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SIMPLICIAL METHOD AND BAER-INVARIANTS

BY

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Introduction. The Baer-invariants of order 0 and 1, as derived functors have been introduced by Frohlich [1]. Modi [7] introduces those of higher order by using the simplicial method of derivation of functors. Furtado-Coelho [2] considered them in Ω-groups and gave a five term exact sequence. In this paper, following Modi, we extend to Ω-groups the higher order relative Baer-invariants and we obtain a long exact sequence of those Baer-invariants. We give an interpretation of the first 6 terms of this sequence and, in this way, we obtain a generalization of the one reached by Furtado-Coelho.

Preliminaries. A category of Rinehart is a based category $(\mathcal{C}, \varepsilon)$, [8, p. 296], in which the following statements hold:

i) *If P_1, P_2 are ε-projective, then $P_1 \amalg P_2$ (coproduct) is ε-projective;*

ii) *Every morphism of ε has a kernel;*

iii) *In a diagram with exact rows and $\beta_i \in \varepsilon$ $(i = 1,2)$ $\beta \in \beta$ if and*

$$\begin{tikzcd} K_1 \arrow[r] \arrow[d, "\alpha"] & B_1 \arrow[r, "\beta_1"] \arrow[d, "\beta"] & A \arrow[d, equal] \\ K_2 \arrow[r] & B_2 \arrow[r, "\beta_2"] & A \end{tikzcd}$$

only if $\alpha \in \varepsilon$.

Let $(\mathcal{C}, \varepsilon)$ be a category of Rinehart. We construct $(\mathcal{C}_1, \varepsilon_1)$ wilh $|\mathcal{C}_1| = \varepsilon$. A morphism of $\mathcal{C}_1$ is a pair (h_0, h_1) which makes the square commutative.

$$\varepsilon_1 = \{h = (h_0, h_1) \in \mathcal{C}_1(\alpha, \beta) \mid h_1 \in \varepsilon,\ \mathrm{Ker}\ h_0 \to \mathrm{Ker}\ h_1 \in \varepsilon\}.$$

$(\mathcal{C}_1, \varepsilon_1)$ is a category of Rinehart [7, p. 93].

Let $F : (\mathcal{C}, \varepsilon) \to \mathcal{D}$ be a functor, where $(\mathcal{C}, \varepsilon)$ is a category of Rinehart and let $\mathcal{D}$ be a category with kernels. We define: $F_1 : (\mathcal{C}_1, \varepsilon_1) \to \mathcal{D}$ by:

$$F_1(\alpha) = \mathrm{Ker}\, F(\alpha)$$

$$F_1(h_0, h_1) = F(h_0) \mid F_1(\alpha).$$

Let $\mathcal{V}$ be a variety, of a given type, of Ω-groups and ε the class of surjections of $\mathcal{V}$. Then $(\mathcal{V}, \varepsilon)$ is a category of Rinehart.

Proposition 1.1. *Let $0 \to F' \to F \to F'' \to 0$ be a exact sequence of functors $(\mathcal{C}, \varepsilon) \to \mathcal{V}$ and F' carring ε in ε. Then*

$$0 \to F_1' \to F_1 \to F_1'' \to 0$$

is an exact sequence of functors $(\mathcal{C}_1, \varepsilon_1) \to \mathcal{V}$.

Proposition 1.2. *If $(\mathcal{C}, \varepsilon)$ is a category of Rinehart, then, every object of $\mathcal{C}$ has an augmented simplicial ε-resolution and two augmented simplicial ε-resolution have the same homotopy type.*

Proof. A complete construction of an André ε-resolution and the simplicial theorem of comparation are given in [7, p. 64 and 72]. Here we explicitate the construction of the two first terms. Let $F_1 \xrightarrow{\omega} F_0 \xrightarrow{d_0} G \to 0$ be exact, with F_0, F_1 ε-projective. We construct :

$$\begin{tikzcd} \ldots X_1 = F_1 \sqcup F_2 \arrow[r, shift left, "d_0 = \binom{\omega}{1}"] \arrow[r, shift right, "d_1 = \binom{0}{1}"'] & X_0 = F_0 \arrow[l, bend right=40, "s"'] \arrow[r, "d_0"] & G \end{tikzcd}$$

Proposition 1.3. *Let $\mathcal{V}$ be a variety of Ω-groups. Every simplicial $\mathcal{V}$-object is a pointed Kan complex and every surjective simplicial $\mathcal{V}$-morfism is a fibration* [3, p. 11, 21]. We have a functoral and commutative diagram

$$\begin{tikzcd} \mathcal{V}^{SF} \arrow[rr] \arrow[d, "\pi_n"'] & & KAN^* \arrow[dl, "\pi_n\ (n\geq 1)"'] \arrow[dr, "\pi_0"] & \\ \mathcal{V} \arrow[r] & Gr \arrow[rr] & & Set^* \end{tikzcd}$$

where $\mathcal{V}^{SF}$ is the category of simplicial $\mathcal{V}$-objects and simplicial $\mathcal{V}$-morfism that are fibrations and KAN* is the KAN complex category with base point and fibrations.

For a simplicial $\mathcal{V}$-object

$$\begin{tikzcd} V:\ \ldots V_n \arrow[r, shift left, "d_0"] \arrow[r, shift right, "d_n"'] & V_{n-1} \arrow[l, bend right=40] \arrow[l, bend right=60] \ldots\ldots & \arrow[r, shift left, "d_0"] \arrow[r, shift right, "d_1"'] & V_0 \arrow[l, bend right=40] \end{tikzcd}$$

we define the Moore complex

(MV) by $(MV)_n = \bigcap\limits_{i=1}^{n} \operatorname{Ker} d_i \,(n \geq 1)$, $(MV)_0 ; V_0$, $\delta_n ; = d_0|_{(MV)_n}$

$(M'V)$ by $(M'V)_n = \bigcap\limits_{i=0}^{n-1} \operatorname{Ker} d_i \,(n \geq 1)$, $(M'V)_0 = V_0$, $\delta_n' = d_n|_{(M'V)_n}$

Proposition 1.4. *With the above notations MV and $M'V$ are chain complexes in $\mathcal{V}$(Im δ_n is an ideal in Ker δ_{n-1}, resp. δ_n') and, further,*

$$\Pi_n(V) = H_n(M'V) = H_n(MV), \quad (n \geq 0)$$

[9, p. 37–39].

Let $(\mathcal{C}, \varepsilon)$ be a category of Rinehart, $\mathcal{V}$ a variety of Ω-groups and $F : \mathcal{C} \to \mathcal{V}$ a functor, such that $F(0) = 0$. If A and B are objects of $\mathcal{C}$ and $X \to A$, $Y \to B$ are augmented simplicial ε-resolutions, then, FX and FY are simplicial $\mathcal{V}$-objects. We define :

$$L_n F(A) = \Pi_n(FX), \ (n \geq 0).$$

If $f : A \to B$ is a morphism in $\mathcal{C}$. $L_n F(f) = \Pi_n(F\bar{f})$, $(n \geq 0)$, where $\bar{f}$ is a lifting of f.

Proposition 1.5. i) *If $F : \mathcal{C} \to \mathcal{V}$ is a functor such that $F(0) := 0$ and $G \xrightarrow{\alpha} G'' \in \varepsilon$, then there exists an exact sequence*

$$\ldots . L_n F_1(\alpha) \to L_n F(G) \to L_n F(G'') \to L_{n-1} F_1(\alpha) \ldots . L_0 F(G'') \to 0$$

[7, p. 97].

ii) *If $0 \to F' \to F \to F'' \to 0$ is an exact sequence of functors $\mathcal{C} \to \mathcal{V}$ such that $F'(0) = F(0) ; = F''(0) ; = 0$ and G is an object of $\mathcal{C}$, then there exists an exact sequence*

$$\ldots . L_n F'(G) \to L_n F(G) \to L_n F''(G) \to L_{n-1} F'(G) \ldots . L_0 F''(G) \to 0.$$

Proof. It is an easy application of [6, p. 8].

Baer-invariants. Let $\mathcal{C}$ be a variety of Ω-groups and $\mathcal{V}$ a subvariety of $\mathcal{C}$. Then, we have an exact sequence of functors $\mathcal{C} \to \mathcal{C}$:

$$0 \to V \to 1_{\mathcal{C}} \to \mathcal{V} \to 0$$

where VG is the verbal subobject and $\mathcal{V}G = G/VG$.

If G' is an ideal of an object G of $\mathcal{C}$, we define $V(G', G)$ as the smallest ideal of G which contains the set

$$V(G', G) = \{\mathcal{V}(g_1 + g_1', \ldots, g_k + g_k') - \mathcal{V}(g_1, \ldots, g_k) g_i \in G,\ g_i' \in G',$$
$$i = 1, \ldots, k,\ \mathcal{V} \text{ law of } \mathcal{V}\}.$$

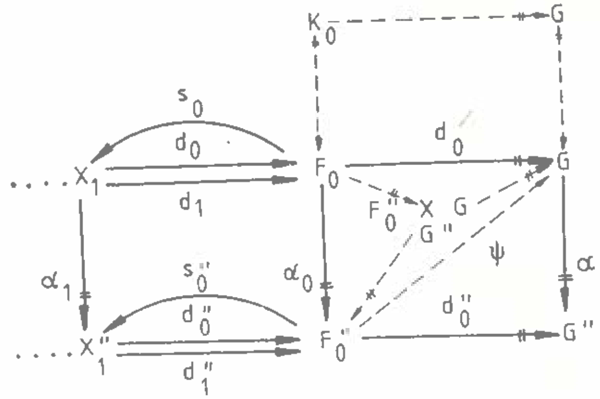
[1, p. 225–226].

Proposition 2.1. *Let $\mathcal{V}, \mathcal{V}'$ be two varieties of $\mathcal{C}$ and let the functor $\mathcal{V}' \xrightarrow{(\mathcal{V}', \mathcal{V})} \mathcal{V}'$ be given by $(\mathcal{V}', \mathcal{V})(G) = G/VG$. For a short exact sequence $G' \to G \to G''$ in $\mathcal{V}'$ we have a long exact sequence ....$L_n(\mathcal{V}', \mathcal{V})(G) \to L_n(\mathcal{V}', \mathcal{V})(G'') \to L_{n-1}(\mathcal{V}', \mathcal{V})_1(\alpha) \ldots . L_0(\mathcal{V}', \mathcal{V})(G'') \to 0$. Further $L_0(\mathcal{V}', \mathcal{V})_1(\alpha) = G'/\mathcal{V}(G', G)$ and $L_1(\mathcal{V}', \mathcal{V})_1(\alpha)$ is a conormal quotient of $R_0 \cap K_0 \cap VF_0$, where $K_0 =$ Ker $(F_0 \to F_0'')$ and $F_0 \to F_0''$ is a presentation $\mathcal{V}'$-free of α.*

Proof. The former afirmation is a consequence of Prop. 1.5.i). By Prop. 1.1. and Prop.1.5.ii), since $\alpha \in |\mathcal{V}_1'|$, we have an exact sequence :

$$\ldots . L_1(1_{\mathcal{V}'})_1(\alpha) \to L_1(\mathcal{V}', \mathcal{V})(\alpha) \to L_0(V_{|\mathcal{V}'})_1(\alpha) \to L_0(1_{\mathcal{V}'})_1(\alpha) \to L_0(\mathcal{V}', \mathcal{V})_1(\alpha) \to 0.$$

Let us consider the André ε_1-resolution of α in $F_1^{\prime}$ (Prop. 1.2).



Since $F_0 \to F_0 \times_{G_0''} G$ is surjective, $K_0 \to G'$ is surjective too. The existence of ψ such that $\alpha\psi = d_0''$ is a consequence of the character $\mathcal{V}'$-free of F_0''.

By Prop. 1.4.

$$L_0(V_{|\mathcal{V}'})_1(\alpha) = \frac{(V_{|\mathcal{V}'})_1(\alpha_0)}{(V_{|\mathcal{V}'})_1(d_0, d_0'')(\mathrm{Ker}\,(V_{|\mathcal{V}'})_1(d_1, d_1''))} =$$

$$= \frac{VF_0 \cap K_0}{d_0(\mathrm{Ker}\, d_1 \cap \mathrm{Ker}\, \alpha_1 \cap VX_1)} .$$

Let $v(x_1, \ldots, x_n)$ be in $VF_0 \cap K_0$. Then : $\mathcal{V}(\psi\alpha_0 x_1, \ldots, \psi\alpha_0 x_n) = \psi\alpha_0(\mathcal{V}(x_1, \ldots, x_n)) = 0$. Thus

$$d_0(v(x_1, \ldots, x_n)) = v(d_0 x_1, \ldots, d_0 x_n) - v(\psi\alpha_0 x_1, \ldots, \psi\alpha_0 x_n) \in V(G', G), \text{ since } -$$
$$-\psi\alpha_0 x_i + d_0 x_i \in G' \ (i = 1, \ldots, n).$$

Since $K_0 \to G'$ is surjective, is also so $L_0(V_{|\mathcal{V}'})_1(\alpha) \to V(G', G)$ and, since $L_0(1_{|\mathcal{V}'})_1(\alpha) = G'$, one has

$$L_0(\mathcal{V}', \mathcal{V})_1(\alpha) = \frac{G'}{V(G', G)} .$$

On the other hand $L_1(1_{\mathcal{V}'})_1(\alpha) = 0$. Therefore

$$L_1(\mathcal{V}', \mathcal{V})_1(\alpha) = \mathrm{Ker}\left(\frac{VF_0 \cap K_0}{d_0(\mathrm{Ker}\, d_1 \cap \mathrm{Ker}\, \alpha_1 \cap VX_1)} \to G'\right) =$$

$$= \frac{R_0 \cap K_0 \cap VF_0}{d_0(\mathrm{Ker}\, d_1 \cap \mathrm{Ker}\, \alpha_1 \cap VX_1)} .$$

Proposition 2.2. *Let $\mathcal{V}, \mathcal{V}'$ be two varieties of $\mathcal{C}$, $G \in |\mathcal{V}'|$ and $R_0 \to F_0 \to G$ a $\mathcal{V}'$-free presentation of G. Then*

i) $L_0(\mathcal{V}', \mathcal{V})(G) = G/VG$.

ii) $L_1(\mathcal{V}', \mathcal{V})(G) = (R_0 \cap VF_0)/V(R_0, F_0)$.

iii) $L_0(V_{|\mathcal{V}'})(G) = VF_0/V(R_0, F_0)$.

Proof. i) Is a consequence of Prop. 1.1, using the André resolution described in Prop. 1. 2

ii) follows from i) and Prop. 2.1, since $L_1(\mathcal{V}', \mathcal{V})(F_0) = 0$, for F_0 is $\mathcal{V}'$-free.

iii) Let us consider the André resolution of Prop. 1.2. Then

$$L_0(V_{|\mathcal{V}'})(G) = \frac{VF_0}{V(d_0)\ (\mathrm{Ker}\, Vd_1)} = \frac{VF_0}{d_0(\mathrm{Ker}\, d_1 \cap V(F_0 \amalg F_1))} =$$

$$= \frac{VF_0}{d_0(\overline{F}_1 \cap V(F_0 \amalg F_1))} ,$$

where $\overline{F}_1$ is the smallest ideal of $F_0 \amalg F_1$, containing F_1.

Let $\beta = \binom{1}{0} : F_0 \amalg F_1 \to F_0 \amalg F_1$ and $v(x_1, \ldots, x_n) \in F_1 \cap V(\overline{F}_0 \amalg F_1)$. Then $v(x_1, \ldots, x_n) = v(x_1, \ldots, x_n) - v(\beta x_1, \ldots, \beta x_n) \in V(\overline{F}_1, F_0 \amalg F_1)$, since $-\beta x_i + x_i \in \overline{F}_i (i = 1, \ldots, n)$. Then $V(\overline{F}_1, F_0 \amalg F_1) = \overline{F}_1 \cap V(F_0 \amalg F_1)$. Thus

$$L_0(V_{|\mathcal{V}'})(G) = \frac{VF_0}{d_0(V(\overline{F}_1, F_0 \amalg F_1))} = \frac{VF_0}{V(d_0 \overline{F}_1, d_0(F_0 \amalg F_1))} = \frac{VF_0}{V(R_0, F_0)} .$$

Let $\mathcal{V}, \mathcal{V}'$ be two varieties of $\mathcal{C}$. We define the relative Baer-invariants of higher order :

$$\mathcal{C}_n(\mathcal{V}', \mathcal{V}) : \mathcal{C} \to \mathcal{C},$$

$$\mathcal{C}_n(\mathcal{V}', \mathcal{V}) : \mathcal{C} \to \mathcal{C}, \ (n \geq 0),$$

$$\mathcal{C}_n(\mathcal{V}', \mathcal{V})(G) = L_n(\mathcal{V}', \mathcal{V})(G/VG),$$

$$\mathcal{C}_n(V', V)(G) = L_n(V|_{\mathcal{V}'})(G/V'G),$$

for $G \in \mathcal{C}$.

By using the above calculations :

$$\mathcal{C}_0(\mathcal{V}', \mathcal{V})(G) = \frac{G}{VG + V'G}, \quad \mathcal{C}_0(V', V)(G) = \frac{V'F_0 + VF_0}{V'F_0 + V(R_0, F_0)},$$

$$\mathcal{C}_1(\mathcal{V}', \mathcal{V})(G) = \frac{(R_0 + V'F_0) \cap (VF_0 + V'F_0)}{V'F_0 + V(R_0, F_0)} ,$$

where $R_0 \to F_0 \to G$ is a $\mathcal{C}$-free presentation of G.

As a consequence of the long exact sequence given by

$$0 \to V_{|\mathcal{V}'} \to 1_{\mathcal{V}'} \to (\mathcal{V}', \mathcal{V}) \to 0,$$

we obtain :

i) $\mathcal{C}_n(\mathcal{V}', \mathcal{V})(G) \cong \mathcal{C}_{n-1}(V', V)(G)$, $n \geq 2$ and

ii) $0 \to \mathcal{C}_1(\mathcal{V}', \mathcal{V})(G) \to \mathcal{C}_0(V', V)(G) \to VG/(VG \cap V'G) \to 0$

is exact.

Special instances of the above Baer-invariants are those considered in [4, p. 108] and [7, M o d i, p. 102] by taking as $\mathcal{C}$ the category of ordinary groups, and [2, p. 599] in Ω-groups.

Proposition 2.3. *If $\mathcal{V} \subset \mathcal{V}'$ are varieties of $\mathcal{C}$, then,*

i) $$\mathcal{C}_1(\mathcal{V}', \mathcal{V})(G) = \text{Coker}\,(L_1\mathcal{V}'(G) \to L_1\mathcal{V}(G)),$$

ii) $$\mathcal{C}_0(V', V)(G) = \text{Coker}\,(L_0V'(G) \to L_0V(G)).$$

Proof. Let $R_0 \to F_0 \to G$ be a $\mathcal{C}$-free presentation

$$\text{i) Coker}\,(L_1\mathcal{V}'(G) \to L_1\mathcal{V}(G)) = \frac{(R_0 \cap VF_0)/V(R_0.F_0)}{(R_0 \cap V'F_0)/(V'F_0 \cap V(R_0F_0))} =$$

$$= \frac{(R_0 \cap VF_0)V(R_0, F_0)}{((R_0 \cap V'F_0) + V(R_0,F_0))V(R_0,F_0)} = \frac{R_0 \cap VF_0}{R_0 \cap (V'F_0 + V(R_0, F_0))} =$$

$$= \frac{(R_0 \cap VF_0) + (V'F_0 + V(R_0'F_0))}{V'F_0 + V(R_0, F_0)} = \frac{VF_0 \cap (R_0 + V'F_0 + V(R_0,F_0))}{V'F_0 + V(R_0, F_0)} =$$

$$= \frac{(R_0 + V'F_0) \cap (VF_0 + V'F_0)}{V'F_0 + V(R_0, F_0)} = \mathcal{C}_1(\mathcal{V}', \mathcal{V})(G):$$

$$\text{ii) Coker}\,(L_0V'(G) \to L_0V(G)) = \frac{VF_0/V(R_0,F_0)}{V'F_0/(V'F_0 \cap V(R_0, F_0))} =$$

$$= \frac{VF_0}{V'F_0 + V(R_0, F_0)} = \frac{V'F_0 + VF_0}{V'F_0 + V(R_0, F_0)} = \mathcal{C}_0(V', V)(G).$$

The second isomorphism theorem and the modular law has been used.

This results allows us to include, as particular instances, the relative Baer-invariants introduced by F r o h l i c h in [1, p. 230], taking as $\mathcal{C}$ the category of associative algebras over a field.

Proposition 2.4. *Let $\mathcal{V}, \mathcal{V}'$ be two subvarieties of $\mathcal{C}$ and $G' \to G \to G''$ a short exact sequence in $\mathcal{C}$. Then, there exists exact sequences*

i) $$\ldots \mathcal{C}_n(\mathcal{V}', \mathcal{V})(G) \to \mathcal{C}_n(\mathcal{V}', \mathcal{V})(G'') \to L_{n-1}(\mathcal{V}', \mathcal{V})_1(\mathcal{V}'\alpha) \ldots\ldots \to$$
$$\to \mathcal{C}_0(\mathcal{V}', \mathcal{V})(G'') \to 0,$$

ii) $$\frac{(K_0 + V'F_0) \cap (R_0 + V'F_0) \cap (VF_0 + V'F_0)}{V'F_0} \to$$

$$\to \frac{(R_0 + V'F_0) \cap (VF_0 + V'F_0)}{V'F_0 + V(R_0, F_0)} \to \frac{(R_0'' + V'F_0'') \cap (VF_0'' + V'F_0'')}{V'F_0'' + V(R_0'', F_0'')} \to$$

$$\to \frac{G' + V'G}{V'G + V(G', G)} \to \frac{G}{VG + V'G} \to \frac{G''}{VG'' + V'G''} \to 0,$$

where $K_0 = \text{Ker}\,(F_0 \to F_0'')$, $F_0 \to F_0''$ *is* $\mathcal{C}_1$*-free presentation of* α, $R_0 = \text{Ker}\,(F_0 \to G)$ *and* $R_0'' = \text{Ker}\,(F_0'' \to G'')$.

Proof. i) By applying Prop. 2.1, to $\mathcal{V}'\alpha : G/V'G \to G''/V'G''$.

ii) The last five terms are a consequence of i) and of the others 0 and 1 Baer-invariants interpretations, since

$$L_0(\mathcal{V}', \mathcal{V})_1(\mathcal{V}'\alpha) = \frac{G'/(G' \cap V'G)}{V(G'/(G' \cap V'G), G/V'G)} = \frac{G'/(G' \cap V'G)}{V(G', G)/(V'G \cap V(G', G))} =$$
$$= \frac{G' + V'G}{V'G + V(G', G)}.$$

Having in account that

$$\begin{tikzcd}
 & K_0 \arrow[r] \arrow[d] & G' \arrow[d] \\
R_0 \arrow[r] \arrow[d] & F_0 \arrow[r] \arrow[d, "\alpha_0"] & G \\
R_0'' \arrow[r] & F_0'' \arrow[r] & G'' \arrow[u, "\alpha"']
\end{tikzcd}$$

is a $\mathcal{C}_1$-free presentation of α, it follows that $R_0 \to R_0''$ is surjective. By applying the functor $\mathcal{V}$ to the presentation, we have the following diagram

$$\begin{tikzcd}
 & K_0/(K_0 \cap V'F_0) \arrow[r] \arrow[d] & G'/(G' \cap V'G) \arrow[d] \\
R_0/(R_0 \cap V'F_0) \arrow[r] \arrow[d] & F_0/V'F_0 \arrow[r] \arrow[d, "\mathcal{V}'\alpha_0"] & G/V'G \arrow[d, "\mathcal{V}'\alpha_0"] \\
R_0''/(R_0'' \cap V'F_0'') \arrow[r] & F_0''/V'F_0'' \arrow[r] & G''/V'G''
\end{tikzcd}$$

Therefore $\mathcal{V}'\alpha_0 \to \mathcal{V}'\alpha$ is a presentation $\mathcal{V}_1'$-free.

By using the Prop. 2.1. $L_1(\mathcal{V}', \mathcal{V})_1(\mathcal{V}'\alpha)$, is a conormal quotient of

$$\frac{K_0}{K_0 \cap V'F_0} \cap \frac{R_0}{R_0 \cap V'F_0} \cap V\left(\frac{F_0}{V'F_0}\right) =$$
$$= \frac{(K_0 + V'F_0) \cap (R_0 + V'F_0) \cap (VF_0 + V'F_0)}{V'F_0}$$

Then the proposition follows.

The five term exact sequence given by Furtado-Coelho [2, p. 600] is a particular case of the one obtained here by taking $\mathcal{V}' = \mathcal{C}$.

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PERIODIC AND QUASI-PERIODIC SOLUTIONS OF ABSTRACT DIFFERENTIAL EQUATIONS

BY

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1. Introduction. The aim of this paper is to investigate the existence of quasi-periodic solutions to the following equations

$$u'(t) + Au(t) + cu(t) = g(t), \tag{1.1}$$

$$-u''(t) + Au(t) + cu(t) = g(t), \tag{1.2}$$

$$u''(t) + au'(t) + Au(t) + cu(t) = g(t), \tag{1.3}$$

where A is a linear (in general unbounded) operator in a Hilbert space H, a, c are real constants and g is a quasi-periodic function. Let us recall that an abstract function $t \in R \to f(t) \in H$ is called quasi-periodic with basic frequencies $\nu_1, \ldots, \nu_m$ if there exists a continuous function $(\theta_1, \ldots, \theta_m) \in R^m \to f^*(\theta_1, \ldots, \theta_m) \in H$ of period 2π in $\theta_1, \ldots, \theta_m$ such that $f^*(\nu_1 t, \ldots, \nu_m t) = f(t)$, $t \in R$.

A similar problem (together with the corresponding weakly non-linear case) for

$$A = -\sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2}$$

is examined by M. Yamaguchi who looks for a solution either in the space of analytic functions ([10], [11]) or in a subspace of continuously differentiable functions that are characterized by certain convergence properties of their Fourier expansions [13]. In all above papers the time-space-Fourier method (see [9]) is used. In [12] he makes use of the space-Fourier method and the spaces of „mixed" type to treat (1.3) where $a = 0$ and A is an elliptic operator of the form

$$A = -\sum_{j,k=1}^{n} \frac{\partial}{\partial x_j}\left(a_{jk}(x)\frac{\partial}{\partial x_k}\right) + a_0(x). \tag{1.4}$$

(In the present paper, we work in the Sobolev spaces of abstract functions and this enables us to weaken the requirements on the smoothness of the right-hand sides). The weakly non-linear problem corresponding to (1.3) with A given by (1.4), $a \neq 0$ and with a small perturbation of the form $\varepsilon f(t, x, u)$ is treated by V. B. Moseenkov [5], [7] also in the framework of Sobolev spaces. In [6] the same author deals with the case $a \to 0$.