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A NOTE ON SINHA'S MIXED TOPOLOGY FOR A FUNCTION SPACE

BY

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In [2] D. Sinha introduced and studied a new concept of mixed topology for a bitopological function space. Some theorems were proved there, which offer certain relations among the different well known topologies on a function space and the mixed topology, constructed out of these topologies.

The purpose of this note is to prove two results by virtue of which literally all the theorems deduced in [2] will follow as immediate consequences. The results proved here, not only reduce the longer and sometimes complicated proof of the theorems in [2], but also offer a considerable insight into the concept of mixed topology for a function space. We shall follow the notations and terminologies of [2] and refer Kelley [1] for relevant details of the known results of function spaces.

In [2] the definition of mixed topology was given as follows:

Definition 1. *Let E and F be topological spaces and $Z \subset F^E$ be endowed with the compact open topology $\mathcal{C}$ and the point open topology $\mathcal{P}$. Consider the bitopological space $(Z, \mathcal{C}, \mathcal{P})$. Let $\mathcal{A}$ be the family of all $\mathcal{P}$-compact sets. Consider the subspace $(A, \mathcal{C}_A)$ for each $A \in \mathcal{A}$. Let τ be the finest topology on Z such that each inclusion map $i_A : A \subset Z$ is continuous for $A \in \mathcal{A}$. Then τ is called the mixed topology on Z.*

Theorem 2. *If τ is the mixed topology for the bitopological space $(Z, \mathcal{C}, \mathcal{P})$ then $\mathcal{P} \subset \mathcal{C} \subset \tau$.*

Proof. Let $V \in \mathcal{C}$. Then for each $\mathcal{P}$-compact set A we have $i_A^{-1}(V) = A \cap V$ to be $\mathcal{C}_A$-open. Since the mixed topology τ is the finest topology such, that the identity map $i_A : (A, \mathcal{C}_A) \to (Z, \tau)$ is continuous for each $A \in \mathcal{A}$, it now follows that $V \in \tau$ and hence $\mathcal{C} \subset \tau$. $\mathcal{P} \subset \mathcal{C}$ is a known result (see [1]).

Theorem 3. *If τ is the mixed topology for $(Z, \mathcal{C}, \mathcal{P})$ then $\mathcal{C}_A = \tau_A$, for all $A \in \mathcal{A}$.*

Proof. Suppose $B (\subset A)$ be a τ_A open set. Then there is a τ-open set F such that $B = F \cap A$. Since $i_A^{-1}(F) = F \cap A = B$ and since $i_A : (A, \mathcal{C}_A) \to (Z, \tau)$ is continuous it follows that B is $\mathcal{C}_A$-open. Then $\tau_A \subset \mathcal{C}_A$. But by Theorem 2, $\mathcal{C}_A \subset \tau_A$. Hence $\mathcal{C}_A = \tau_A$.

We are now in a position to deduce all the theorems in [2] as immediate corollaries to the above two theorems.

We see that if F is Hausdorff, then $(Z, \mathcal{P})$ is also so (see [1]). Then (Z, τ) is Hausdorff by Theorem 2 so that Theorem 3·1 of [2] is proved. Under the conditions of Theorem 3.2 of [2] we have $(Z, \mathcal{C})$ to be regular [1] and hence $(A, \mathcal{C}_A)$ is regular. Then by Theorem 3 above, Theorem 3.2

of [2] follows. As to Theorem 3.3 of [2] we notice that $\mathcal{P}$ is Hausdorff, since F is Hausdorff. Hence $A \in \mathcal{A} \Rightarrow A$ is $\mathcal{P}$-closed $\Rightarrow A$ is $\mathcal{C}$-closed (since $\mathcal{P} \subset \mathcal{C}) \Rightarrow (A, \mathcal{C}_A)$ is T_4, since $(Z, \mathcal{C})$ is $T_4 \Rightarrow (A, \tau_A)$ is T_4, by Theorem 3. This proves Theorem 3.3 of [2].

Now if F is compact then F^E is compact and hence $(Z, \mathcal{P})$ is compact (see [1]). That is, $Z \in \mathcal{A}$ and hence $\tau = \mathcal{C}$ on Z by Theorem 3 and this establishes Theorem 3.4 of [2]. Here we remark that the condition „F is compact" of Theorem 3. 4 [2] could have been replaced by „$\overline{Z(x)}$ is compact in F, for all $x \in E$".

Again, the hypotheses of Theorem 3.5 of [2] imply that $\mathcal{C}$ is a topology of joint continuity and those of Theorem 3.6 of [2] are sufficient for $\mathcal{C}$ to be jointly continuous on compacta (see [1]). Now since $\mathcal{C} \subset \tau$ both the theorems trivially follow. Now, we know that if Z is an evenly continuous family of functions on a topological space E to a topological space F then both the topologies $\mathcal{P}$ and $\mathcal{C}$ on Z are jointly continuous and also if Z is an equicontinuous family then $(Z, \mathcal{P})$ is jointly continuous [1]. Now since $\mathcal{C}$, $\mathcal{P} \subset \tau$ we obtain Theorems 3.7 and 3.8 of [2] at once.

In § 4. of [2] topology of uniform convergence is discussed. Here a family Z of functions on a topological space E to a uniform space $(F, \mathcal{V})$ has been taken and $\tau(\mathcal{U})$ has been taken to denote the topology of uniform convergence where $\mathcal{U}$ is the uniformity of uniform convergence. τ has been used to stand for the mixed topology on Z, constructed from $\tau(\mathcal{U})$ and $\mathcal{P}$ for the bitopological space $(Z, \tau(\mathcal{U}), \mathcal{P})$ exactly in the same way as done in Definition 1. Now, proceeding similarly as in Theorems 2 and 3 we have $\tau(\mathcal{U}) \subset \tau$ and the topologies $\tau(\mathcal{U})$ and τ coincide on every $\mathcal{P}$-compact subset.

Now we know that if each member of Z is continuous then $\tau(\mathcal{U})$ is jointly, continuous (see [1]). Since $\tau(\mathcal{U}) \subset \tau$ we obtain Theorem 4.1 of [2]. Finally, from the hypotheses of Theorem 4.2 of [2] we immediately have $\tau(\mathcal{U})/\mathcal{R}) = \mathcal{C}$ (see [1]). Since the topologies τ and $\tau(\mathcal{U}/\mathcal{R})$ are identical on every $\mathcal{P}$-compact subset A of Z, we have at once $\tau_A = \mathcal{C}_A$ which is Theorem 4.2 of [2]. We further note here that from the conditions assumed in Theorem 4.2 of [2] it is evident that τ is jointly continuous on compacta if E is Hausdorff or regular. In fact, here $\tau(\mathcal{U}/\mathcal{R}) = \mathcal{C}$ and the members of Z are continuous on compact subsets, of E. Hence $\mathcal{C}$ is jointly continuous on compacta [1]. Since $\mathcal{C} = \tau(\mathcal{U}/\mathcal{R}) \subset \tau$, τ is also jointly continuous on compacta.

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REFERENCES

1. K e l l e y J.L. — *General Topology*, New York (1955)
2. S i n h a D. — *Mixed topology for a function space*, An. st. Univ. Iași, Tomul XXVII, s. I a, 1981, f. 1, p. 43—47

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SYSTEMES D'EQUATIONS AUX DERIVEES PARTIELLES ANALYTIQUES DANS DES ESPACES DE BANACH

PAR

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Dans [4, 5] A. H a i m o v i c i a étudié l'existence d'une intégrale analytique pour des systèmes d'équations aux dérivées partielles ayant la forme

$$
\text{(a)} \quad
\begin{aligned}
\frac{\partial u_i}{\partial x} &= f_i\left(x,\ y,\ z,\ u_p,\ v_q,\ \frac{\partial u_p}{\partial y},\ \frac{\partial u_p}{\partial z},\ \frac{\partial v_q}{\partial x},\ \frac{\partial v_q}{\partial z}\right),\\
\frac{\partial v_j}{\partial y} &= g_j\left(x,\ y,\ z,\ u_p,\ v_q,\ \frac{\partial u_p}{\partial y},\ \frac{\partial u_p}{\partial z},\ \frac{\partial v_q}{\partial x},\ \frac{\partial v_q}{\partial z}\right),
\end{aligned}
$$

$$u_i(0,\ y,\ z)=0,\ v_j(x,\ 0,\ z)=0,\ i,\ p=\overline{1,\ h},\ j,\ q=\overline{1,\ k},$$

où les fonctions f_i et g_j sont analytiques de tous leurs arguments dans un voisinage de $(0, \ldots, 0) \in K^{3(h+k+1)}$.

En partant des résultats obtenus dans ces travaux, nous étudions un système qui généralise dans des espaces de Banach, le système (a). A ce but on utilise la dérivée de Fréchet et la notion de fonction analytique introduite à l'aide des polynômes homogènes dont nous rappelons la définition brièvement dans 1.

1. **Fonctions analytiques dans des espaces de Banach. Définition 1.** *Soit $X_1, \ldots, X_k$ des espaces linéaires normés et Y un espace de Banach sur le même corps Γ (R ou C). Une fonction $P_{i_1 \ldots i_k}(x_1, \ldots, x_k)$ définie sur $X_1 \times \ldots \times X_k$ à valeurs dans Y, est par définition un polynôme homogène de degré i_1 en $x_1, \ldots, i_k$, en x_k s'il existe une fonction $i_1 + \ldots + i_k$-linéaire, $h(x_1^1, \ldots, x_{i_1}^1, \ldots, x_1^k, \ldots, x_{i_k}^k)$, définie sur $X_1^{i_1} \times \ldots \times X_k^{i_k}$ et à valeurs dans Y, symétrique dans les variables $(x_1^1, \ldots, x_{i_1}^1)$; … ; $(x_1^k, \ldots, x_{i_k}^k)$ respectivement de manière que*

$$P_{i_1 \ldots i_k}(x_1, \ldots, x_k) = h(x_1, \ldots, x_1 ;\ \ldots ;\ x_k, \ldots, x_k).$$

La fonction h est appelée la „forme polaire" du polynôme.

Nous allons noter par „0" les zéros des espaces qui vont apparaître par la suite.

Proposition. *La forme polaire d'un polynôme homogène $P_{i_1 \ldots i_k}$ est unique. Si le polynôme $P_{i_1 \ldots i_k}$ est continu en $0 \in X_1 \times \ldots \times X_k$, alors la forme polaire h est continue en $0 \in X_1^{i_1} \times \ldots \times X_k^{i_k}$* [1, 6].