$$+\int_B \int_{t_0}^{t} [(\mathbf{S}_t+\mathbf{S}_{t_0}-2\mathbf{S}_\lambda)\cdot\nabla\dot{\mathbf{u}}_\lambda+(v_t+v_{t_0}-2v_\lambda)\dot{\eta}_\lambda]\,d\lambda\,da.$$

Next,

$$\int_{t_0}^{t} [(\mathbf{S}_t+\mathbf{S}_{t_0}-2\mathbf{S}_\lambda)\cdot\nabla\dot{\mathbf{u}}_\lambda+(v_t+v_{t_0}-2v_\lambda)\dot{\eta}_\lambda]\,d\lambda= \tag{3.8}$$

$$=(\mathbf{S}_t+\mathbf{S}_{t_0})\cdot(\nabla\mathbf{u}_t-\nabla\mathbf{u}_{t_0})+(v_t+v_{t_0})\,(\eta_t-\eta_{t_0})-2\int_{t_0}^{t}(\mathbf{S}\cdot\nabla\mathbf{u}+\theta\dot{\eta})\,d\lambda.$$

By (3.5)

$$\mathbf{S}\cdot\nabla\dot{\mathbf{u}}+\theta\dot{\eta}=\nabla\dot{\mathbf{u}}\cdot\mathbf{C}\nabla\mathbf{u}+a v\dot{v}=\dot{w}/2, \tag{3.9}$$

where

$$w=\mathbf{S}\cdot\nabla\mathbf{u}+\theta\eta=\nabla\mathbf{u}\cdot\mathbf{C}\nabla\mathbf{u}+av^2. \tag{3.10}$$

Thus, we have

$$\int_{t_0}^{t}(\mathbf{S}\cdot\nabla\dot{\mathbf{u}}+v\dot{\eta})d\lambda=\mathbf{S}_t\cdot\nabla\mathbf{u}_t-\mathbf{S}_{t_0}\cdot\nabla\mathbf{u}_{t_0}+v_t\eta_t-v_{t_0}\eta_{t_0}. \tag{3.11}$$

In view of (3.5) we can write $\mathbf{S}_t\cdot\nabla\mathbf{u}_{t_0}+v_t\eta_{t_0}=\nabla\mathbf{u}_{t_0}\cdot\mathbf{C}\nabla\mathbf{u}_t+av_tv_{t_0}$.

Since $\mathbf{C}$ is symmetric, we obtain

$$\mathbf{S}_t\cdot\nabla\mathbf{u}_{t_0}+v_t\eta_{t_0}=\mathbf{S}_{t_0}\cdot\nabla\mathbf{u}_t+v_{t_0}\eta_t. \tag{3.12}$$

From (3.8), (3.11), (3.12) we arrive at $(3.6)_{1,2}$. The relation $(3.6)_3$ is a direct consequence of $(3.6)_1$ and Theorem 3 of [4].

REFERENCES

1. G u r t i n, M.E. — *Thermodynamics and the Griffith criterion for brittle fracture*, Int. J Solids Struct 15 (1979), 553—560
2. G u r t i n, M.E. — *Thermodynamics and the cohesive zone in fracture*. ZAMP 30 (1979), 991—1003
3. G u r t i n, M.E. — *On the energy release rate in quasi-static elastic crack propagation*. J. Elasticity, 9 (1979), 187—195
4. G u r t i n, M.E. and Y a t o m i C. — *On the energy release rate in elastodynamic crack propagation*, Arch. Rational Mech. Anal. 74 (1980), 231—247
5. C a r l s o n, D.E. — *Linear Thermoelasticity*, Handbuch der Physik, Vol. VI a/2, Berlin—Heidelberg—New York, Springer—Verlag, 1972
6. F r e u n d, L.B. — *Energy flux into the tip of an extending crack in an elastic solid*, J. Elasticity 2 (1972), 341—349
7. R i c e, J.R. — *An examination of the fracture mechanic energy balance from the point of view of continuum mechanics*, Proc of the First Int Conference in Fracture, Sendai, 1965, 309—315

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STRESS DISTRIBUTION IN AN INFINITE REGION BOUNDED BY A CIRCLE UNDER BIAXIAL FORCES

BY

M. LAL and R. S. SIKARWAR

1. Introduction. Complex variable approach for the solution of two-dimensional problems in elasticity has proved to be very useful. This technique was propounded by K o l o s o v [1] and M u s k h e l i s h v i l i [2]. It was later developed and adopted by A d k i n s, G r e e n et al. [3, 4] to be used in solving problems in second order elasticity. Subsequently C a r l s o n and S h i e l d [5] have used this technique to study of second and higher order effects in a class of problems in plane finite elasticity. Of late, B h a r g a v a and P a n d e [6, 7] have further modified this technique to make it suitable for the solution of more general problems in plane strain. Recently L a l and P a n d e [8, 9] have used his technique to solve several problems in the second order elasticity. A brief resume of the same is given below. Let

$$\zeta=x_1+ix_2 \text{ and } z=\xi+i\eta$$

be the coordinates of a point of an elastic body in its undeformed state $\mathcal{B}_0$ and the deformed state $\mathcal{B}$ respectively. The Airy's stress function φ and the displacement vector $D=u_1+iu_2$, being analytic, can be expanded as

$$\begin{aligned}\varphi&=\mu\varepsilon\{\varphi_0(z,\bar z)+\varepsilon\varphi_1(z,\bar z)+\ldots\},\\ D&=\varepsilon\{D_0(z,\bar z)+\varepsilon D_1(z,\bar z)+\ldots\},\end{aligned} \tag{1.1}$$

where ε is a parameter depending upon the physical condition of the problem.

Equilibrium equations are satisfied if

$$T^{11}=\overline{T}^{22}=-4\frac{\partial^2\varphi}{\partial\bar z^2}\,;\ T^{12}=4\frac{\partial^2\varphi}{\partial z\partial\bar z}, \tag{1.2}$$

where

$$T^{11}=\overline{T}^{22}=p_{\xi\xi}-p_{\eta\eta}-2ip_{\xi\eta},\quad T^{12}=p_{\xi\xi}+p_{\eta\eta}. \tag{1.3}$$

If the resultant stress along the boundary is zero, then

$$\frac{\partial\varphi}{\partial z}=\frac{\partial\varphi}{\partial\bar z}=0, \tag{1.4}$$

The entire elastic fields is determined by a set of potential functions $\{\Omega(z), \geqslant\omega(z)\}$ in the infinitesimal case and $\{\Omega(z), \geqslant\omega(z)\}$ and $\{\Delta(z), \delta(z)\}$ in the second order case.

We have

$$\begin{aligned}\varphi_0(z,\bar z)&=\bar z\Omega(z)+z\bar\Omega(\bar z)+\omega(z)+\bar\omega(\bar z),\\ D_0(z,\bar z)&=\varkappa\Omega(z)+\bar\omega'(\bar z)-z\bar\Omega'(\bar z),\end{aligned}\tag{1.5}$$

$$\frac{\partial\varphi_1}{\partial\bar z}=\Delta(z)+z\bar\Delta'(\bar z)+\bar\delta'(\bar z)+\gamma\Gamma(z,\bar z)+B_3\Omega(z)\bar\Omega'(\bar z)-B_1'z\{\bar\Omega'(\bar z)\}^2,\tag{1.6}$$

$$\begin{aligned}D_1(z,\bar z)&=\varkappa\Delta(z)-z\bar\Delta'(\bar z)-\bar\delta'(\bar z)-\gamma\Lambda(z,\bar z)-B_3\Omega(z)\bar\Omega'(\bar z)+\\ &+B_1''z\{\bar\Omega'(\bar z)\}^2+B_1'\int^{\bar z}\bar\Omega'(\bar z)\bar\omega''(\bar z)d\bar z-B_4\int^{z}\{\Omega'(z)\}^2dz,\end{aligned}\tag{1.7}$$

where $(\varkappa=(5\lambda+6\mu)/(3\lambda+2\mu)$ for the plane stress, λ and μ are Lamé's constants ; γ, B_0, B_1' and B_1'' are elastic constants defined in [4].

2. The problem. Let us consider the region $|z|\geqslant R$ and map it on the unit circle in the ζ-plane by means of

$$z=w(\zeta)=R/\zeta.\tag{2.1}$$

where R is a real constant. The potential functions $\Omega(z)$, $\omega'(z)$, $\Delta(z)$ and $\delta'(z)$ will be transformed as following

$$\begin{aligned}\Omega(z)&=\Omega[w(\zeta)]=\Omega_1(\zeta), &\quad \Delta(z)&=\Delta[w(\zeta)]=\Delta_1(\zeta),\\ \omega'(z)&=\omega'[w(\zeta)]=\omega_1(\zeta), &\quad \delta'(z)&=\delta'[w(\zeta)]=\delta_1'(\zeta).\end{aligned}\tag{2.2}$$

As pointed out earlier, for the infinitesimal case the expressions of the potential functions $\Omega_1(\zeta)$, $\omega_1'(\zeta)$ are essential. The expressions for $\Omega_1(\zeta)$ and $\omega_1'(\zeta)$ are obtained by

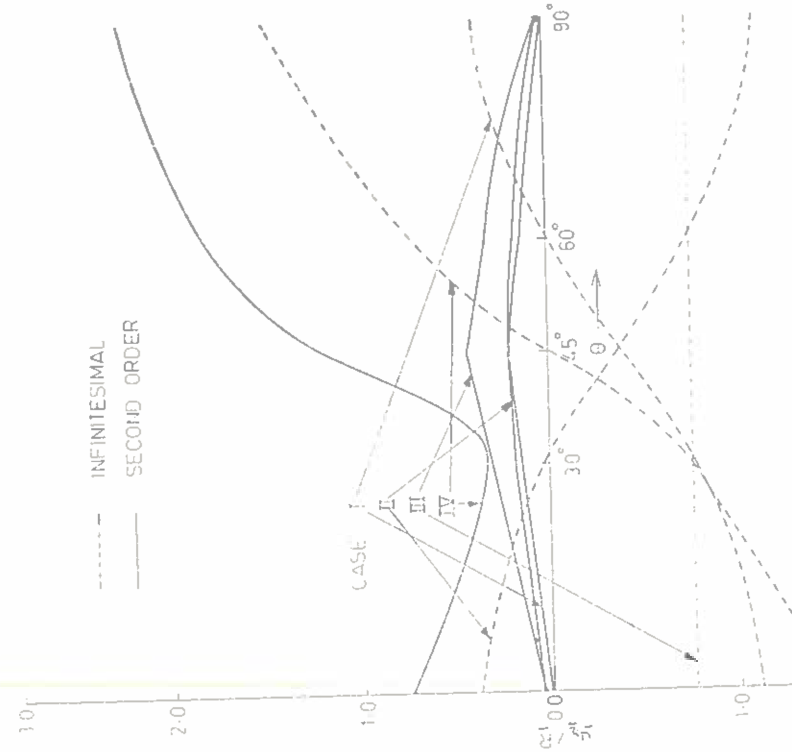
$$\Omega_1(\zeta)=\frac{1}{2}\varphi_1(\zeta)\ \text{and}\ \omega_1'(\zeta)=\frac{1}{2}\psi_1(\zeta),\tag{2.3}$$

where $\varphi_1(\zeta)$ and $\psi_1(\zeta)$ are analytic functions and found out as stated by Sokolnikoff [10] as following :

$$\begin{aligned}\varphi_1(\zeta)&=\frac{X_1+iX_2}{2\pi(1+\varkappa)}\log\zeta+(B+iC)R/\zeta+\varphi^0(\zeta),\\ \psi_1(\zeta)&=-\frac{\varkappa(X_1-iX_2)}{2\pi(1+\varkappa)}\log\zeta+(B'+iC')R_0/\zeta+\psi^0(\zeta).\end{aligned}\tag{2.4}$$

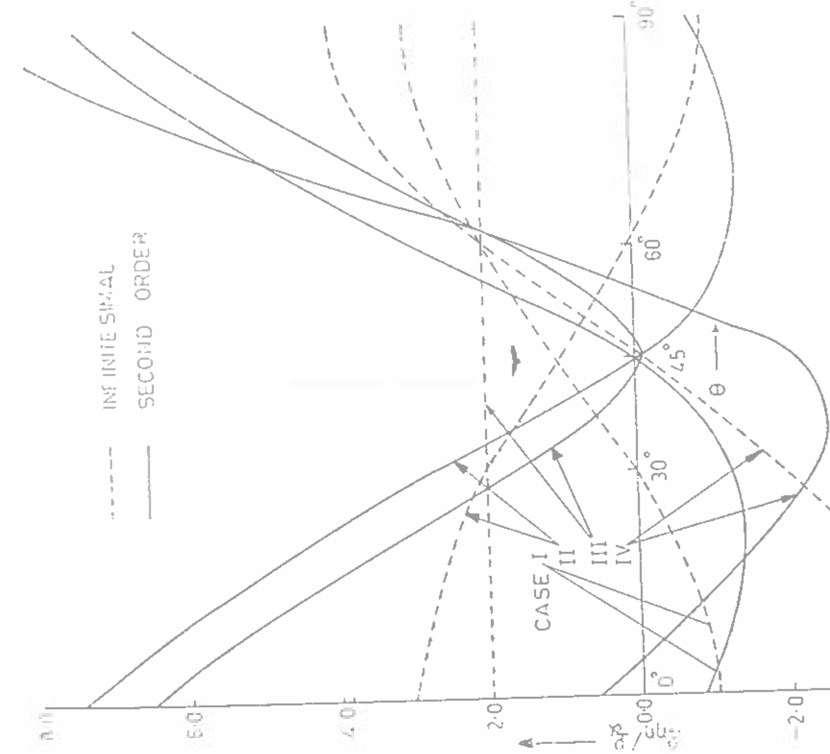
We recall that X_1+iX_2 is the resultant force acting related to the stresses and the constants B, B', C, C' are related to the stresses and rotation at infinity as following

$$2B-B'=\tau_{11}(\alpha),\ 2B+B'=\tau_{22}(\alpha),\ C'=\tau_{12}(\alpha),$$



VARIATION OF RADIAL DISPLACEMENT

Fig. 2



VARIATION OF HOOP STRESS UNDER BI-AXIAL TENSION

Fig. 1

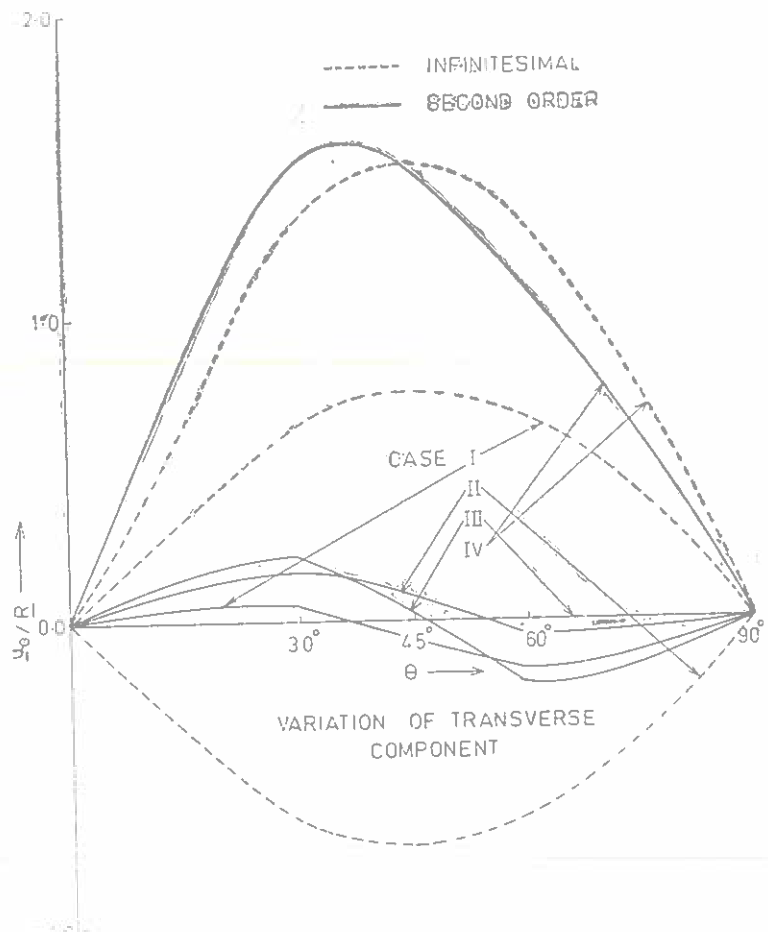


Fig. 3

where $\tau_{\alpha\beta}(\alpha)$, $(\alpha, \beta=1, 2)$ represents the limiting value of $\tau_{\alpha\beta}(a)$ as the point a recedes to infinity. The constant C has no effect on the state of plane stress and can always be set equal to zero.

It is worth mentioning that the requirement for the $\tau_{\alpha\beta}$ to be bounded at infinity does not imply that the displacements u_α remain bounded. If the displacements are to be bounded at infinity, then taking $\tau_{\alpha\beta}(\infty)=0$, $x_1+ix_2=0$ and $C=0$, we have

$$B=[\tau_{11}(\alpha)+\tau_{22}(\alpha)]/4\,;\quad B'=[\tau_{22}(\alpha)-\tau_{11}(\alpha)]/2,\quad C'=\tau_{12}(\alpha). \tag{2.5}$$

For the determination of the single-valued analytic functions $\varphi^0(\zeta)$ and $\psi^0(\zeta)$, the integro-differential equation of Muskhelishvili [2] has been used, as

$$\varphi^0(\sigma)-\overline{\varphi}'(\sigma)/\sigma^0+\overline{\psi}^0(\overline{\sigma})=F^0(\sigma), \tag{2.6}$$

where $\sigma=\exp(-i\theta)$ is the value of ζ on the boundary of the unit circle C and $F^0(\sigma)$ is given by

$$\begin{aligned} F^0(\sigma)=F(\sigma)-\{(X_1+iX_2)\log \tau\}/2\pi-BR/\sigma+ \\ +[(X_1-iX_2)\tau/2\pi(1+\varkappa)-BR\sigma^2]/\sigma^3-(B'-iC')R\sigma, \end{aligned} \tag{2.7}$$

and $F(\sigma)=f_1(\theta)+if_2(\theta)$, determined by the specified stress distribution on the circular boundary. Setting $\varphi^0(\zeta)=\sum_{k=1}^{\infty} a_k\zeta^k$, $\psi^0(\alpha)=\sum_{k=1}^{\infty} b_k\zeta^k$, in (2.6) and writing in the right hand expression the Fourier series representation for the single-valued function.

$$F(\sigma)-\frac{X_1+iX_2}{2\pi}\log\sigma\equiv\sum_{k=-D}^{\infty}A_k\sigma^k-\frac{X_1+iX_2}{2\pi}\left[i\pi+\sum_{k=1}^{\infty}(\overline{\sigma}^k-\sigma^k)/k\right],$$

we obtain

$$\begin{aligned} \sum_{k=1}^{\infty} a_l\sigma^k-\sum_{k=1}^{\infty} k\overline{a}_k\overline{\sigma}^{k-2}+\sum_{k=0}^{\infty}\overline{b}_k\overline{\sigma}^k=\sum_{k=-\infty}^{\infty}A_k\sigma^k-\frac{x_1+ix_2}{2\pi}\Big[i\pi+ \\ +\sum_{k=1}^{\infty}(\overline{\sigma}^k-\sigma^k)/k\Big]-2BR/\sigma-(B'-iC')R\sigma+(X_1-iX_2)/2\pi(1+\varkappa)\sigma^2, \end{aligned} \tag{2.8}$$

On compairing the same powers of σ, we obtain

$$\begin{aligned} &a_1=A_1-(B'-iC')R+(X_1+iX_2)/2\pi,\quad \overline{b}_0=A_0-(X_1+iX_2)/2\pi, \\ &a_k=A_k+(X_1+iX_2)/2\pi R,\ k\geqslant 2,\qquad \overline{b}_1=A_1-2BR-(X_1+iX_2)/2\pi \\ &\overline{b}_2=A_{-2}-(X_1+iX_2)/4\pi+(X_1-iX_2)/2\pi(1+\varkappa), \\ &\overline{b}_k=A_{-k}+(k-2)\overline{a}_{k-2}-(k-2)\overline{a}_{k-2}-(X_1+iX_2)/2k\pi,\ k\geqslant 3. \end{aligned} \tag{2.9}$$

Now we discuss the effect of the stress concentration for different cases in the neighbourhood of the hole $|z|=R$, located in a plane subjected to the action of constant loads at a great distance from the hole

Case I. Let us first suppose that the plane is stretched by the tensile forces acting in the X_1-direction. We take $\tau_{11}(\alpha)=P_1$, $\tau_{22}(\alpha)=\tau_{12}(\alpha)=0$.

Since the hole is free from stress, so $X_1+iX_2=0$, and $F(\sigma)=0$. The constants B, B', C' in (2.9) are determined with the help of (2.5) as $B=P_1/4$, $B'=-P_1/2$, $C'=0$. Equations (2.9) then yield,

$$a_1=P_1R/2,\ a_k=0,\ k\geqslant 2,\ \bar{b}_0=0,\ \bar{b}_1=-P_1R/2,\ \bar{b}_2=0,$$

$$\bar{b}_3=-P_1R/2,\ b_k=0,\ k>3.$$

Thus $\varphi^0(\zeta)=P_1R\zeta/2$; $\psi^0(\zeta)=-P_1R(\zeta-\zeta^3)/2$, and from formulas (2.4), we have

$$\varphi_1(\zeta)=P_1R[\zeta+(2\zeta)^{-1}]/2,\quad \psi_1(\zeta)=-P_1R[\zeta^{-1}-\zeta-\zeta^3]/2. \tag{2.10}$$

Now, using the relations (2.3) and (2.10), it yield

$$\Omega_1(\zeta)=P_1R[\zeta+(2\zeta)^{-1}]/4,\quad \omega_1'(\zeta)=-P_1R[\zeta^{-1}-\zeta-\zeta^3]/4, \tag{2.11}$$

thus the first set of potential functions, which determines the linear elasticity, are known.

Since the resultant force along the boundary is zero, equation (1.6) gives

$$\frac{\partial\varphi_1}{\partial\bar{z}}=\Delta(z)+z\bar{\Delta}'(\bar{z})+\bar{\delta}'(\bar{z})+\gamma\Gamma(z,\bar{z})+B_3\Omega(z)\bar{\Omega}'(\bar{z})-B_1z\{\bar{\Omega}'(\bar{z})\}^2=0. \tag{2.12}$$

Substituting for $\Gamma(x,\bar{z})$, $\Omega(z)$, $\bar{\Omega}'(\bar{z})$ in equation (2.12) and multiplying by $\zeta/\sigma(\sigma-\zeta)$, integration round C gives

$$\Delta_1(\zeta)=-\frac{P_1^2R\zeta}{32}(2B_1\zeta^2+B_3). \tag{2.13}$$

Again taking conjugate complex of (2.12), multiplying by $1/(\sigma-\zeta)$ and integration round C, we have

$$\delta_1'(\zeta)=-\frac{P_1^2R\zeta}{64}[(12\zeta^4+4\zeta^2+3)B_1-3B_3]. \tag{2.14}$$

Since $\{\Omega_1(\zeta),\omega_1'(\zeta)\}$ and $\{\Delta_1(\zeta),\delta_1'(\zeta)\}$ are known and $p_{\xi\xi}=p_{\xi\eta}=0$ in this case due to the uniaxial tension, taking $\varepsilon=1/\mu$, the hoop-stress on the boundary is given by

$$\begin{aligned}p_{\eta\eta}=&P_1(\sigma^4-\sigma^2+1)/\sigma^2+P_1^2[8\gamma\sigma^8-2(7-\varkappa)\sigma^6+\\&+\{5B_3-B_1+6(3-\varkappa)\gamma\}\sigma^4-2(7-\varkappa)\gamma\sigma^2+8\gamma]/16\mu\sigma^4,\end{aligned} \tag{2.15}$$

this assumes a maximum value $(p_{\eta\eta})_{max}=3P_1$ at $\theta=\pi/2$ and $3\pi/2$ in infinitesimal case. The radial and transverse components of displacement are as following : $D(z,\bar{z})=\exp(i\psi)(u_r+iu_\theta)$, where $\exp(i\psi)=\sigma w'(\sigma)/|w'(\sigma)|$

$$\begin{aligned}u_r=&-P_1R(1+\varkappa)(1+2\cos 2\theta)/8\mu+P_1^2R[15\{B_4+(\varkappa+2)B_1+6B_1'-\\&-B_1''\}-10\{6B_1+2B_1'-6B_2''-3(\varkappa+1)B_3-6B_4\}\cos 2\theta+\\&4\{15B_1+9B_1'-15B_1''-5B_4\}\cos 4\theta]/960\mu^2,\end{aligned} \tag{2.16}$$

$$\begin{aligned}u_\theta=&[P_1R(1+\varkappa)\sin 2\theta]/4\mu-P_1^2R[5\{6B_1+8B_1'-6B_1''+\\&3(\varkappa+1)B_3+6B_4\}\sin 2\theta-2\{15B_1+9B_1'-\\&-15B_1''+5B_4\}\sin 4\theta]/480\mu^2.\end{aligned} \tag{2.17}$$

Case II : If we put $\tau_{11}(\infty)=0$, $\tau_{12}(\infty)=0$ and $\tau_{22}(\infty)=P_2$ and proceed as above, the following results are obtained :

$$\Omega_1(\zeta)=-P_2R(2\zeta^2-1)/8,\quad \omega_1'(\zeta)=-P_2R(\zeta^4+\zeta^2-1)/4\zeta, \tag{2.18}$$

for infinitesimal case, and

$$\begin{aligned}&\Delta_1(\zeta)=-P_2^2R[2B_1\zeta^3-B_3\zeta]/32,\\&\delta_1'(\zeta)=-P_2^2R[12B_1\zeta^5-4B_1\zeta^3+3(B_1-B_3)\zeta]/64,\end{aligned} \tag{2.19}$$

for second order case. The hoop-stress and displacements on the boundary are as follows :

$$\begin{aligned}p_{\eta\eta}=&P_2(1+2\cos 2\theta)+P_2^2[16\gamma\cos 4\theta+4(7-\varkappa)\gamma\cos 2\theta+\\&5B_3-B_1+6(3-\varkappa)\gamma]/16\mu,\end{aligned} \tag{2.20}$$

$$\begin{aligned}u_r=&P_2R(1+\varkappa)(2\cos 2\theta-1)/8\mu+P_2^2R[4\{15B_1+9B_1'-15B_1''-\\&-5B_4\}\cos 4\theta+10\{6B_1+2B_1'-6B_1''-3(\varkappa+1)B_3-6B_4\}\cos 2\theta+\\&+15\{6B_1'+(\varkappa+2)B_1-B_1''+B_4\}]/960\mu^2,\end{aligned} \tag{2.21}$$

$$\begin{aligned}u_\theta=&-[P_2R(1+\varkappa)\sin 2\theta]/4\mu+P_2^2R[2\{15B_1+9B_1'-15B_1''+\\&+5B_4\}\sin 4\theta+5\{6B_1+8B_1'-6B_1''+3(\varkappa+1)B_3+\\&+6B_4\}\sin 2\theta]/480\mu^2.\end{aligned} \tag{2.22}$$

Case III. By superposition the expressions (2.15)—(2.17) and (2.20)—(2.22), we find the uniform biaxial tension with $P_1=P_2=P_0$,

$$p_{\eta\eta}=2P_0+P_0^2[16\gamma\cos 4\theta+5B_3-B_1+6\gamma(3-\varkappa)]/8\mu, \tag{2.23}$$

the radial and transverse component of displacements are

$$\begin{aligned}u_r=&-P_0R(1+\varkappa)/4\mu+P_0^2R[15\{6B_1'+(\varkappa+2)B_1-B_1''+B_4\}+\\&4\{15B_1-9B_1'-15B_1''-5B_4\}\cos 4\theta]/480\mu^2,\end{aligned} \tag{2.24}$$

$$u_\theta=P_0^2R[(15B_1+9B_1'-15B_1''+15B_4)\sin 4\theta]/120\mu^2. \tag{2.25}$$

Case IV. If we put $\tau_{11}(\infty)=P_0$ and $\tau_{22}(\infty)=-P_0$ in (2.15)—(2.17) and (2.20) -(2.22), by combining the results, we shall get the solution in the state of pure shear and the results are as follows :

$$p_{\tau\tau} = -4P_0 \cos 2\theta + P_0^2[16\gamma \cos 4\theta + 5B_3 - B_1 + 6\gamma(3-\varkappa)]/0\mu, \tag{2.26}$$

$$\begin{aligned} u_r = &-[P_0(1+\varkappa)R \cos 2\theta]/2\mu + P_0^2 R[15\{(\varkappa+2)B_1 + 6B_1 - B_1'' + \\ &+ B_4\} + 4\{15B_1 + 9B_1' - 15B_1'' - 5B_4\} \cos 4\theta]/480\mu^2, \end{aligned} \tag{2.27}$$

$$\begin{aligned} u_\theta = &[P_0 R(1+\varkappa)\sin 2\theta]/2\mu + P_0^2 R[\{15B_1 + 9B_1' - 15B_1'' + \\ &+ 5B_4\} \sin 4\theta]/120\mu^2, \end{aligned} \tag{2.28}$$

which has an absolute maximum $|p_{\tau\tau}| = 4P_0$ at $\theta = 0, \pi/2, \pi, 3\pi/2$ in the infinitesimal case.

In the hoop-stress expressions, the first expression gives the hoop-stress in infinitesimal case and which is free from elastic constants. Figures 1, 2 and 3 show the variation of hoop-stresses, radial and transverse components of displacemet for different values of θ, treating P_i/μ, $(i=0, 1, 2)$ is unity for all cases.

REFERENCES

1. Kolosov, G.V. – *On an Application of complex Function Theory to a plane Problem of the Mathematical Theory of Elasticity*. Yuriev, 1909
2. Muskhelishvili, N.I. – *Some Basic Problems of Mathematical Theory of Elasticity* P. Noordhoff, Greningen – The Netherlands, 1963
3. Adkins, J.E., Green, A.E. and Shield, R.T. – Phil. Trans. Roy. Soc., London, (A) *246* (1953), 181
4. Adkins, J.E., Green, A.E. and Nicholas, G.C. – Phil. Trans. Roy. Soc., London (A) *247* (1954), 279
5. Carlson, D.E. and Shield, R.T. – Arch. Rat Mech. and Anal., *19* (1965) 189
6. Bhargava, R.D. and Pande, D. – Acta Mech. *14* (1966), 355
7. Bhargava, R.D. and Pande, D. – Int. J. Non-Linear Mech. *6*, (1971), 327
8. Lal, M. and Pande, D. – J. Mathe. Phys. Sci., *13* (1979), 411
9. Lal, M. – An. şt. Univ. Iaşi, *27* (1981), 195
10. Sokolnikoff, I.S. – *Mathematical Theory of Elasticity*, McGraw Hill, (1953)

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SKEW DIVISION BELTS

BY

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Let E be a finite set with two operations „$\oplus$" and „$\otimes$". We say that E is a *skew division belt* if the following properties hold: (a) $(E, \oplus)$ is an idempotent semigroup, (b) $(E, \otimes)$ is a group, (c) for any $x, y, z \in E$, $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$ and $(y \oplus z) \otimes x = (y \otimes x) \oplus (z \otimes x)$.

We call E a skew division belt by analogy with [1] where for a division belt $(E, \oplus)$ is a commutative idempotent semigroup. The aim of this work is to analyze such an algebra.

Let $\varnothing$ be the *unit element* [1] of $(E, \otimes)$ and for every $x \in E$ let $x^{-1} \in E$ be the inverse of x in $(E, \otimes)$. Let $x \in E$ and $y = x \oplus \varnothing$. It follows that $y \oplus \varnothing = x \oplus \varnothing \oplus \varnothing = x \oplus \varnothing = y$. Then E contains at least an element y such that $y + \varnothing = y$. Similarly, E contains at least an element z for which $\varnothing \oplus z = z$.

Let $F = \{y \in E : y \oplus \varnothing = y\}$ be and $G = \{z \in E : \varnothing \oplus z = z\}$.

Theorem. *$(F, \otimes)$ and $(G, \otimes)$ are normal subgroups of $(E, \otimes)$ and $E = F \otimes G$.*

Proof. Let $y_1, y_2 \in F$. From $y_1 \oplus \varnothing = y_1$ on right-multiplying by y_2 we obtain $(y_1 \otimes y_2) \oplus y_2 = y_1 \otimes y_2$ whence $(y_1 \otimes y_2) \oplus \varnothing = [(y_1 \oplus \varnothing) \otimes y_2] \oplus \varnothing = (y_1 \otimes y_2) \oplus y_2 \oplus \varnothing = (y_1 \otimes y_2) \oplus y_2 = y_1 \otimes y_2$. Hence $y_1 \otimes y_2 \in F$. Let $n = |E|$. Obviously, $y^{-1} = y^{n-1}$ for every $y \in E$.

If $y \in F$ then (by the above proved property) $y^{n-1} \in F$ and therefore $y^{-1} \in F$. Let $x \in E$ and $y \in F$. From $y \oplus \varnothing = y$ we have $x^{-1} \otimes y \otimes x = x^{-1} \otimes (y \oplus \varnothing) \otimes x = [(x^{-1} \otimes y) \oplus x^{-1}] \otimes x = (x^{-1} \otimes y \otimes x) \oplus \varnothing$, i. e., $x^{-1} \otimes y \otimes x \in F$. Hence $(F, \otimes)$ is a normal subgroup of $(E, \otimes)$. Similarly, $(G, \otimes)$ is a normal subgroup of $(E, \otimes)$. Let $y \in F$. Then $y^{-1} \in F$, i.e., $y^{-1} \oplus \varnothing = y^{-1}$ and therefore on left-multiplying by y yields $\varnothing \oplus y = \varnothing$. Similarly, if $z \in G$, then $z \oplus \varnothing = \varnothing$. Hence if $x \in F \cap G$, then $x \oplus \varnothing = x = \varnothing \oplus x = \varnothing$, i.e., $F \cap G = \{\varnothing\}$. Since F and G are normal subgroups for every $f \in F$ and $g \in G$ $f \otimes G = G \otimes f$ and $g \otimes F = F \otimes g$ hold. So there exist unique $\tilde{f} \in F$ and $\tilde{g} \in G$ such that

$$f \otimes g = \tilde{g} \otimes f \text{ and } g \otimes f = \tilde{f} \otimes g. \tag{1}$$

Let now $e \in E$ and $y = e \oplus \varnothing$. Then $y \in F$. On the other hand $y = (\varnothing \oplus e^{-1}) \otimes e$ and $\varnothing \oplus e^{-1}$ belongs to G. Therefore $e = (\varnothing \oplus e^{-1})^{-1} \otimes y \in G \otimes F$ or $e = \tilde{y} \otimes (\varnothing \oplus e^{-1})^{-1} \in F \otimes G$. Hence $E = G \otimes F = F \otimes G$. (Q.E.D.).

Corollary (R.A. Cuninghame-Green [1]). *The only division belt having a finite number of elements is the singleton division belt $\{\varnothing\}$.*