TABLE 1

The constituent terms terms of the formulation ; A in 10^{-3} cal/day/cm^2 ; B in cal/day/cm^2/^{0} C^{-1}

X	C	A	B	Φ
0.0872	6 140	539	204	6.8
0.1737	6 031	526	989	6.1
0.2588	5 915	508	2 460	5.6
0.3420	5 734	485	4 438	5.4
0.4226	5 492	452	6 656	5.2
0.5000	5 287	417	8 911	5.2
0.5736	5 028	380	10 951	5.1
0.6428	4 825	341	12 622	5.1
0.7071	4 363	295	13 423	5.0
0.7660	4 055	248	13 431	5.0
0.8192	3 718	207	12 966	0.1
0.8660	3 270	164	11 577	5.1
0.9063	2 828	123	9 622	5.0
0.9397	2 375	87	7 381	5.0
0.9659	1 996	57	5 187	5.2
	1 744	35	3 358	5.8

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SEMANTICS IN COMPLETE LATTICES FOR RELATIONAL DATABASE FUNCTIONAL DEPENDENCIES

BY

VALERIU BREAZU

§ 1. Functional dependencies were introduced in relational database analisis and design by Codd [1]. The functional dependency calculus (FD calculus) as an abstract manipulation of functional dependencies was axiomatized by Armstrong [2] and Beeri et al. [3]. The surprising relationships existing between the FD calculus and the study of certain particular boolean functions asociated to the functional dependencies first appears in Delobel and Casey [4] and is further cleared in Fagin [5].

Like the propositional calculus the FD calculus raises two problems : a proof or deduction problem of syntactical nature and a truth or validity problem of semantic nature.

Let U be a finite set whose elements are called *symbolic attributes* (this is the syntactic appearance of the concept of attribute). By *set of attributes* we mean a subset of U. A functional dependency statement (FD statement) between two sets of attributes is simply a pair (X, Y) of subsets of U, that we write $X \leftarrow Y$ and read „X depends on Y". The deductions in FD calculus are based on the following axioms (the set $\{A\} = A$ is denoted simply by A) :

A1. $\forall A \in X \subseteq U,\ A \leftarrow X$;

A2. $\forall X, Y, Z \subseteq U$, *from* $X \leftarrow Y$ *and* $Y \leftarrow Z$ *deduce* $X \leftarrow Z$;

A3. $\forall X, Y \subseteq U$, *from* $A \leftarrow Y$ *for all* $A \in X$ *deduce* $X \leftarrow Y$.

Note that only $A1$ is an axiom scheme while A 2 and $A3$ are inference rules.

Given a set π of functional dependencies called premises, a ***deduction*** from π is a finite sequence of FD statements so that each of them is either a premise or an axiom from $A1$, or can be inferred using $A2$ or $A3$ from some of the FD statements preceding it in the sequence. We say that the last FD statement in the sequence may be deduced from premises π (or that π syntactically implies it). Denote by Ded (π) the set of FD's that can be deduced from π. Note that $A3$ works as an inferrence rule only because U is finite. In order to check for the validity of a FD we use two kinds of models for U : table-like models and 2-value models.

A table-like model φ consists of a set of *entries* (rows), E, and an interpretation that maps every symbolic attribute A into a *table attribute* (column) defined for every entry in E, that is, a map A_φ with range rg (A_φ) : (this is the semantic appearance of the concept of attribute). These tables are a small generalization of database relations because two rows can have identical values in all columns. The FD statement $X \leftarrow Y$ is *valid* in the table-like model φ if

$$\forall e, e' \in E \text{ if } \forall A \in Y \ \ A_\varphi(e) = A_\varphi(e') \text{ then } \forall A \in X \ \ A_\varphi(e) = A_\varphi(e').$$

Given a set π of FD statements we say that the FD statement d is a *table consequence* of π (or that π semantically implies d) if d is valid in every table-like model in which all the FD statem nts from π are valid. Denote by TCon (π) the set of table consequences of π.

A 2-*value model* consists of an interpretation that maps every symbolic attribute into one of the values 0 and 1, that is, a map $\beta : U \rightarrow \mathbf{B}$ (where $\mathbf{B} = \{0, 1\}$).

The FD statement $X \leftarrow Y$ is valid in the 2-values model β if the boolean expression : boolexp $(X) \Rightarrow$ boolexp (Y) takes the value 1 when the variables from U take the values assigned to them by β where

$$\text{boolexp}\,(Z) = \begin{cases} A_1 \vee \ \ldots \vee A_n & \text{if } Z = \{A_1, \ldots, A_n\}, \\ 0 & \text{if } Z = \varnothing. \end{cases}$$

Note that the finiteness of U is essential in the above definition. The concept of 2-*value consequence* is defined analogously to that of table consequence. Denote by 2 Con (π) the set of 2-value consequences of the FD statement set π.

The FD calculus enjoys the following property :

1.1. Double Completeness Theorem. For any set π of FD statement

$$\text{Ded}\,(\pi) = \text{TCon}\,(\pi) = 2\text{Con}\,(\pi).$$

The proof of this theorem will be given in § 4.

§2. **Lattice models for the functional dependency calculus.** Consider again the FD calculus related to a finite set U of symbolic attributes. Let φ be a table-like model with set of entries E. One can check that $X \leftarrow Y$ is valid in φ iff : $\bigcap_{A \in X} \ker A_\varphi \supseteq \bigcap_{A \in Y} \ker A_\varphi$ where $\ker A_\varphi$ is the equivalence relation on E determined by A_φ : $\ker A_\varphi = \{(e, e') \in E \times E \mid A_\varphi(e) = A_\varphi(e')\}$. The set of all equivalence relations on E forms a closure system on $\mathcal{P}(E \times E)$ and therefore a complete lattice. We denote by $(E_q(E), \supseteq)$ the *dual* of this complete lattice. If $\ker_\varphi : U \rightarrow E_q(E)$ such that $\ker \varphi(A) = \ker A_\varphi$ then $X \leftarrow Y$ is valid in φ iff $\overline{\ker}_\varphi(X) \supseteq \overline{\ker}_\varphi(Y)$, where $\overline{\ker}_\varphi : \mathcal{P}(U) \rightarrow E_q(E)$ is the unique extension of $\ker_\varphi$ to a join complete homomorphism (see B i r k h o f f [6]).

One can also check that for any 2-value model $\beta : U \rightarrow \mathbf{B}$ the FD $X \leftarrow Y$ is valid in β iff $\overline{\beta}(x) \leqslant \overline{\beta}(Y)$. where $\overline{\beta}: \mathcal{P}(U) \rightarrow \mathbf{B}$ is the unique extension of β to a join complete homomorphism. Note that this equivalent definition of validity in 2-value models does not require the finiteness of U in order to have sense.

A *lattice model* μ consists of a complete lattice $(L, \leqslant)$ and an interpretation $\mu : U \rightarrow L$. A FD statement $X \leftarrow Y$ is *valid* in μ if

$$\overline{\mu}(X) \leqslant \overline{\mu}(Y)$$

where $\overline{\mu}: \mathcal{P}(U) \rightarrow L$ is the unique extension of μ to a join complete homomorphism.

Given a set π of FD statements the FD d is a consequence of π if d is valid in every lattice model in which all the FD statement, from π are valid Denote by Con (π) the set of consequences of π. The 2-value models are obviously lattice models.

An *equivalence lattice model* is a lattice model in which the lattice is the dual of the complete lattice of the equivalence relations on some set. To each table-like model φ with entries E we associate the equivalence lattice model $\ker_\varphi$ with lattice $(E_q(E), \supseteq)$ and we have seen that validity is the same in these two models. On the other hand every equivalence lattice model can be associated to a table-like model in the above manner because any equivalence relation is determined by a function (the canonical surjection onto the equivalence class set, for instance) which we can think as a table attribute, the set of entries being the set in which the equivalence relations are defined. Also every 2-values model can be regarded as an equivalence lattice model because there are exactly 2 equivalence relations on $\mathbf{B}$ and there exists an isomorphism of partially ordered sets between $(\mathbf{B}, \leqslant)$ and $(E_q(\mathbf{B}), \supseteq)$, the validity being the same in the two models.

We can now state :

Lemma 2.1. *For any set π of FD statement's*

$$\text{Con}\,(\pi) \subseteq \text{TCon}\,(\pi) \subseteq 2\ \text{Con}\,(\pi).$$

The proof was indicated in the above considerations.

§3. **Semicongruences.** In this paragraph we shall conserve the terminology of § 1. However, the finiteness assumption for U is not necessary in the sequel. We shall use it only since it is necessary to see how our new concepts generalize those already introduced.

Definition 3.1. *A binary relation $\leftarrow$ on $\mathcal{P}(U)$ is called semicongruence if* :

(i) $\forall A \in X \subseteq U$, $\quad A \leftarrow X$;

(ii) $\forall X, Y, Z \subseteq U$, if $\quad X \leftarrow Y$ and $Y \leftarrow Z$ then $X \leftarrow Z$ (transitivity);

(iii) $\forall X, Y \subseteq U$, $\quad$ if $\quad A \leftarrow Y$, $\forall A \in X$ then $X \leftarrow Y$.

Remark 3.2. One should make a distinction between the FD statement $X \leftarrow Y$ which means just the pair (X, Y) and the set theory statement $X \leftarrow Y$ which means $(X, Y) \in \leftarrow$ when $\leftarrow$ is a binary relation $\leftarrow \subseteq \mathcal{P}(U) \times \mathcal{P}(U)$.

Remark 3.3. The name „semicongruence" has the following justification : let us call *congruence* an equivalence relation $\sim$ on $\mathcal{P}(U)$ satisfying

$$\forall \{X_i\}, \{Y_i\}, \ i \in I, \ X_i, Y_i \subseteq U \text{ if } X_i \sim Y_i \ \ \forall i \in I \text{ then } \bigcup_{i \in I} X_i \sim \bigcup_{i \in I} Y_i,$$

One can check that there exists a one to one correspondence between the set of semicongruences on $\mathcal{P}(U)$ and the set of congruences on $\mathcal{P}(U)$ given by:

(i) to any semicongruence $\leftarrow$ corresponds the congruence $X \sim Y$ iff $X \leftarrow Y$ and $Y \leftarrow X$,

(ii) to any congruence corresponds the semicongruence $X \leftarrow Y$ iff $X \cup \cup Y \sim Y$.

Now, in order to connect semicongruences with the FD calculus, observe first that the set of all semicongruences on $\mathcal{P}(U)$ is a closure system. We can therefore speak about the semicongruence generated by a binary relation ρ, denoted ρ^* which is the intersection of all semicongruences containing ρ.

Lemma 3.4. *Suppose that U is a finite set. Then $\pi^* = \mathrm{Ded}\ (\pi)$ for any set π of FD's,i.e. for any binary relation π on $\mathcal{P}(U)$.*

If $(L, \leqslant)$ is a complete lattice and $h: \mathcal{P}(U) \to L$ is a join complete-homomorphism, then the binary relation $\leftarrow$ on $\mathcal{P}(U)$ defined by $X \leftarrow Y$ *iff* $h(X) \leqslant \leqslant h(Y)$ *is a semicongruence, as it can be checked, which we shall denote by* scgr_h.

Proposition 3.5. *For any set π of FD's*

a) $\mathrm{Con}\ (\pi) = \bigcap_{\pi \subseteq \mathrm{segr}\,\bar{\mu}} \mathrm{s\ gr}\,\bar{\mu}$ — where μ ranges over all lattices models and $\bar{\mu}$ is the unique extension of μ to a join complete-homomorphism.

b) $2\ \mathrm{Con}\ (\pi) = \bigcap_{\pi \subseteq \mathrm{scgr}_{\bar{\beta}}} \mathrm{scgr}_{\bar{\beta}}$ where β ranges over all 2-value models and $\bar{\beta}$ is the unique extension of β to a join complete-homomorphism. The proof is straightforward from the definition.

Corollary 3.6. *For any set π of FD's* $\pi^* \subseteq \mathrm{Con}\ (\pi)$

Remark 3.7. Any semicongruence can be regarded as scgr_h, where h is the canonical surjection associated with the corresponding congruence. So, in fact $\pi^* = \mathrm{Con}\ (\pi)$.

Let „$\leftarrow$“ be a semicongruence on $\mathcal{P}(U)$. The attribute set $K \subseteq U$ is said to be *dependency closed* regarding „$\leftarrow$“ if $\forall A \in U$ if $A \leftarrow K$ then $A \in K$.

Proposition 3.8. *The set $\mathcal{C}$ of all dependency closed (regarding a given semicongruence $\leftarrow$) parts of U is a closure system. The corresponding closure operator is* $c_{\leftarrow} : \mathcal{P}(U) \to \mathcal{P}\ (U)$, $c(X) = \{A \in U \mid A \leftarrow X\}$.

Moreover one can check that $\forall X, Y \subseteq U, X \leftarrow Y$ iff $c(X) \subseteq c(Y)$. On the other hand, given any closure system $\mathcal{C}$ on $\mathcal{P}(U)$ and the corresponding closure operator $c: \mathcal{P}(U) \to \mathcal{P}(U)$ consider its corestriction $c: \mathcal{P}(U) \to \mathcal{C}$ which is a join complete homomorphism. Denoting scgr_c by „$\leftarrow$“ one can check that $c(X) = \{A \in U \mid A \leftarrow X\}$. We have established a one to one correspondence between the set of all semicongruence on $\mathcal{P}(U)$ and the set of all closure systems on $\mathcal{P}(U)$: $\leftarrow$ corresponds to $\mathcal{C}$ and inversely $\mathcal{C}$ and c corresponds to scgr_c. This correspondence is also antitone : $\leftarrow \subseteq \leftarrow$ iff $\mathcal{C} \supseteq \mathcal{C}$ (it is therefore an isomorphism between the complete lattice of all semicongruences and the dual of the complete lattice of all closure systems).

4. Atomic semicongruences. Let $\beta : U \to \mathbf{B}$ be a 2-values model. One can check that only U and $\beta^{-1}(0)$ can be dependency closed regarding $\mathrm{scgr}_{\bar{\beta}}$ (of course it is possible that $U = \beta^{-1}(0)$. This justifies our next definition.

Definition 4.1. *An atomic semicongruence is a semicongruence whose corresponding closure system consists of at most two attribute sets.* Any closure system $\mathcal{C}$ on $\mathcal{P}(U)$ contains U so if card $(\mathcal{C}) \leqslant 2$ then $\mathcal{C} = \{U, Z\}$ with $Z \in \mathcal{P}(U)$ (card $(\mathcal{C}) = 1$ iff $\mathcal{C} = \{U\}$ and this is the case $Z = U$). We shall denote by $\leftarrow_Z$ the atomic semicongruence corresponding to $\{U, Z\}$.

One can easily check that:

$$\forall X, Y \subseteq U\ \ X \leftarrow_Z Y \quad \text{iff}\ \ Y \subseteq Z \Rightarrow X \subseteq Z$$

To $\leftarrow_Z$ we can associate the 2-value model $\beta : U \to \mathbf{B}\beta(A) = \begin{cases} 0 & \text{if } A \in Z \\ 1 & \text{otherwise} \end{cases}$

One can check that $\leftarrow_Z = \mathrm{scgr}_{\bar{\beta}}$. Therefore there exists a one to one correspondence between atomic semicongruences and 2-value models.

We shall now turn to our main result, a structure theorem for the complete lattice of all semicongruences.

Lemma 4.2. *If $\leftarrow$ is a semicongrunce and $\leftarrow_Z$ is an atomic semicongruence then $\leftarrow \subseteq \leftarrow_Z$ iff Z is dependency closed regarding $\leftarrow$.*

Proof. Assume $\leftarrow \subseteq \leftarrow_Z$. Then if $A \leftarrow Z$ it follows $A \leftarrow_Z Z$ and because $Z \subseteq Z$ we have $\{A\} \subseteq Z$ that is $A \in Z$. Therefore Z is dependency closed. Conversely, $\forall X, Y \subseteq U$ if $X \leftarrow Y$ then if $Y \subseteq Z$ it follows $Y \leftarrow Z$ and then $X \leftarrow Z$. For all $A \in X$, so $A \leftarrow Z$ and Z being dependency closed, $A \in Z$. So $X \subseteq Z$ and therefore $X \leftarrow_Z Y$.

Theorem 4.3. *Any semicongruence is equal to the intersection of all atomic semicongruences containing it.*

Proof. It suffices to prove that the intersection is included in the semicongruence. If $X \leftarrow_Z Y$ for all Z so that $\leftarrow \subseteq \leftarrow_Z$ then by Lemma 4.2. $X \leftarrow \leftarrow_Z Y$ for all $Z \in \mathcal{C}$. But $c(Y) \in \mathcal{C}$ and so because $Y \subseteq c(Y)$ we have $X \subseteq c(Y)$. Now $c(X) \subseteq c(c(Y)) = c(Y)$ and therefore $X \leftarrow Y$.

An immediate consequence is:

Corollary 4.4. *For any binary relation* $\pi \subseteq \mathcal{P}(U) \times \mathcal{P}(U)$ $\pi^* = \bigcap_{\leftarrow_Z \subseteq \pi} \leftarrow_Z$ *where $\leftarrow_Z$ ranges over all atomic semicongruences.*

We shall now complete the proof of Theorem 1.1.

Lemma 4.5. *For any set π of FD's.*

$$\pi^* = 2\mathrm{Con}\ (\pi).$$

The proof follows from. Lemma 4.4, the considerations from the beginning of this paragraph and Proposition 3.5. b)

Now, Theorem 1.1 follows from Lemma 4.5, Lemma 2.1, Lemma 3.4, and Corollary 3.6.

5. Final Remarks. The reader might have noticed that the boolean expressions involved in defining validity in our 2-values models are the boolean duals of those from [4] and [5]. We made this choice seeking clarity of exposition. Using the original expressions however would not have changed the truth of the Double Completeness Theorem. Indeed, the value models would still have been lattice models but the complete lattice would have been the dual of $(\mathbf{B}, \leqslant)$, which is isomorphic to $(\mathbf{B}, \leqslant)$ and therefore all facts about atomic semicongruences remain unchanged.

We have defined semicongruences in $\mathcal{P}(U)$. It is possible to give a general definition in some complete lattice. Let $(L, \leqslant)$ be a complete lattice. The binary relation $\leftarrow$ on L is called a *semicongruence* if :

(i) $\forall x, y \in L$, if $x \leqslant Y$, then $x \leftarrow Y$;

(ii) $\forall x, y, z \in L$, if $x \leftarrow y$ and $y \leftarrow z$ then $x \leftarrow z$ (transitivity) :

(iii) $\forall x \in L, A \subseteq L$, if $a \leftarrow x$, $\forall a \in A$ then $\sup A \leftarrow x$.

In the special case of $(\mathcal{P}(U), \subseteq)$ this definition is equivalent to that of §3. Note also that any semicongruence is a preorder. Remark 3.3, connection with join complete homomorphisms and closure systems, definition of atomic semicongruences. Lemma 4.2. and Theorem 4.3, which all can be generalized to this situation (Moore parts take the place of closure systems).

This approach might be interesting regarding the lattice theory problem of characterizing the complete lattice of all closure operators on a given complete lattice.

We believe that the tools we used can also be applied to other topics in relational databases such as key finding and normalization.

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RECENZII

SZOLEM MANDELBROJT : *Selecta*. Gauthier-Villars, Paris, 1981, 639 p.

Comitetul Naţional Francez al Matematicienilor a iniţiat publicarea unor lucrări de teoria analiticităţii, ale unuia din marii analişti ai secolului nostru, S. Mandelbrojt, a cărui operă întinsă pe mai bine de jumătate de secol cuprinde peste 200 titluri. În acest volum sînt incluse următoarele lucrări : 1. La série de Taylor et son prolongement analytique (avec J. Hadamard). 2. Modern researches on the singularities of functions defined by Taylor's series. 3. Analytic functions and classes of infinitely differentiable functions. 4. Dirichlet series. 5. General theorems of closure. 6. Composition theorems. Cu excepţia primei lucrări, apărută în „Scientia", nr. 41 (1926), celelalte au apărut în „The Rice Institute Pamphlet" în volumele 14 (1927), 29 (1942), 31 (1944), 38 (1951) şi 45 (1958). Prima lucrare, excelentă sinteză a rezultatelor cunoscute în teoria funcţiilor pînă la acel moment, fundamentează problematica şi metodele de studiu al seriilor Taylor şi prelungirilor analitice cu ajutorul şirului coeficienţilor. Următoarele lucrări au la bază cursuri postuniversitare ţinute de S. Mandelbrojt la Rice Institute în mai mulţi ani academici. Ele se constituie ca teorii generale şi unitare bazate pe metode introduse de autor, metode ce au impulsionat studiul funcţiilor analitice. Volumul se încheie cu lista completă pe ani a lucrărilor lui S. Mandelbrojt, reprezentant de frunte al şcolii matematice franceze.

Teodor Precupanu

GEORGES DE RHAM : *Oeuvres Mathématiques*, L'enseignement mathématique, Université de Genève, 1981, 748 p. ; sFr. 110.

Acest volum, editat de P. Jeanquartier şi M. Kervaire, cuprinde toate lucrările matematice ale lui Georges de Rham, cu excepţia monografiei sale şi a unui curs policopiat. Prima parte, intitulată „Topologie şi analiză", conţine majoritatea lucrărilor, în ordine cronologică. Alte 7 lucrări, cu caracter elementar, sînt reunite în partea a doua, sub titlul „Curbe şi funcţii definite prin ecuaţii funcţionale". Este inclus şi raportul lui Élie Cartan asupra tezei lui de Rham. Volumul reprezintă un omagiu adus unuia dintre puţinii matematicieni, încă în viaţă, care au adus contribuţii esenţiale la fundamentarea teoriilor moderne ale topologiei algebrice, topologiei diferenţiale şi geometriei diferenţiale.

Jack Weinstein

DAVID A. KLARNER (Editor) : *The Mathematical Gardner*. Prindle, Weber & Schmidt, Boston, Mass. ; Wadsworth International, Belmont, Ca., 1980, VIII+382 p., \$ 22.45.

Volumul este dedicat lui Martin Gardner cu ocazia împlinirii vîrstei de 65 de ani. Personalitate unică în comunitatea matematică, acesta redactează de două decenii o rubrică din „Scientific American", în care popularizează rezultatele noi obţinute în diversele ramuri ale matematicii. În volum au fost incluse 30 articole, grupate pe următoarele teme : teoria jocurilor (R. K. Guy, N. G. de Bruijn, C. Berge, J. Boyce, A. Shamar & R. L. Rivest & L. M. Adelman, V. Chvátal, D. Berengut) ; geometrie (S. W. Golomb, H. Taylor, R. Connelly, S. Burr, H. Eves, L. Bankoff) ; parchetări bidimensionale (R. L. Graham, W. T. Tutte, Doris Schattschneider, B. Grünbaum & G. C. Shephard, H. S. M. Coxeter) ; parchetări tridimensionale (D. G. Hoffman, R. M. Robinson, C. J. Bouwkamp, D. A. Klarner); probleme distractive (D. E. Knuth, S. Kim, Ch. W. Trigg, R. Honsberger, W. T. Laaser & L. Ramshaw) ; numere şi teoria codurilor (D. E. Knuth, R. Read, N. J. A. Sloane). Remarcăm, printre numeroasele figuri şi ilustraţii, unele reproduceri după opere de artă ale lui Marjorie Rice şi M. C. Escher.

Jack Weinstein