$$u(x^1,\ldots,x^n)=h(a_1x^1+\ldots+a_nx^n),$$

où h est une fonction arbitraire.

Remarque 3.5. 3.5.1. Evidemment, la condition (3.9) est nécessaire et suffisante pour que l'espace M conformément euclidien admette comme courbes presque spéciales des droites parallèles.

3.5.2. La condition nécessaire et suffisante afin qu'un espace de Riemann M conformément euclidien admette comme courbes presque spéciale: des droites parallèles est que sa métrique puisse être réduite, par une transformation linéaire, à la forme canonique:

$$ds^2=e^{2f(x^1)}\,[(dx^1)^2+\ldots+(dx^n)^2], \tag{3.9}$$

où f est une fonction arbitraire. La métrique (3.9′) est l'une des formes données par Schapiro pour la métrique d'un espace de Riemann n—2 fois projectif ([9] p. 39).

3.5.3. Si dans (3.9′) on a $f(x^1)=a_1x^1$, où a_1 est une constante non nulle, alors:

$$ds^2=e^{2a_1x^1}\,[(dx^1)^2+\ldots+(dx^n)^2],$$

est la métrique de Vagner des espaces à connexion constante [9]. Il résulte que: *Tout espace de Riemann M, conformément euclidien qui est un espace de Vagner a comme courbes presque spéciales des droites parallèles.*

3.5.4. Si dans (3.9′) on prend $f(x^1)=(1/2)ln[1/[-K(x^1)^2]]$, où K est une constante négative, alors on obtient la forme canonique de Beltrami

$$ds^2=\frac{(dx^1)^2+\ldots+(dx^n)^2}{-K(x^1)^2}.$$

Il résulte que: *Tout espace de Riemann M à courbure constante négative, rapporté à un système de coordonnées dans lequel la métrique a la forme de Beltrami, a les courbes presque spéciales formées de droites parallèles.*

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DEFORMABLE SOLIDS WITH MICROSTRUCTURE HAVING A SYMMETRIC STRESS TENSOR

BY

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In the last years, various models for bodies with microstructure were elaborated and some boundary problems were solved. Among the papers existing in the scientific literature, concerning this subject we quote [1], [4], [5], [6], [8], [10], [12], [15], [17]. Other results and references to this problem can be found in [9], [11], [14], [15], [16], [18].

Generally, till now, only models with microstructure having an asymmetric streess tensor were considered. But, the asymmetry of the stress tensor is not a consequence of the presence of microstructure in the body, it is rather a constitutive supposition. Then, we may imagine bodies with microstructure having a symmetric stress tensor and, because such a model seems to have remarkable properties, we intend to construct, in the following, a model for a deformable body with microstructure, which has a symmetric stress tensor for all processes of deformation.

First, a more general model will be built and afterwards a model with symmetric stress tensor will be derived.

We refer the motion of the body to fixed rectangular Cartesian axes.

Let V be the region occupied by the body and P an arbitrary part of V; $P\subset V$.

For a body with microstructure we take the usual definition. Then, for such a body there exists a vector field φ, called the microrotations field and a tensor $J_{ik}=J_{ki}$ $(J_{ik}\,\xi_i\,\xi_k>0,\ \xi_i\neq 0)$, called the microinertia tensor, defined on V, such that, the angular momentum $\mathbf{K}(P)$ and the kinetic energy $E(P)$ of the mass contained in the region $P\subset V$ are given by

$$\mathbf{K}(P)=\int_P\rho(x)[\mathbf{r}\times\mathbf{v}+J_{ik}\,\dot\varphi_k\mathbf{e}_i]dX,\qquad E(P)=\frac{1}{2}\int_P\rho(x)[\mathbf{v}^2+J_{ik}\,\dot\varphi_i\,\dot\varphi_k]dX,$$

where $\rho(x)$ is the density of the medium, $\mathbf{r}$ is the position of the point $x\in P$ with respect to the coordinate system, $\mathbf{v}$ is the velocity of this point, $\varphi=$ $=\varphi_k\mathbf{e}_k$, $\dot\varphi=\dot\varphi_k\mathbf{e}_k=\dfrac{d\varphi}{dt}$. $\mathbf{e}_k$ are the orthogonal unit vectors parallel to the coordinate axes and dX is the volume element.

It is known that the models with microstructure have some connections with nonlocal theories. For this reason, in order to construct the mentioned model, we shall use the same scheme as in [2], [3], but adapted to a local description of the deformation.

In brief, this scheme is as follows : The external influences are modeled by external surface tractions and external surface couples, external body forces and external body couples, external body heat sources, external magnetic flux, etc. The body reply to external actions are among internal surface tractions and internal surface couples, internal body forces and internal body couples, internal body heat sources etc.

Here, we will consider that the external influences received by the body are represented by external surface tractions and external surface couples, external body forces and external body couples, external heat body sources and an external surface heat flux. As reply, the body opposes internal body forces and internal body couples, internal surface tractions and internal surface couples, internal body heat sources and an internal heat flux.

We suppose that for our model the following principles are valid :

I. *Principle of conservation of mass* :

$$\frac{d}{dt}\int_P \rho(x)\,dX=0. \tag{3}$$

II. *Principle of internal body actions* :

a) There exists a vector function $\mathbf{g}(x)$ defined on V, called internal body forces density per unit mass, such that, the total internal body force $\mathbf{g}(P)$ acting on the mass contained in a region $P\subset V$ is given by

$$\mathbf{g}(P)=\int_P \rho(x)\,\mathbf{g}(x)\,dX,\qquad \mathbf{g}(x)=\mathbf{g}_P(x)+\mathbf{g}_{V-P}(x)=\mathbf{g}_V(x), \tag{5}$$

where $\mathbf{g}_P(x)$ is the contribution to $\mathbf{g}(x)$ of the mass contained in P and $\mathbf{g}_{V-P}(x)$ is the contribution to $\mathbf{g}(x)$ of the mass located in $V—P$.

We have

$$\int_P \rho(x)\,\mathbf{g}_P(x)\,dX=0. \tag{6}$$

b) There exists a vector function $\mathbf{m}(x)$ defined on V, called internal body couples density per unit mass, such that, the total body couple acting on the mass contained in P is given by

$$\mathbf{m}(P)=\int_P \rho(x)\,\mathbf{m}(x)\,dX,\qquad \mathbf{m}(x)=\mathbf{m}_P(x)+\mathbf{m}_{V-P}(x)=\mathbf{m}_V(x) \tag{7}$$

and

$$\int_P \rho(x)\,[\mathbf{r}\times\mathbf{g}_P(x)+\mathbf{m}_P(x)]\,dX=0. \tag{8}$$

c) There exists a scalar function $w(x)$ defined on V, called internal body heat sources density per unit mass, such that, the total internal body heat contained in the region P is given by

$$w(P)=\int_P \rho(x)\,w(x)\,dX,\quad w(x)=w_P(x)+w_{V-P}(x)=w_V(x), \tag{9}$$

and

$$\int_P \rho(x)[\mathbf{g}_P(x)\,\mathbf{v}+\mathbf{m}_P(x)\dot{\varphi}+w_P(x)]\,dX=0. \tag{10}$$

In the following, instead of (10), we take as valid the equation

$$\mathbf{g}(x)\,\mathbf{v}+\mathbf{m}(x)\dot{\varphi}+w(x)=0, \tag{11}$$

which is, evidently, more restrictive.

In the above formulae $\mathbf{m}_P$, $\mathbf{m}_{V-P}$ and w_P, w_{V-P} have similar meaning as $\mathbf{g}_P$, $\mathbf{g}_{V-P}$.

III. *Principle of linear momentum* :

$$\frac{d}{dt}\int_P \rho(x)\,\mathbf{v}(x)\,dX=\int_P \rho(x)\,[\mathbf{f}(x)+\mathbf{g}_{V-P}(x)]\,dX+\int_{\partial P}\mathbf{t}\,d\sigma, \tag{12}$$

where $\mathbf{f}(x)$ are the external body forces per unit mass, $\mathbf{t}$ are the surface tractions per unit area of ∂P, the boundary of P.

IV. *Principle of angular momentum* :

$$\begin{aligned}\frac{d}{dt}\int_P \rho(x)[\mathbf{r}\times\mathbf{v}+J_{ik}\varphi_k\mathbf{e}_i]\,dX&=\int_P \rho(x)[\mathbf{r}\times(\mathbf{f}+\mathbf{g}_{V-P})+\mathbf{M}+\mathbf{m}_{V-P}]\,dX+\\&+\int_{\partial P}(\mathbf{r}\times\mathbf{t}+\mathbf{c})\,d\sigma,\end{aligned} \tag{13}$$

where $\mathbf{M}(x)$ are the external body couples per unit mass, and $\mathbf{c}$ is the surface couple stress vector per unit area across the surface ∂P.

V. *Principle of energy* : There exists a specific internal energy ε per unit mass, defined on V, such that, the balance of energy can be stated as

$$\begin{aligned}&\frac{d}{dt}\int_P \rho(x)\left[\varepsilon+\frac{1}{2}\mathbf{v}^2+\frac{1}{2}J_{ik}\dot{\varphi}_i\dot{\varphi}_k\right]dX=\\&=\int_P \rho(x)\,[(\mathbf{f}+\mathbf{g}_{V-P})\mathbf{v}+(\mathbf{M}+\mathbf{m}_{V-P})\dot{\varphi}+r+w_{V-P}]\,dX+\int_{\partial P}(\mathbf{t}\mathbf{v}+\mathbf{c}\dot{\varphi}-h)\,d\sigma,\end{aligned} \tag{14}$$

where r is the external body heat sources per unit mass and h is the surface heat flux per unit area across the surface ∂P.

VI. *Principle of entropy.* There exists a specific entropy S per unit mass, defined on V, such that, for any process of deformation we have

$$\frac{d}{dt}\int_P \rho S\,dX-\left\{\int_P \rho\,\frac{r}{T}\,dX-\int_{\partial P}\frac{h}{T}\,d\sigma\right\}\geqslant 0, \tag{15}$$

where $T>0$ is the absolute temperature.

Using the above principles, we can derive the equations of motion and the constitutive equations for our body.

We shall use the formulae

$$\mathbf{t}=t_k\mathbf{e}_k=t_{jk}n_j\mathbf{e}_k=\mathbf{t}_jn_j,\qquad \mathbf{f}=f_i\mathbf{e}_i,\quad \mathbf{g}=g_i\mathbf{e}_i,$$
$$\mathbf{c}=c_k\mathbf{e}_k=c_{jk}n_j\mathbf{e}_k=\mathbf{c}_jn_j,\qquad \mathbf{M}=M_i\mathbf{e}_i,\quad \mathbf{m}=m_i\mathbf{e}_i,\ \text{etc.}$$

where t_{ij} are the stress tensor components, c_{ij} are the couple stress components and $\mathbf{n}=n_k\mathbf{e}_k$ is the outward unit normal to the surface ∂P.

The principle of conservation of the mass gives us the equation of continuity

$$\frac{d\rho}{dt}+\rho\,\mathrm{div}\,\mathbf{v}=0. \tag{16}$$

From the principle of linear momentum with (6), (16) and the divergence theorem, we get

$$\int_P[\mathbf{t}_{i,i}+\rho(\mathbf{f}+\mathbf{g})-\rho\mathbf{a}]\,dX=0, \tag{17}$$

where $\mathbf{a}$ is the acceleration field and the index j after comma indicates partial differentiation with respect to x_j.

Since the integrand is supposed to be continuous and (17) must hold for arbitrary P, it results that

$$t_{ji,j}+\rho(f_i+g_i)=\rho a_i \tag{18}$$

Taking into account the equations (16), (18) and (8), from the principle of angular momentum, at last, we have

$$c_{ji,j}+\varepsilon_{ijk}\,t_{jk}+\rho(M_i+m_i)=\rho J_{ik}\ddot{\varphi}_k+\rho\dot{J}_{ik}\dot{\varphi}_k, \tag{19}$$

where ε_{ijk} is the alterning symbol.

The model which satisfies the equations (18) and (19) is similar to the micropolar models (with free rotations), but the presented model has a different behaviour and we call it *model with natural rotations.*

The postulate of energy after some calculus takes the form

$$\begin{aligned}\int_P\rho(\dot{\varepsilon}+v_i\dot{v}_i+J_{ik}\dot{\varphi}_i\ddot{\varphi}_k)\,dX&=\int_P\rho[(f_i+g_i)\,v_i+(M_i+m_i)\,\dot{\varphi}_i+r+w]\,dX+\\&+\int_P(t_{ji,j}v_i+t_{ji}v_{i,j}+c_{ji,j}\,\dot{\varphi}_i+c_{ji}\,\dot{\varphi}_{i,j}-q_{i,i})\,dX,\qquad J_{ik}\dot{\varphi}_i\dot{\varphi}_k=0.\end{aligned} \tag{20}$$

where q_i are the components of the heat flux.

Because (20) holds for arbitrary P and the equations (18), (19) are valid, we obtain the equation of energy in the form

$$\rho\dot{\varepsilon}-t_{ji}\,v_{i,j}-c_{ji}\,\dot{\varphi}_{i,j}+\varepsilon_{ijk}\,t_{jk}\,\dot{\varphi}_i+q_{i,i}-\rho(r+w)=0. \tag{21}$$

The inequality of entropy can be reduced to

$$\rho T\dot{S}-\rho r-q_{i,i}-\frac{q_iT_{,i}}{T}\geqslant 0. \tag{22}$$

Making use of the specific Helmholtz free energy $\Psi=\varepsilon-ST$, and taking into account (11), the equation of energy and the inequality of entropy become

$$\begin{aligned}&\rho(\dot{\psi}+\dot{S}T+S\dot{T})-t_{ji}\,v_{i,j}-c_{ji}\,\dot{\varphi}_{i,j}+\varepsilon_{ijk}\,t_{jk}\,\dot{\varphi}_i+\\&\quad+\rho(g_i\,v_i+m_i\,\dot{\varphi}_i)+q_{i,i}-\rho r=0,\end{aligned} \tag{23}$$

$$\begin{aligned}&-\rho(\dot{\psi}+S\dot{T})+t_{ji}\,v_{i,j}+c_{ji}\,\dot{\varphi}_{i,j}-(\rho m_i+\varepsilon_{ijk}\,t_{jk})\dot{\varphi}_i-\\&\quad-\rho g_i\,v_i-\frac{q_i\,T_{,i}}{T}\geqslant 0.\end{aligned} \tag{24}$$

Now, it remains to prescribe appropriate constitutive equations for ψ, S, t_{ij}, c_{ij}, g_i m_i and q_i, the internal body sources w being given by (11). The constitutive equations must be chosen in accordance with the principle of objectivity.

It remains to see if a local model is or is not compatible with the presence of internal body forces and internal body couples.

In the following we suppose $g_i=0$, $m_i=0$, and, in addition, the stress tensor is symmetric*.

Consequently, for the linear theory, we will take as independent constitutive variables. $\varepsilon_{ij}=1/2\,(u_{i,j}+u_{j,i})$, $\chi_{ij}=\varphi_{i,j}$, T and $T_{,j}$, where u_i are the displacements.

Then, supposing the body homogeneous, we propose a set of constitutive equations in the form

$$\begin{cases}\psi=\psi(\varepsilon_{ij},\chi_{ij},T,T_{,i}), & c_{ij}=c_{ij}(\varepsilon_{ij},\chi_{ij},T,T_{,i}),\\ S=S(\varepsilon_{ij},\chi_{ij},T,T_{,i}), & q_i=q_i(\varepsilon_{ij},\chi_{ij},T,T_{,i})\\ t_{ij}=t_{ij}(\varepsilon_{ij},\chi_{ij},T,T_{,i}),\end{cases} \tag{25}$$

and the inequality of entropy becomes

$$\begin{aligned}&-\left(S+\frac{\partial\psi}{\partial T}\right)\dot{T}+\left(t_{ij}-\rho\,\frac{\partial\psi}{\partial\varepsilon_{ij}}\right)\dot{\varepsilon}_{ij}+\\&+\left(c_{ij}-\rho\,\frac{\partial\psi}{\partial\chi_{ij}}\right)\dot{\chi}_{ij}-\rho\,\frac{\partial\psi}{\partial T_{,i}}\,\dot{T}_{,i}-\frac{q_iT_{,i}}{T}\geqslant 0.\end{aligned} \tag{26}$$

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* Continuous media which, in addition to the stated postulates, satisfy also the conditions $g_i=m_i=0$, are only the micropolar media and media with symmetric stress tensor.

Because (26) must be satisfied for arbitrary $\dot{T}$, $\dot{\varepsilon}_{ij}$, $\dot{\chi}_{ij}$, $\dot{T}_{,i}$, we have

$$S=-\frac{\partial\psi}{\partial T},\quad t_{ij}=\rho\frac{\partial\psi}{\partial\varepsilon_{ij}},\quad \frac{\partial\psi}{\partial\varepsilon_{ij}}=\frac{\partial\psi}{\partial\varepsilon_{ji}},\quad c_{ij}=\rho\frac{\partial\psi}{\partial\chi_{ij}},\quad \frac{\partial\psi}{\partial T_{,i}}=0,$$

ε_{ij} and ε_{ji} being formally considered as independent variables.

The inequality of entropy and the equation of energy reduce to

$$-q_i\,T_{,i}\geq 0,\quad \rho\dot{S}T+q_{i,i}-\rho r=0.$$

In order to satisfy the inequality of entropy we can take for q_i a law of Fourier's type

$$q_i=-K_{ij}\,T_{,j},$$

such that $K_{ij}=K_{ji}$ are the coefficients of a positive definite quadratic form.

Now, let us suppose that in the reference configuration, the body is free from stresses and couple stresses and take ψ as a polynomial of second degree of ε_{ij}, χ_{ij}, and $\theta=T-T_0$, hence we have

$$\begin{aligned}2\rho\psi=\;&C_{ijmn}\,\varepsilon_{ij}\,\varepsilon_{mn}+2H_{ijmn}\,\varepsilon_{ij}\,\chi_{mn}+2\alpha_{ij}\,\varepsilon_{ij}\,\theta+\\&+M_{ijmn}\,\chi_{ij}\,\chi_{mn}+2\beta_{ij}\,\chi_{ij}\theta+K\theta^2,\end{aligned}\tag{27}$$

where T_0 is the uniform temperature of the natural state, and C_{ijmn}, H_{ijmn}, ... are the tensors of elastic moduli which describe the mechanical properties of the medium.

The constitutive equations are

$$\left\{\begin{aligned}&t_{ij}=C_{ijmn}\,\varepsilon_{mn}+H_{ijmn}\,\chi_{mn}+\alpha_{ij}\theta,\\&c_{ij}=H_{ijmn}\,\varepsilon_{mn}+M_{ijmn}\,\chi_{mn}+\beta_{ij}\theta,\\&-\rho S=\alpha_{mn}\,\varepsilon_{mn}+\beta_{mn}\,\chi_{mn}+K\theta.\\&C_{ijmn}=C_{jimn}=C_{ijnm}=C_{mnij},\qquad H_{ijmn}=H_{jimn},\ \alpha_{ij}=\alpha_{ji},\end{aligned}\right.\tag{28}$$

To the obtained equations we must add initial conditions and boundary conditions.

The model having the constitutive equations given by (28) may possess remarkable properties worth to be studied.

As an example, let us consider the case when

$$\begin{aligned}2\rho\psi=\;&\lambda\varepsilon_{mm}\,\varepsilon_{nn}+2\mu\varepsilon_{ij}\,\varepsilon_{ij}+2\omega\varepsilon_{mm}\,\chi_{nn}+2\pi\varepsilon_{ij}(\chi_{ij}+\chi_{ji})+\\&+\alpha\chi_{mm}\,\chi_{nn}+\beta\chi_{ij}\,\chi_{ij}+\gamma\chi_{ij}\,\chi_{ji}\end{aligned}$$

so that

$$t_{ij}=(\lambda\varepsilon_{mm}+\omega\chi_{mm})\,\delta_{ij}+2\mu\varepsilon_{ij}+\pi(\chi_{ij}+\chi_{ji}),$$

$$c_{ij}=(\omega\varepsilon_{mm}+\alpha\chi_{mm})\,\delta_{ij}+2\pi\varepsilon_{ij}+\beta\chi_{ij}+\gamma\chi_{ji}.$$

In connection with these constitutive equations, let us examine the extent and the torsion of a cylindrical beam. The beam is limited by two

planes $x_3=0$, $x_3=h$ $(h>0)$ and by a cylindrical surface $\mathcal{F}$. The domain of a cross-section of the beam and its area will be denoted by S and the boundary of S by Γ. We take the axis of x_3 to be central-line of the beam and the axes of x_1 and x_2 to be principal axes of the end $x_3=0$.

We suppose that there are no body forces and no body couples, so that the equilibrium equations are given by

$$t_{ji,j}=0,\qquad c_{ji,j}=0.$$

The beam is in equilibrium under following boundary conditions

$$t_{ji}\,n_j=0,\qquad c_{ji}\,n_j=0\quad\text{on }\mathcal{F};$$

$$\int\limits_S t_{3i}\,d\sigma=R_3\,\delta_{i3},\quad \int\limits_S t_{33}x_\alpha d\sigma=0\ \ (\alpha=1,2),$$

$$\int\limits_S(x_1\,t_{23}-x_2\,t_{13})\,d\sigma=\mathcal{M}_3,\quad \int\limits_S c_{3i}\,d\sigma=C_3\delta_{i3}\ \text{ on }\ x_3=h;$$

where R_3, $\mathcal{M}_3$ and C_3 are given.

The above problem can be solved if we take the displacements u_i and the rotations φ_i in the form

$$u_1=-\tau x_2x_3+a_1x_1,\quad u_2=\tau x_1x_3+a_2x_2,\quad u_3=\tau\varphi(x_1,x_2)+a_3\,x_3,$$

$$\varphi_1=b_1\,x_1,\quad \varphi_2=b_2\,x_2,\quad \varphi_3=b_3\,x_3,$$

where $\varphi(x_1,x_2)$ is the classical function of torsion defined by

$$\varphi_{,ii}=0\ \text{ in }S,$$

$$\varphi_{,1}\,n_1+\varphi_{,2}\,n_2=x_2\,n_1-x_1\,n_2\ \text{ on }\Gamma.$$

The constants a_i and b_i are taken such that

$$(\lambda+2\mu)a_1+\lambda a_2+\lambda a_3+(\omega+2\pi)b_1+\omega b_2+\omega b_3=0,$$

$$\lambda a_1+(\lambda+2\mu)a_2+\lambda a_3+\omega b_1+(\omega+2\pi)b_2+\omega b_3=0,$$

$$\lambda a_1+\lambda a_2+(\lambda+2\mu)a_3+\omega b_1+\omega b_2+(\omega+2\pi)b_3=\frac{R_3}{S},$$

$$(\omega+2\pi)a_1+\omega a_2+\omega a_3+(\alpha+\beta+\gamma)b_1+\alpha b_2+\alpha b_3=0,$$

$$\omega a_1+(\omega+2\pi)a_2+\omega a_3+\alpha b_1+(\alpha+\beta+\gamma)b_2+\alpha b_3=0,$$

$$\omega a_1+\omega a_2+(\omega+2\pi)a_3+\alpha b_1+\alpha b_2+(\alpha+\beta+\gamma)b_3=\frac{C_3}{S},$$

and τ is determined as in the classical problem of torsion.

The determinant of the above equations is different from zero because ψ is a positive definite quadratic form. The necessary and sufficient conditions for ψ to be nonnegative can be put into evidence if we remark that we may write:

$$2\rho\psi=(\lambda-\omega)\,\varepsilon_{mm}\,\varepsilon_{nn}+2(\mu-\pi)\,\varepsilon_{ij}\,\varepsilon_{ij}+$$
$$+(\alpha-\omega)\,\gamma_{mm}\,\gamma_{nn}+(\beta-2\pi)\,\gamma_{ij}\,\gamma_{ij}+\gamma\gamma_{ij}\,\gamma_{ji}+$$
$$+\omega(\varepsilon_{mm}+\gamma_{mm})\,(\varepsilon_{nn}+\gamma_{nn})+\pi(\varepsilon_{ij}+\gamma_{ij})\,(\varepsilon_{ij}+\gamma_{ji})+$$
$$+\pi(\varepsilon_{ij}+\gamma_{ji})\,(\varepsilon_{ij}+\gamma_{ji}).$$

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THERMAL STRESSES IN ANISOTROPIC MICROPOLAR ELASTIC CYLINDERS

BY

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1. **Introduction.** In the classical theory of thermoelasticity the problem of thermal stresses in right cylinders has been considered in various papers (see e.g. [1], [2]). For the case of homogeneous and isotropic micropolar elastic cylinders this problem was studied in [3]. In this paper we consider the problem of thermal stresses for inhomogeneous and anisotropic micropolar elastic cylinders. The coefficients which characterize the thermoelastic properties of the cylinder are assumed to be independent of the axial coordinate. We suppose the cylinder to be in equilibrium under the action of a temperature distribution which is a polynomial in the axial coordinate. The method is used in order to study the deformation of a circular cylinder.

2. **Statement of the problem** We consider a right cylinder of lenght l, bounded by plane ends perpendicular to the generators. The generic cross-section Σ is assumed to be bounded by the curve L. In this paper a rectangular Cartesian system $0x_k$, $(k=1, 2, 3)$, is used. The x_3 — axis is parallel to the generators of the cylinder and $x_1 0 x_2$ — plane contains one of terminal cross-sections. We shall employ the usual summation and differentiation conventions: Greek subscripts are understood to run over the integers (1, 2), whereas Latin subscripts-unless otherwise specified — to the range (1, 2, 3,); summation over repeated subscripts is used and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate.

We assume that the cylinder is occupied by an anisotropic micropolar elastic material. We suppose that the body loadings and the lateral loadings are absent. The cylinder is assumed to be in equilibrium under the action of a given temperature, the loadings applied on each ends being statically equivalent to zero.

The basic equations of the linear theory of micropolar thermoelastostatics are [4]:

— the equilibrium equations

$$t_{ji,j}=0, \qquad m_{ji,j}+\varepsilon_{ijk}\,t_{jk}=0, \tag{2.1}$$

— the constitutive equations

$$\begin{aligned} t_{ij}&=A_{ijkl}\,e_{kl}+B_{ijkl}\,\gamma_{kl}-D_{ij}\,T,\\ m_{ij}&=B_{klij}\,e_{kl}+C_{ijkl}\,\gamma_{kl}-E_{ij}\,T, \end{aligned} \tag{2.2}$$