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FIXED POINT THEOREMS IN UNIFORM SPACES

BY

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It is the purpose of this paper to point out the fact that in many cases with a fixed point theorem in metric spaces is (rather often) associated, by generalization, a fixed point theorem in uniform spaces.

To illustrate the assertion above, we recall our result given by theorem 1 (see [1], [2], [3]), which represents the natural generalization to uniform spaces of the famous Banach theorem on contraction mappings in metric spaces.

Furthermore, a generalization of Maia's theorem ([4], [3], Theorem 2) in uniform spaces endowed with two uniform structures is given. Finally, by theorem 3 we generalize the theorem of Kannan from metric spaces [5], [3]. This chapter of fixed point theorem in uniform spaces is little developed. It is important since any structure of a locally convex space (or even of topological vector space) is a uniform structure.

Theorem 1. *Let X be a sequentially complete, Hausdorff separated, uniform space. Let also $\{\rho_\alpha\}_{\alpha\in A}$ be a separated, saturated set of pseudometrics defining the uniform structure of X. If the function $f:X\to X$ is contractive with respect to the set $\{\rho_\alpha\}_{\alpha\in A}$, then f has a unique fixed point which can be found by the method of successive approximations (starting from any element of X).*

Remark 1. By f-contraction with respect to $\{\rho_\alpha\}_{\alpha\in A}$, we mean that there exist a function $\varphi: A\to A$ and the positive numbers $\lambda_\alpha>0$ such that

$$(1)\qquad \rho_\alpha(f(x),f(y))\leqslant\lambda_\alpha\rho_{\varphi(\alpha)}(x,y),\quad \forall\alpha\in A \text{ and } x,y\in X$$

and the series

$$(2)\qquad \rho_\alpha(x,y)+\sum_{n=1}^{\infty}\lambda_\alpha\lambda_{\varphi(\alpha)}\dots\lambda_{\varphi^{n-1}(\alpha)}\rho_{\varphi^n(\alpha)}(x,y)$$

are convergent for each $\alpha\in A$ and $x, y\in X$. Here φ^n is the n-th iteration of φ.

Remark 2. The proof of theorem 1 as well as some applications can be found in [1], [2].

Theorem 2. *Let X be a set endowed with two Hausdorff separated uniform structures σ, τ. Let $\mathcal{P}=\{p_\alpha\}_{\alpha\in A}$, $Q=\{q_\beta\}_{\beta\in B}$) be a separated saturated set of pseudo-matrics which defines the structure $\sigma(\tau$, respectively).*

If $f:X\to X$ is a function satisfying the conditions:

I. *There is a function $\varphi: A\to B$ and the real numbers $m_\alpha>0$ such that*

$$p_\alpha(x,y)\leqslant m_\alpha q_{\varphi(\alpha)}(x,y),\quad \forall\alpha\in A,\ x,y\in X\ ;$$

II. (X, σ) *is a sequentially complete uniform space*;
III. $f:(X,\sigma)\to(X,\tau)$ *is continuous*;
IV. f *is Q-contractive (in the sense of remark 1)*.
then f has a unique fixed point which can be obtained by successive approximations (starting from any element of X).

Proof. If one takes a sequence $\mathcal{S}$ of successive approximations, one proves (as in [1], [3]) that $\mathcal{S}$ is a Cauchy sequence in (X,τ). By I, $\mathcal{S}$ is also a Cauchy sequence in (X,σ), hence it is convergent (in vertue of II). The limit of this sequence is a fixed point of f (since f is continuous, as indicated by III). The uniqueness follows from IV as in [1], [3].

Remark 3. In the particular case in which both $\mathcal{P}$ and Q are made up by a single element, one obtains M a i a 's theorem.

Theorem 3. *Let X be a uniform space satisfying the properties of theorem 1 and let $f: X\to X$ be a function. If there exist real numbers λ_α (with $0<\lambda_\alpha<1/2$) such that*

$$\rho_\alpha(f(x),f(y))\leqslant\lambda_\alpha[\rho_\alpha(x,f(x))+\rho_\alpha(y,f(y))] \tag{3}$$

for all $\alpha\in A$ and $x, y\in X$, then f has a unique fixed point, which can be obtained via the method of successive approximations (starting from any element of X).

Proof. If one takes a sequence $\mathcal{S}$ of successive approximations, then by making use of (3) (as in [5]) one shows that $\mathcal{S}$ is a Cauchy sequence (for each fixed $\alpha\in A$), hence it is convergent to an element $x\in X$. It follows from (3) (as in [5]) that $\rho_\alpha(x,f(x))=0$ for each $\alpha\in A$ (i.e. $x=f(x)$) hence the existence of the fixed point is proved. The uniqueness can be proved as in [5] (observing that if x' and x'' are two fixed points then $\rho_\alpha(x',x'')=0$ for each $\alpha\in A$).

Remark 4. In the case in which the uniform structure of X is given by a single distance one obtains Kannans theorem.

Remark 5. In general, with each problem related to fixed point theorems in metric spaces is associated a similar problem (related to fixed point theorems) in uniform spaces.

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DISTRIBUTIONS FOR SET FUNCTIONS

BY

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In the Schwartz theory of distributions, every distribution is a linear continuous functional on the strict inductive limit of the spaces $\mathcal{D}_K(\Omega)$, where Ω is a domain of R^n, K ranges over all compact subsets of Ω and $\mathcal{D}_K(\Omega)$ is the set of all functions $f\in C_0^\infty(\Omega)$ such that $\operatorname{supp}(f)\subseteq K$ (see [1], p. 19).

In this paper we extend the notion of distribution to set functions.

We shall consider $\Omega=(-\infty, 0)$, but we shall see that this assumption is not a restriction because our considerations are also valid when Ω is a non-void open set of R^n.

We denote by $C^\infty(\Omega)$ the linear space of all complex-valued functions defined on Ω which have continuous derivatives of every order. For any compact subset $Q\subset\Omega$ and any integer $n\geqslant 1$ we define (1)

$$p_{Q,n}(x)=\sup_{\substack{t\in Q\\0\leqslant k\leqslant n}}|x^{(k)}(t)|,\quad x\in C^\infty(\Omega). \tag{1}$$

The family of the semi-norms of type (1) induces on $C^\infty(\Omega)$ a structure of Hausdorff locally convex space.

We denote by $\mathcal{P}(\Omega)$ the family of all subsets of Ω and for every compact $Q\subset\Omega$ let us consider

$$\mathcal{D}_Q=\left\{F:\mathcal{P}(\Omega)\to\bar{R}:\ \textit{there exists}\ f\in C^\infty(\Omega)\ \textit{with}\ \operatorname{supp}(f)\subseteq Q\ \textit{such that}\ F(A)=\int_{\bar{A}} f\,d\mu,\ \forall A\in\mathcal{P}(\Omega)\right\} \tag{2}$$

where μ is the Lebesgue measure.

Every $\mathcal{D}_Q$ with the topology induced by the semi-norms

$$q_{Q,n}(F)=p_{Q,n}(f),\quad n=0,1,2,\ldots \tag{3}$$

is a Hausdorff locally convex space.

The strict inductive limit of these spaces denoted by

$$\mathcal{D}=\bigcup_{Q\subset\Omega}\mathcal{D}_Q\qquad Q - a\ compact \tag{4}$$

is the space of the test functions on which we shall define the distributions.

We remark that if we replace the family $\mathcal{P}(\Omega)$ by the family $\mathcal{A}$ of measurable subsets of Ω, then we take in (2) $F(A)=\int_A f\,d\mu,\ \forall A\in\mathcal{A}$.