau)]}{r(\tau)}\, d\tau \leq L + \int_{t_0}^{t} \Phi'(\tau)\, w(\tau)\, d\tau - \tag{14}$$

$$-\int_{t_0}^{t} \Phi(\tau)\, p(\tau)\, d\tau, \text{ where } L = \Phi(t_0)\, w(t_0).$$

Next, (14) is multiplied by $\omega_0(t) = \psi_1(t)$ and integrated from t_0 to t. The obtained result is multiplied by $\omega_1(t)$ and integrated again from t_0 to t and so on. After $n-1$ such steps, multiplying the obtained result by $\omega_{n-1}(t)$ and integrating from t_0 to t we obtain that for every $t \geq t_0$

$$Q_n(t) + T_n(t) \leq \left(\int_{t_0}^{t} \psi_n(s)\, ds\right)[L + D_n(t) - A_n(t)],$$

where

$$Q_n(t) = \int_{t_0}^{t} \omega_{n-1}(s_{n-1}) \int_{t_0}^{s_{n-1}} \omega_{n-2}(s_{n-2}) \ldots \int_{t_0}^{s_1} \psi_1(s)\, \Phi(s)\, w(s)\, ds\, ds_1 \ldots ds_{n-1}$$

$$T_n(t) = \int_{t_0}^{t} \omega_{n-1}(s_{n-1}) \int_{t_0}^{s_{n-1}} \omega_{n-2}(s_{n-2}) \ldots$$

$$\ldots \int_{t_0}^{s_1} \psi_1(s) \int_{t_0}^{s} \frac{\Phi(\tau)\, w^2(\tau) f'[x(\tau)]}{r(\tau)}\, d\tau\, ds\, ds_1 \ldots ds_{n-1},$$

and

$$D_n(t) = \left(\int_{t_0}^{t} \psi_n(s)\, ds\right)^{-1} \int_{t_0}^{t} \omega_{n-1}(s_{n-1}) \int_{t_0}^{t} \omega_{n-2}(s_{n-2}) \ldots$$

$$\ldots \int_{t_0}^{s_1} \psi_1(s) \int_{t_0}^{s} \Phi'(\tau)\, w(\tau)\, d\tau\, ds\, ds_1 \ldots ds_{n-1}.$$

The rest of the proof can be carried out by using the same arguments with those in [6].

Theorem 5. *Let the conditions* (i)—(iii, ε), (vi), (II) *and* (C_5) *be satisfied. If, in addition,* (C_6) *and*

$$\lim_{t \to \infty} B_n(t) = \infty \tag{C_7}$$

hold, then the differential inequality (**) *has no positive solutions.*

Proof. We assume the existence of a positive solution x of the inequality (**) with domain $[t_0, \infty)$. As in Theorem 1, we define the function w by the formula (12) and prove (13) and (14). Using the Bonnet second mean-value theorem and taking into account the increasing character of the function $\Phi' r$, from (14) we obtain that for some $c > 0$ and for every $t \geq t_0$

$$\Phi(t)\, w(t) + \int_{t_0}^{t} \frac{\Phi(\tau)\, w^2(\tau) f'[x(\tau)]}{r(\tau)}\, d\tau \leq L + c\Phi'(t) r(t) - \int_{t_0}^{t} \Phi(\tau)\, p(\tau)\, d\tau$$

and consequently

$$\frac{\Phi(t)\, w(t)}{\Phi'(t)\, r(t)} + \frac{1}{\Phi'(t)\, r(t)} \int_{t_0}^{t} \frac{\Phi(\tau) w^2(\tau) f'[x(\tau)]}{r(\tau)}\, d\tau \leq L_1 + c - \tag{15}$$

$$-\frac{1}{\Phi'(t)\, r(t)} \int_{t_0}^{t} \Phi(\tau)\, p(\tau)\, d\tau, \quad L_1 = \frac{L}{\Phi'(t_0)\, r(t_0)}.$$

The proof can be completed by successive multiplications and integrations of (15) exactly in the same way as in the proof of Theorem 4.

As applications of Theorems 4 and 5 one can easily derive the following corollaries concerning the differential equation (*).

Corollary 4. *Let the conditions* (i)—(iii, ε), (iv), (v), (I) *and* (C_6) *be satisfied. Then under the assumptions of Theorem 4 all the solutions of the differential equation* (*) *are oscillatory.*

Corollary 5. *Let the conditions* (i)—(iii, ε), (iv), (II) *and* (C_5) *be satisfied. Then under the assumptions of Theorem 5, all the solutions of the differential equation* (*) *are oscillatory.*

Remark. The technique used in this paper for the study of the inequality (**) (Theorems 1—5) is proved to a very useful tool. This technique allowed us to obtain as direct applications corresponding general results for the equation (*) (Corollaries 1—5) which include as special cases the equations (1)—(3). Due to the form of the equation (*) and the assumptions imposed on the functions r, p, f, H and Φ, our results obtained for the equation (*) generalize and extend the corresponding ones given in [1]—[12] even in the cases of the equations (1)—(3). More precisely, Corollaries 1 and 2 generalize and extend the corresponding results given in [4] and [8] for the equations (3) and (2) respectively. Moreover, these corollaries treat the case where Φ is monotone and $|\Phi'|\, r$ is strictly increasing on $[t_0, \infty)$ which have been not done before. Corollary 3 is new and, as far as we know, results of this type are not appeared in the litterature. Corollary 4 generalize and extend the previous result given in [6] for the equation (2). Corollary 5 concerns the case where Φ is strictly monotone and $|\Phi'|r$ is increasing on $[t_0, \infty)$. Such a case has not been treated before.

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REFERENCES

1. Belohorec, S. — *Two remarks on the properties of solutions of nonlinear differential equations*, Acta F.R.N. Univ. Comen. - Mathematica, 22 (1969), 19—26.
2. Coles, W. J. — *An oscillatory criterion for second order linear differential equations*, Proc. Amer. Math. Soc., 19(1968), 755—759;
3. Coles W. J. and Willett, D. — *Summability criteria for oscillation of second order linear differential equations*, Ann. Mat. Pura Appl., 79 (1968), 391—398.
4. Kamenev, I. V. — *On the oscillation of solutions of second order nonlinear differential equations with an alternating coefficient*, Differencial'nye Uravnenija, 6 (1970), 1718-1721 (Russian).
5. Kamenev, I. V. — *Some specific nonlinear oscillation theorems*, Matem. Zam., 10 (1971), 129—136 (Russian).
6. Kamenev, I. V. — *Oscillation criteria related to averaging of solutions of ordinary differential equations of second order*, Differencial'nye Uravnenija, 10 (1974), 246—252 (Russian).
7. Kiguradze, I. T.— *A note on the oscillation of solutions of the equation $u'' + a(t)|u|^n sgn u = 0$*, Casopis Pestov. Mat., 92 (1967), 343—350 (Russian).
8. Kiguradze, I. T. — *Some singular boundary value problems for ordinary differential equations*, University of Tbilisi, Tbilisi 1975 (Russian).
9. Onose, H. — *Oscillation theorems for nonlinear second order differential equations*, Proc. Amer. Math. Soc., 26 (1970), 461—464.
10. Waltman P. — *An oscillation criterion for a nonlinear second order equation*, J. Math. Anal. Appl. 10 (1965), 172—187.
11. Wintner, A. — *A criterion of oscillatory stability*, Quart. Appl. Math., 7 (1949), 115—117.
12. Zlamal, M. — *Oscillations criterions*, Casopis Pestov. Mat., 75 (1950), 213—218.

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СУЩЕСТВОВАНИЕ И ЕДИНСТВЕННОСТЬ ГЛОБАЛЬНОГО РЕШЕНИЯ НЕКОТОРЫХ ИНТЕГРАЛЬНО - ФУНКЦИОНАЛЬНЫХ УРАВНЕНИЙ В ПРОСТРАНСТВЕ

В. Г. АНГЕЛОВ, Д. Д. БАЙНОВ

В настоящей работе рассматривается Вопрос с существовании и
единственности решения некоторых интегрально функциональных урав-
нении. связанные с дифференциально-функциональными уравнениями
нейтрального типа.

Рассмотрим начальную задачу

(1) $$\dot{y}(t) = X(t,\ y(t),\ y(\Delta_1(t)),\ \dot{y}(\Delta_2(t))),\ t > 0,$$

(2) $$y(t) = \psi(t),\ \dot{y}(t) = \dot{\psi}(t),\ t \leqq 0,$$

где неизвестная функция $y(t)$ принимает значения в некотором банахо-
вом пространстве B. Без ограничения общности можно предполагать, что
$\psi(0) = 0$ (см. [1], стр. 254). Положив $x(t) = \dot{y}(t)$ при $t > 0$ и $\varphi(t) = \dot{\psi}(t)$
при $t \leqq 0$, получаем :

(3) $$x(t) = X\left(t, \int_0^t x(s)\,ds, \int_0^{\Delta_1(t)} x(s)\,ds,\ x(\Delta_2(t))\right),\ t > 0,$$

(4) $$x(t) = \varphi(t), \qquad t \leqq 0.$$

Дальше будем заниматься исследованием задачи (3), (4). Методом
аккретивных операторов, докажем теорему с существования и единст-
венности глобального решения начальной задачи (3), (4) в пространстве
L^p суммируемых по Бохнеру с p ой степенью функций, где $1 \leqq p < +\infty$.
Отметим, что в [2] другим методом получены достаточные условия су-
ществования и единственности решения задачи (3), (4) на конечном ин-
тервале при $p = 1$. Результат получен в [2] является самым общим резуль-
татом в фундаментальной теории начальных задач типа (3), (4).

Введем некоторые обозначения. Через R^1 обозначим множество
действительных чисел, через R^1_+-множество неотрицательных действитель-
ных чисел, через R^1_- - множество неположительных действительных чисел,
через B - фиксированное банаховое пространство с нормой $\|\cdot\|$, в которое
искомые функции принимают свои значения. Через $L^p(I\ ;\ B)$ будем обо-
значать множество всех функций $f(t) : \mathrm{I} \to B$(I — интервал действительной