we obtain

$$([m]\times B)^{-1}\circ \underset{PA\times f}{\exists}(\in_A)=\underset{1\times f}{\exists}(([m]\times A)^{-1}(\in_A)=$$

$$=\underset{1\times f}{\exists}(A'\xrightarrow{p_m^{-1}\cdot m}1\times A)=p_A^{-1}\circ\exists_f(A'\xrightarrow{m}A)=p_B^{-1}(B'\xrightarrow{m'}B)=$$

$$=B'\xrightarrow{p_B^{-1}\cdot m}1\times B.$$

c) Let $A'\xrightarrow{m}A$ be a subobject of $A$, let $\varphi_m: A\to\Omega$ be its characteristic morphism, let $\lambda_m=\varphi_m p_A: 1\times A\to\Omega$ and $[m]: 1\to PA$ be the unique morphism such that $e_A\circ([m]\times A)=\lambda_m$. Let $B'\xrightarrow{m'}B=\forall_f(A'\xrightarrow{m}A)$ be a morphism and $\varphi_{m'}: B\to\Omega$ be its characteristic morphism, let $\lambda_{m'}=\varphi_{m'}p_B: 1\times B\to\Omega$ be a morphism and let $[m']: 1\to PB$ be the unique morphism such that $e_B([m']\times B)=\lambda_{m'}$. To show that $\forall_f[m]=[m']$, it is enough to prove that $\forall f([m]\times B)=\lambda_m$, and for that, to show they are the characteristic morphisms of a same subobject $B'\xrightarrow{p_B^{-1}\cdot m'}1\times B$ of $1\times B$. Applying the Proposition 1.02 to the pullbacks *1 we have

$$([m]\times B)^{-1}\circ\underset{PA\times f}{\forall}(\in_A)=\underset{1\times f}{\forall}(([m]\times A)^{-1}(\in_A))=\underset{1\times f}{\forall}(A'\xrightarrow{p_m^{-1}\circ m}1\times A)=$$

$$=p_B^{-1}\circ\forall_f(A'\xrightarrow{m}A)=p_B^{-1}(B'\xrightarrow{m'}B)=B'\xrightarrow{p_B^{-1}m'}1\times B.$$

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# THE PRECISE ANNULI CONTAINING THE ZEROS OF POLYNOMIALS

BY

ALEXANDER ABIAN

Let $p(z)=a_0+\ldots+a_nz^n$ with $a_0\neq 0$ be a polynomial over complex numbers. The mathematical literature is replete [1] with results concerning the bounds for the moduli of zeros of $p(z)$. Below we give formulas for evaluating the precise minimum $r$ and the precise maximum $R$ of the moduli of zeros of $p(z)$. Thus, $r\leqslant|z|\leqslant R$ is the precise annulus which contains all the zeros of $p(z)$. Perhaps, in general, the actual evaluation of our formulas is impractical. However, it seems that many of the known bounds are actually various approximations of our formulas. Let

$$A(z)=\frac{1}{a_0+\ldots+a_nz^n}\qquad\text{and}\qquad B(w)=\frac{w^n}{a_0w^n+\ldots+a_n}.$$

Based on the notations introduced above, we prove:

**Theorem.** *The minimum $r$ and the maximum $R$ of the moduli of zeros of the polynomial $p(z)$ are given by:*

$$r=\frac{1}{\overline{\lim\limits_{n\to\infty}}\sqrt[n]{\left|\frac{A^{(n)}(0)}{n!}\right|}}\qquad\text{and}\qquad R=\overline{\lim_{n\to\infty}}\sqrt[n]{\left|\frac{B^{(n)}(0)}{n!}\right|}$$

*Proof.* Clearly, the radius of convergence of the Taylor series of $A(z)$ which is valid in the disk about 0 is equal to $r$. But then, the formula for $r$ mentioned in the Theorem is derived via the Cauchy-Hadamard formula for the radius of convergence of that Taylor series.

Again, clearly, the inner radius of convergence of the Laurent series of $A(z)$ which is valid in the infinite annulus about 0 is equal to $R$. For $k=0, 1, 2,\ldots$ if we replace in that Laurent series $z^{-k}$ by $w^k$, we obtain the Taylor series (about 0) of $B(w)$. But then the formula for $R$ mentioned in the Theorem is the inverse of the Cauchy-Hadamard formula for the radius of convergence of that Taylor series.

**Corollary.** *Let $p(z)=a_0+\ldots+a_nz^n$ with $a_0\neq 0$ be a polynomial over complex numbers and let $A(z)$ and $B(w)$ be defined as in the above. Then the precise annulus which contains all the zeros of $p(z)$ is given by:*

$$\frac{1}{\overline{\lim\limits_{n\to\infty}}\sqrt[n]{\left|\frac{A^{(n)}(0)}{n!}\right|}} \leqslant |z| \leqslant \overline{\lim\limits_{n\to\infty}}\sqrt[n]{\left|\frac{B^{(n)}(0)}{n!}\right|}.$$

*Remark.* Let us observe that our Theorem and its proof once more exhibit a persistent pattern in handling questions related to polynomials. After all, we were interested in the moduli of the zeros of a polynomial $p(z)$, and yet (contrary to expectation), we considered its inverse $1/p(z)$ and achieved our goal by means of analytic methods.

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# SUR UNE APPLICATION DE LA MESURE DE NON-COMPACITE

PAR

OLGA COSTINESCU

En 1930, C. K u r a t o w s k i a introduit la notion de mesure de non-compacité, [3], et il a utilisé cette notion dans l'étude des espaces complets. D'ailleurs, la notion mentionée est un outil très efficace dans la recherche mathématique. En ce qui suit, nous allons utiliser cette „mesure" pour caractériser une importante classe d'opérateurs.

Soient $X$ un espace de Banach réel, $B(X)$ l'algèbre des opérateurs $T: X\to X$ linéaires et bornés, $K(X)\subset B(X)$ l'idéal des opérateurs compacts, $S_X$ la sphère unité dans l'espace $X$, $M(X)$ la classe des ensembles bornés dans l'espace $X$ et $\mathcal{X}$ la famille des ensembles compacts dans l'espace $X$.

**Définition 1.** *Soit $E\in M(X)$ ; on appelle mesure de non-compacité de $E$, l'élément $q(E)$ donné par* $\inf\{r\in R_+^*, E\subset \mathfrak{U}\,(S(x_i, r), I$ *est une famille finie d'indices*$\}$ ; *$S(x_i, r)$ est la sphère de centre $x_i$ et rayon $r$.*

On démontre, sans difficulté, la

**Proposition 1.** *L'opérateur $q: M(X)\to R_+$ satisfait aux conditions suivantes :*

*a) $q(E)=0$ si et seulement si $\bar{E}\in\mathcal{X}$.*
*b) $q$ est une application isothone,*
*c) quel que soient $E_1, E_2\in M(X)$, on a $q(E_1+E_2)\leqslant q(E_1)+q(E_2)$*
*d) quel que soient $c\in R$ et $E\in M(X)$, on a $q(cE)=|c|q(E)$.*

*Remarque 1.* Du fait que dim $X=\infty$ si et seulement si $\bar{S}_X\notin\mathcal{X}$, et vu qu'on a la propriété (*a*), proposition 1, il résulte que dim $X=\infty$ si et seulement si $q(S_X)>0$ ; on démontre de même, que dans le cas où dim $X=\infty$, on a $q(S_X)=1$.

La notion de mesure de non-compacité introduite pour les ensembles bornés nous donne la possibilité d'introduire une notion analogue pour les opérateurs $T\in B(X)$ ;

**Définition 2.** *Soit $T\in B(X)$; on appelle mesure de non-compacité de l'opérateur $T$, la mesure de non-compacité de l'ensemble $T(S_X)$; on note la mesure de non-compacité de $T$ par le symbole $\|T\|_q$.*

On prouve aisément la

**Proposition 2.** *L'application $\|\cdot\|_q: B(X)\to R_+$ satisfait aux conditions :*