Under the assumption (i), the system (23) has a unique solution.

As a majorant scalar equation we can consider the equation

$$z^s \frac{d^s w}{dz^s} = Aw + b_n z^n + P^*(z, w), \quad \text{where} \tag{24}$$

$$P^*(z, w) = \sum_{\substack{i, j = \overline{0, \infty} \\ i + kj > n}} \|p_{ij}\| z^i w^j .$$

6. Briot-Bouquet Differential Equations in Banach Algebras. Let $X = Y$ be a comutative Banach algebra with unit. In this case, we can consider other equations which generalize the equations from [3] and which have analytical solutions in the sense of the definition 2 from 1.

We shall give, for simplicity, a generalization of equation (*).

Theorem 6. *Consider the equation*

$$x^s \frac{d y^s (x, x_0)}{d x^s} = A y + x^n b_n + P(x, y), \quad \textit{where} \tag{25}$$

a) $x_0 \in X$, x_0 *with inverse* x_0^{-1},

b) s *and* n *are integers* $\geqslant 1$, $s \leqslant n$,

c) A *is a continuous linear operator, defined over* X *with values in* X,

d) $P(x, y)$ *is an analytical function in a neighbourhood of the origin of* X^2 *with values in* X, *of the form*

$$P(x, y) = \sum_{\substack{i, j = \overline{0, \infty} \\ i + nj > n}} x^i y^j a_n, \quad \textit{with } a_n \in X \textit{ being constants values.}$$

If (i) $\dfrac{(n-s)!}{n!} \|x_0^{-1}\|^s \|A\| < 1$, *then the equation* (25) *has a unique analytical solution of the form*

$$y(x) = x^n c_n + x^{n+1} c_{n+1} + \ldots \tag{26}$$

The proof is similar to that of the theorem 2.

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ON A STOCHASTIC PROCESS WITH DISCRET TIME AND IMPOSED LENGTH OF TIME

BY

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In this paper we propose a model suggested by many situations we meet frequently in practice. As for instance in the viticulture and fruitgrowing. An event (the appearance of a fertile offshoot) occurs from time to time (here and there on the cord) and the length of time (the length of the cord) is imposed. The probability of such an event is a function of time t and the total length of the time, T (the length of the cord).

We consider that an event may occurs at $t = 0, 1, \ldots, T$, and we denote by $P_T(t)$ the probability that the event occurs at the time t. Let $Q_T(t) = 1 - P_T(t)$ be the probability that the event does not occur at the time t. We consider a random variable X_t which associates the number 1 when the event occurs at the moment t, and the number 0 when the event does not occur at the instant t

$$X_t \begin{pmatrix} 1 & 0 \\ P_T(t) & Q_T(t) \end{pmatrix}, \quad t = 0, 1, \ldots, T. \tag{1}$$

The total number of the occurences of the event until the moment T is

$$\xi_T = \sum_{t=0}^{T} X_t . \tag{2}$$

We suppose that the following property holds

$$P(X_t = k / X_m = i_m,\ m = 0, 1, \ldots, t-1) = P(X_t = k), \quad k, i_m = 0, 1, \tag{3}$$

that is, the appearance of the event at the moment t does not depend on the occurences of the event at earlier instants.

An important problem is to determine T such that the expectation of ξ_T be maximum. This expected number of ξ_T is

$$N(T) = M(\xi_T) = \sum_{t=0}^{T} M(X_T) = \sum_{t=0}^{T} P_T(t). \tag{4}$$

The practice shows that the probabilities $P_T(t)$ are increasing functions on an interval $[0, t_0]$, and decreasing on $[t_0, T]$, where $t_0 \in [0, T]$.

We will consider two expressions for the probability $P_T(t)$.

Case 1. We suppose that

$$P_T(t) = -a_T t^2 + b_T t + c_T , \tag{5}$$

where $0 \leqslant P_T(t) \leqslant 1$ for each $t \in [0, T]$.

The following conditions can be obtained immediately

$$(6)\quad \begin{gathered} a_T > 0,\ T > \frac{b_T}{2a_T} > 0,\ P_T(0) = c_T \geqslant 0, \\ P_T(T) = -a_T T^2 + b_T T + c_T \geqslant 0,\ 0 \leqslant \frac{b_T^2 + 4a_T c_T}{4a_T} \leqslant 1. \end{gathered}$$

If we substitute (5) into the expression (4) we obtain

$$(7)\quad N(T) = -a_T \frac{T(T+1)(2T+1)}{6} + b_T \frac{T(T+1)}{2} + Tc_T.$$

If we consider that $a_T = a$, $b_T = b$, $c_T = c$, where a, b, c are constants, we get the following integer programming problem

$$(8)\quad \begin{gathered} N(T) = -a \frac{T(T+1)(2T+1)}{6} + b \frac{T(T+1)}{2} + Tc = \max \\ \text{subject to } T > \frac{b}{2a},\ T \text{ is a natural number,} \end{gathered}$$

where $a > 0$, $b > 0$, $c \geqslant 0$ and $b^2 \leqslant 4a(1-c)$.

The maximum point fulfils the restriction $T > b/2a$, if the following inequality holds

$$(9)\quad 2a^2 \leqslant 3b^2 + 12ac.$$

The natural condition $P_T(0) = c_T = 0$ implies the maximum conditions

$$(10)\quad a > 0,\ b > 0,\ 2a^2 < 3b^2 \leqslant 12a.$$

We obtain from (10) the inequalities $a < 6$ and $b \leqslant 2\sqrt{6}$.

The average number of occurences of the event in the unit time is

$$(11)\quad f(T) = \frac{N(T)}{T} = -\frac{a}{6}(T+1)(2T+1) + \frac{b}{2}(T+1) + c.$$

The maximum point of $f(T)$ is

$$(12)\quad T_{\max} = \frac{3(b-a)}{a}.$$

The restriction $T_{\max} > b/2a$ implies $5b > 6a$ and therefore we point out the problem

$$(13)\quad \begin{gathered} f(T) = -\frac{a}{6}(T+1)(2T+1) + \frac{b}{2}(T+1) + c = \max \\ \text{subject to } T > \frac{b}{2a},\ T \text{ natural,} \end{gathered}$$

where $a > 0$, $b > 0$, $6a < 5b$, $b^2 \leqslant 4a(1-c)$.

If we have $c = 0$, then we find $3b^2 \leqslant 12a \leqslant 10b$, hence $a < 25/9$, $b < 10/3$.

Case 2. Let us suppose the following form of the probability $P_T(t)$

$$(14)\quad P_T(t) = c_t \frac{t^{T-1}}{\Gamma(T)} e^{-t},\ t = 0,\ 1,\ 2,\ \ldots,\ T.$$

In this formula we consider that the nonnegative coefficients c_t depend only on the time length T.

Now, the average number of succeses until T is

$$(15)\quad N(T) = \frac{1}{\Gamma(T)} [c_0 + c_1 e^{-1} + c_2 2^{T-1} e^{-2} + \ldots + c_T T^{T-1} e^{-T}].$$

If the coefficient c_1 is not zero, then we have $N(1) < N(2)$. The sequence of values of the function $N(T)$ may be written as follows

$$\begin{aligned} N(1) &= P_1(0) + \underline{P_1(1)} \\ N(2) &= P_2(0) + P_2(1) + \underline{P_2(2)} \\ N(3) &= P_3(0) + P_3(1) + P_3(2) + \underline{P_3(3)} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ N(T) &= P_T(0) + P_T(1) + P_T(2) + P_T(3) + \ldots + \underline{P_T(T)} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

We show in the following theorem that the series $\sum_{T=1}^{\infty} P_T(T)$ is important for the boundedness of the function $N(T)$.

Theorem. *If the series* $\sum_{n=1}^{\infty} P_n(n)$ *is convergent, then the function* $N(T)$ *is bounded.*

Proof. The following relations hold for $T > 2$:

$$\begin{aligned} N(T+1) &= \frac{1}{\Gamma(T+1)} [c_0 + c_1 e^{-1} + c_{T+1}(T+1)^T e^{-(T+1)}] + \\ &+ \frac{1}{\Gamma(T)} \left[\frac{2}{T} c_2 2^{T-1} e^{-2} + \ldots + \frac{T}{T} c_T T^{T-1} e^{-T}\right] \leqslant \frac{1}{\Gamma(T+1)} [c_0 + c_1 e^{-1} + \\ &+ c_{T+1}(T+1)^T e^{-(T+1)}] + \frac{1}{\Gamma(T)} [N(T)\Gamma(T) - c_0 - c_1 e^{-1}] \leqslant N(T) + \\ &+ \frac{1}{\Gamma(T+1)} [c_{T+1}(T+1)^T e^{-(T+1)} - (T-1)(c_0 + c_1 e^{-1})], \text{ so that we have} \end{aligned}$$

$$(16)\quad N(T+1) \leqslant N(T) + \frac{1}{\Gamma(T+1)} [c_{T+1}(T+1)^T e^{-(T+1)} - (T-1)(c_0 + c_1 e^{-1})],$$

or

$$(16')\quad N(T+1) \leqslant N(T) + \left[P_{T+1}(T+1) - \frac{T-1}{\Gamma(T+1)} (c_0 + c_1 e^{-1})\right].$$

By using the inequality (16'), we obtain

$$
(17)\qquad N(n) \leqslant P_2(2) + P_3(3) + \ldots + P_n(n) + \left[1 - \left(\frac{1}{2\,!} + \frac{2}{3\,!} + \ldots + \frac{n-2}{(n-1)\,!}\right)\right](c_0 + c_1 e^{-1}).
$$

The inequality (17) and the equality $\sum_{n=2}^{\infty} \frac{n-2}{(n-1)\,!} = 1$ impliy the theorem.

The expression (14), the inequality (16) and the above theorem lead to the following remarks:

a) If there exists a number $T_0 > 3$ such that

$$
(18)\qquad c_T \leqslant \frac{T-2}{T^{T-1}}(c_0 + c_1 e^{-1}) e^T \text{ for } T \geqslant T_0,
$$

then, the function $N(T)$ has a maximum point $T \in [2, T_0]$.

b) If there exists $\lim_n c_{n+1}/c_n$ and $\lim_n c_{n+1}/c_n < 1$, then the function $N(T)$ is bounded.

c) If there exists a number $q \in (0, 1)$ and a natural number n_0 such that $c_{n+1} < q c_n$ for $n \geqslant n_0$, then the function $N(T)$ is bounded.

d) If $c_0 \neq 0$, $c_1 \neq 0$, the sequence $\{c_n\}$ decreases and there exists a number $n_0 \geqslant 3$ such that

$$
c_{n_0} \leqslant \frac{n_0 - 2}{n_0^{n_0 - 1}} e^{n_0}(c_0 + c_1 e^{-1}),
$$

then, the function $N(T)$ is maximum for $\bar{n} \leqslant n_0$.

e) If $P_n(n) = 0$ for $n \geqslant n_0$, then the function $N(T)$ is bounded for any time.

f) The average number of the occurences of the event in the unit time satisfies the inequality

$$
(19)\qquad \frac{N(T)}{T} \leqslant \frac{P_2(2) + \ldots + P_T(T)}{T} + \frac{1}{T}\left[1 - \left(\frac{1}{2\,!} + \frac{2}{3\,!} + \ldots + \frac{T-2}{(T-1)\,!}\right)\right](c_0 + c_1 e^{-1}).
$$

Therefore, if there exists $\lim_n N(n)/n$ and the sequence $P_n(n)$ is convergent, then we have

$$
(20)\qquad \lim_n \frac{N(n)}{n} \leqslant \lim_n P_n(n) \leqslant 1.
$$

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THERMAL INFLUENCES ON THE SHEAR ACCELERATION WAVES IN INCOMPRESSIBLE SIMPLE FLUIDS

BY

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1. Introduction. An exact theory of the growth and decay of surfaces of discontinuity in non-linear, incompressible, simple fluids with memory undergoing steady plane-rectilinear shearing flows, is developed in [1], but no account has been taken about the dependence of the constitutive functional upon the temperature or entropy. Thus, most of those results are to be thought appropriate to circumstances when either θ or η is kept constant, or to fluids for which the response functionals are insensitive to changes in these variables.

In this paper we shall consider nonsteady shearing flows and we shall allow our constitutive functionals to be affected not only by the history of the strain, but also by the histories of the thermodynamic variables θ^t and $\mathbf{g}^t$. The main results obtained here are:

1) a homothermal shear wave propagating in a region undergoing a nonsteady shearing flow, is damped out or it blows up[1] (i.e. its amplitude can become infinite in a finite time).

2) a shear wave in a linear simple fluid undergoing a nonsteady shearing flow, cannot grow, but it is always damped out.

2. Constitutive equations and smooth assumptions. The fluids to be considered here, will be characterized by constitutive equations of the form [2—4].

$$
(2.1)\qquad \begin{aligned} (\mathbf{S}, \psi, \eta) &= (\underset{s=0}{\overset{\infty}{\mathbf{S}, \psi,}} \eta)(\mathbf{G}(s), \theta^t(s), \mathbf{g}_r^t(s)), \\ \mathbf{q} &= \underset{s=0}{\overset{\infty}{\mathbf{q}}}(\mathbf{G}(s), \theta^t(s), \mathbf{g}^t(s)), \quad \mathbf{G}(\cdot) = \mathbf{F}^t(\cdot)^T \mathbf{F}^t(\cdot) - \mathbf{1}, \end{aligned}
$$

in which $\mathbf{S}$, ψ, η and $\mathbf{q}$ are respectively, the extra-stress tensor, the free energy, the entropy and the heat flux, while $\mathbf{F}^t(\cdot)$, $\theta^t(\cdot)$ and $\mathbf{g}^t(\cdot)$ are the histories of the relative deformation gradient, of the temperature and of its spatial gradient. By $\mathbf{g}_r^t(\cdot)$ we have denoted the past history of $\mathbf{g}^t(\cdot)$.

For a rectilinear shearing flow, whose velocity field in a Cartesian co-ordinate system x, y, z is given by

$$
(2.2)\qquad \dot{x} = 0, \quad \dot{y} = v(x, t), \quad \dot{z} = 0,
$$

[1] Cf. Coleman and Gurtin [1], for mechanical waves.