On en déduit que :

$$(16)\qquad H^{n-1}(X;\mathcal{F})\cong\left\{(\tilde{g}_{i_0\cdots i_{n-1}})\,\middle|\,\sum_{p=0}^{n}(-1)^p f_{01\cdots n}^{0\cdots\hat{p}\cdots n}(g_{0\cdots\hat{p}\cdots n})=0\right\}$$

$$(17)\qquad H^n(X;\mathcal{F})\cong\operatorname{Im}\delta^{n-1}=\frac{G_{01\cdots n}}{G_0^n+\ldots+G_n^n}.$$

Les considérations précédentes démontrent le résultat suivant :

Théorème 2.1. *Les groupes de cohomologie d'un simplexe euclidien X de dimension n, à valeurs dans un faisceau cellulaire simple $\mathcal{F}$, sont donnés par les formules*

$$(18)\qquad H^q(X;\mathcal{F})=\left\{(\tilde{g}_{i_0\cdots i_q})\in\bigoplus_{0\le i_0<\cdots<i_q\le n}\frac{G_{i_0\cdots i_q}}{G_0^q+\ldots+G_q^q}\,\middle|\,\sum_{p=0}^{q+1}(-1)^p f_{i_0\cdots\cdots\cdots i_{q+1}}^{i_0\cdots\hat{i}_p\cdots i_{q+1}}(g_{i_0\cdots\hat{i}_p\cdots i_{q+1}})=0\right\}$$

pour $q=0,1,\ldots,n$ et ils sont nuls pour tout $q>n$.

Remarque 2.2. Si les homomorphismes $f_{i_0\cdots i_q}^{i_0\cdots\hat{i}_p\cdots i_q}$ sont injectifs (situation que l'on rencontre dans [2] et [3]), les formules (18) deviennent beaucoup plus simples : $H^0(X;\mathcal{F})=\bigcap_{i=0}^{n}G_i$;

$$H^n(X;\mathcal{F})=\frac{G_{01\cdots n}}{G_{12\cdots n}+G_{02\cdots n}+\ldots+G_{01\cdots n-1}},$$

et dans $H^q(X;\mathcal{F})$ (pour $q=1,2,\ldots,n-1$) parmi les C_{n+1}^{q+2} relations de la forme : $\sum_{p=0}^{q+1}(-1)^p g_{i_0\cdots\hat{i}_p\cdots i_{q+1}}=0$ seulement C_n^{q+1} relations sont indépendantes; il suit que parmi les C_{n+1}^{q+1} facteurs de $H^q(X;\mathcal{F})$ on considère uniquement $C_n^q(=(C_{n+1}^{q+1}-C_n^{q+1})$ facteurs qui remplissent les C_n^{q+1} relations indépendantes.

Pour les simplexes euclidiens de dimension 1, 2, et 3 on retrouve les résultats obtenus dans [3].

BIBLIOGRAPHIE

1. B r e d o n G. — *Sheaf theory*, Mc. Graw-Hill Book Company 1967
2. C o s t i n e s c u C.N. — *Şirul spectral Atiyah-Hirzebruch în K_G-teorie şi aplicaţii*, St. cerc. mat., 27,4 (1975) 425—442
3. C o s t i n e s c u C.N. — *Contribuţii în topologia algebrică. Şirul spectral Atiyah-Hirzebruch şi K-teorie*, Thèse de doctorat, 1982, Bucureşti
4. D o g a r u O. — *Fascicule celular simple*, St. cerc. mat., 27,5 (1975), 535—545
5. G o d e m e n t R. — *Topologie algébrique et théorie des faisceaux*, Hermann, Paris, 1958
6. M i r o n R., P o p I. — *Topologie algebrică*, Ed. Acad. R.S.R. Bucureşti, 1974
7. S p a n i e r E. — *Algebraic topology*, Mc Graw-Hill New York 1966

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SEMI-INVARIANT SUBMANIFOLDS OF $W_1\oplus W_2\oplus W_4$-MANIFOLDS

BY

DOMINGO CHINEA

In [2] D. E. B l a i r, G. D. L u d d e n and K. Y a n o gave conditions for a semiinvariant submanifold of a complex manifold to be normal, and in [3] we studied the case of semi-invariant submanifolds of a quasi Kaehlerian manifold.

The purpose of this paper is to study semi-invariant submanifolds of manifolds in the classes. W_4, $W_1\oplus W_4$, $W_2\oplus W_4$ and $W_1\oplus W_2\oplus W_4$, [5]. First of all we recall in §1 the definition and properties of a semiinvariant submanifold of an almost complex manifold and give some relations among different elements associated to the structures defined in the manifold and in the submanifold. In §2 we study different types of almost contact metric structures which are induced in a semi-invariant submanifold by the different types of almost Hermitian structures above. For that, we use the G r a y-H e r v e l l a's classification of almost Hermitian manifolds [5] and the classification obtained by J. A. O u b i ñ a [7] for almost contact structures.

I wish to express my hearty thanks to L. A. C o r d e r o and J. A. O u b i ñ a for several comments useful in the preparation of this paper.

1. Preliminaries. A $(2r+1)$-dimensional real differentiable manifold M of classe C^∞ is said to have a (φ, ξ, η)-structure or an almost contact structure if it admits a field φ of endomorphisms of the tangent spaces, ia vector field ξ, and a 1-form η satisfying

$$(1.1)\qquad \eta(\xi)=1, \qquad\qquad (1.2)\qquad \varphi^2=-I+\eta\otimes\xi,$$

where I denotes the identity transformation [8]. Then $\varphi\cdot\xi=0$ and $\eta\cdot\varphi=0$; moreover, the endomorphisms φ has rank $2r$, [1].

If a manifold M with a (φ, ξ, η)-structure admits a Riemannian metric g such that

$$(1.3)\qquad g(\varphi X, \varphi Y)=g(X, Y)-\eta(X)\,\eta(Y),$$

where $X, Y\in\chi M$, then M is said to have a (φ, ξ, η, g)-structure or an almost contact metric structure and g is called a compatible metric [8]. An immediate consequence is $\eta(X)=g(X, \xi)$. The 2-form Φ on M defined by

$$(1.4)\qquad \Phi(X, Y)=g(X, \varphi Y)$$

s called the fundamental 2-form of the almost contact metric structure.

We recall the definitions of the types of almost contact metric structures here involved, (see [7]):
(φ, ξ, η, g) *is trans-Sasakian* (tS) if

$$(1.5)\qquad (\nabla_X\Phi)(Y, Z) = -(1/2r)\big(g(X, Y)\,\delta\Phi(Z) - g(X, Z)\,\delta\Phi(Y) + g(X, \varphi Y)\eta(Z)\,\delta\eta - g(X, \varphi Z)\eta(Y)\,\delta\eta\big)$$

and

$$(1.6)\qquad \delta\Phi(\varphi X) = 0\,;$$

(φ, ξ, η, g) *is nearly-trans-Sasakian* (ntS) *if*

$$(1.7)\qquad (\nabla_X\Phi)(X, Y) = -[1/(2r)]\,\big(g(X, X)\,\delta\Phi(Y) - g(X, Y)\,\delta\Phi(X) + g(\varphi X, Y)\eta(X)\,\delta\eta\big).$$

and

$$(1.8)\qquad (\nabla_X\eta)\,Y = -[1/(2r)]\,\big(g(\varphi X, \varphi Y)\,\delta\eta + g(\varphi X, Y)\,\delta\Phi(\xi)\big)\,;$$

(φ, ξ, η, g) *is almost-trans-Sasakian* (atS) *if*

$$(1.9)\qquad d\Phi = \Phi\wedge\omega, \quad \text{where } \omega \text{ is the 1-form on } M \text{ given by } \omega(X) = (1/r)\big(\delta\Phi(\varphi X) - \eta(X)\,\delta\eta\big).$$

and

$$(1.10)\qquad d\eta = [1/(2r)]\,\big(\delta\Phi(\xi)\,\Phi - 2\eta\wedge\varphi(\delta\Phi)\big)\,;$$

(φ, ξ, η, g) *is quasi-trans-Sasakian* (qtS) *if*

$$(1.11)\qquad \begin{aligned}(\nabla_X\Phi)(Y, Z) + (\nabla_{\varphi X}\Phi)(\varphi Y, Z) + \eta(Y)(\nabla_{\varphi X}\eta)Z = {}&-(1/r)\,\big(g(X, Y)\,\delta\Phi(Z) - g(X, Z)\,\delta\Phi(Y) - g(X, \varphi Y)\,(\delta\Phi(\varphi Z) - \\ &-\eta(Z)\,\delta\eta) + g(X, \varphi Z)\,(\delta\Phi(\varphi Y) - \eta(Y)\,\delta\eta)\big),\end{aligned}$$

for all $X, Y, ZZ \in (M)$.

The relations among these structures are represented in the following diagram (where $\to$ denotes strict inclusion), [7].

$$\begin{tikzcd} & ntS \arrow[dr] & \\ tS \arrow[ur] \arrow[dr] & & qtS \\ & atS \arrow[ur] & \end{tikzcd}$$

Let M be a submanifold immersed in an almost complex manifold $\tilde M$ with almost complex structure J.

Definition 1.1. *Let $\tilde U$ be an non-vanishing vector field on $\tilde M$ such that*

(1.12) $JX = \varphi X - \eta(X)\,\tilde U$, $X \in \chi(M)$, $\xi = J\tilde U_{|M} \in \chi(M)$, *$\tilde U$ is never tangent to M, where φX is the tangential component to M of JX. Then we say that M is a semi-invariant submanifold of $\tilde M$ with respect to $\tilde U$.*

It is well known that any semi-invariant submanifold M of an almost complex manifold $\tilde M$ with induced structure (φ, ξ, η) is an almost contact manifold.

Furthermore, if $\tilde M$ is an almost Hermitian manifold with structure $(J, \tilde g)$ and $\tilde U$ is a unit vector field normal to M then (φ, ξ, η, g) is an almost contact metric structure on M, where g is the induced metric on M, [2]. Hereafter, $\tilde M$ will be an almost Hermitian manifold and M a semi-invariant submanifold of $\tilde M$.

Comparing the fundamental 2-forms Φ and $\tilde\Phi$ of the structures in M and $\tilde M$, respectively, we have

$$(1.13)\quad \Phi(X, Y) = \tilde\Phi(X, Y), \qquad (1.14)\quad d\Phi(X, Y, Z) = d\tilde\Phi(X, Y, Z),$$

for all $X, Y, Z \in \chi(M)$.

Let $\mathcal{L}$, $\tilde\nabla$, ∇ denote the Lie differentiation and the Riemannian connections of the metrics $\tilde g$ and g, respectively. Then, for $\tilde X, \tilde Y \in \chi(\tilde M)$, $X, Y \in \chi(M)$, we have

$$(1.15)\qquad (\mathcal{L}_{\tilde U}\tilde g)(\tilde X, \tilde Y) = \tilde g(\tilde\nabla_{\tilde X}\tilde U, \tilde Y) + \tilde g(\tilde X, \tilde\nabla_{\tilde Y}\tilde U),$$

$$(1.16)\qquad (\mathcal{L}_{\tilde U}\tilde g)(X, Y) = 2\tilde g(\tilde\nabla_X\tilde U, Y) = 2\tilde g(\tilde\nabla_Y\tilde U, X),$$

$$(1.17)\qquad (\mathcal{L}_{\tilde U}J)\,J\tilde X = -J(\mathcal{L}_{\tilde U}J)\,\tilde X,$$

$$(1.18)\qquad (\mathcal{L}_{\tilde U}\tilde g)(J\tilde X, J\tilde Y) = (\mathcal{L}_{\tilde U}\tilde g)(\tilde X, \tilde Y) - \tilde g((\mathcal{L}_{\tilde U}J)\,\tilde X, J\tilde Y) - \tilde g(J\tilde X, (\mathcal{L}_{\tilde U}J)\,\tilde Y),$$

$$(1.19)\qquad \begin{aligned}(\mathcal{L}_{\tilde U}\tilde g)(\varphi X, \varphi Y) = {}&(\mathcal{L}_{\tilde U}\tilde g)(X, Y) - \tilde g((\mathcal{L}_{\tilde U}J)\,X, JY) - \tilde g(JX, (\mathcal{L}_{\tilde U}J)\,Y) + \\ &+\eta(X)\,(\mathcal{L}_{\tilde U}\tilde g)(\tilde U, \varphi Y) + \eta(Y)\,(\mathcal{L}_{\tilde U}\tilde g)(\varphi X, \tilde U),\end{aligned}$$

$$(1.20)\qquad (\mathcal{L}_{\tilde U}\tilde\Phi)(\tilde X, \tilde Y) = (\mathcal{L}_{\tilde U}\tilde g)(\tilde X, J\tilde Y) + \tilde g(\tilde X, (\mathcal{L}_{\tilde U}J)\,\tilde Y).$$

Also, we obtain

Proposition 1.1. *For all $X, Y, Z \in \chi(M)$,*

$$(1.21)\qquad (\tilde\nabla_X\tilde\Phi)(Y, Z) = (\nabla_X\Phi)(Y, Z) - (1/2)\,\big(\eta(Z)\,(\mathcal{L}_{\tilde U}\tilde g)(X, Y) - \eta(Y)(\mathcal{L}_{\tilde U}\tilde g)(X, Z)\big),$$

$$(1.22)\qquad (\tilde\nabla_{\tilde U}\tilde\Phi)(X, Y) = (1/2)\,\big[\tilde g((\mathcal{L}_{\tilde U}J)\,Y, X) - \tilde g((\mathcal{L}_{\tilde U}J)\,X, Y) + \eta(X)\,(\mathcal{L}_{\tilde U}\tilde g)(\tilde U, Y) - \eta(Y)\,(\mathcal{L}_{\tilde U}\tilde g)(\tilde U, X)\big],$$

$$(1.23)\qquad (\tilde\nabla_{\tilde U}\tilde\Phi)(\tilde U, X) = (1/2)\big[-(\mathcal{L}_{\tilde U}\tilde g)(\tilde U, \varphi X) - \tilde g((\mathcal{L}_{\tilde U}J)\tilde U, X) + \tilde g(\tilde U, (\mathcal{L}_{\tilde U}J)X)\big],$$

$$(\tilde{\nabla}_{\tilde{U}}\tilde{\Phi})(JX, Y)=(1/2)\,[\tilde{g}((\mathcal{L}_{\tilde{U}}J)Y, JX)-\tilde{g}((\mathcal{L}_{\tilde{U}}J)JX, Y)+ \tag{1.24}$$
$$\eta(X)(\mathcal{L}_{\tilde{U}}\tilde{g})(\tilde{U}, \varphi Y)-\eta(Y)(\mathcal{L}_{\tilde{U}}\tilde{g})(\tilde{U}, \varphi X)].$$

$$(\tilde{\nabla}_X\tilde{\Phi})(\tilde{U}, Y)=-(\nabla_X\eta)Y-(1/2)(\mathcal{L}_{\tilde{U}}\tilde{g})(X, \varphi Y). \tag{1.25}$$

Proof. (1.21) follows from (1.12), (1.13) and (1.16). From the relations (1.12) and (1.16), we deduce (1.22). (1.23) is proved in a similar way. The expression (1.24) follows from (1.12), (1.15), (1.16), (1.17) and (1.18). Finally, from (1.12), (1.16) and the relation $(\nabla_X\eta)Y=g(Y, \nabla_X\xi)$, we obtain (1.25).

Now, we shall relate the coderivative δ in M with the coderivative $\tilde{\delta}$ in $\tilde{M}$.

If we take a φ-basis $\{X_1, \ldots, X_r, \varphi X_1, \ldots, \varphi X_r, \xi\}$ defined on an open subset of M, with $\dim M=2r+1$, and by completing it to obtain a J-basis $\{\tilde{X}_1, \ldots, \tilde{X}_n, J\tilde{X}_1, \ldots, J\tilde{X}_n, \tilde{U}, \xi\}$, with $\dim \tilde{M}=2n+2$ and $\tilde{X}_i=X_i$, $J\tilde{X}_i= =\varphi X_i$ $i=1, \ldots, r$, we have

Proposition 1.2. *For any $X\in\chi(M)$,*

$$\tilde{\delta}\tilde{\Phi}(X)=\delta\Phi(X)-\sum_{j=r+1}^{n}((\tilde{\nabla}_{\tilde{X}_j}\tilde{\Phi})(\tilde{X}_j, X)+(\tilde{\nabla}_{J\tilde{X}_j}\tilde{\Phi})(J\tilde{X}_j, X))+$$
$$+(1/2)\,\eta(X)\,[\sum_{i=1}^{r}((\mathcal{L}_{\tilde{U}}\tilde{g})(X_i, X_i)+(\mathcal{L}_{\tilde{U}}\tilde{g})(\varphi X_i, \varphi X_i))+ \tag{1.26}$$
$$+(\mathcal{L}_{\tilde{U}}\tilde{g})(\xi, \xi)]+(\mathcal{L}_{\tilde{U}}\tilde{g})(\tilde{U}, \varphi X)+g((\mathcal{L}_{\tilde{U}}J)\tilde{U}, X).$$

Proof. Let $X\in\chi(M)$. The coderivative of the fundamental 2-form $\tilde{\Phi}$ with respect to a J-basis is expressed by

$$\tilde{\delta}\tilde{\Phi}(X)=-\sum_{i=1}^{n}((\tilde{\nabla}_{\tilde{X}_i}\tilde{\Phi})(\tilde{X}_i, X)+(\tilde{\nabla}_{J\tilde{X}_i}\tilde{\Phi})(J\tilde{X}_i, X))-(\tilde{\nabla}_{\tilde{U}}\tilde{\Phi})(\tilde{U}, X)-(\tilde{\nabla}_{\xi}\tilde{\Phi})(\xi, X).$$

Analogously, the coderivative of the fundamental 2-form Φ with respect to a φ-basis is expressed by

$$\delta\Phi(X)=-\sum_{i=1}^{r}((\nabla_{X_i}\Phi)(X_i, X)+(\nabla_{\varphi X_i}\Phi)(\varphi X_i, X))-(\nabla_{\xi}\Phi)(\xi, X).$$

Using these expressions, (1.17), (1.19), (1.21) and (1.23), we obtain (1.26).

2. Semi-invariant Submanifolds of $W_1\oplus W_2\oplus W_4$-Manifolds. We shall divide this paragraph, according to the type of almost Hermitian structure considered in $\tilde{M}$, in different subparagraphs.

2.1. Semi-invariant submanifolds of W_4-manifold

Theorem 2.1. *If $\tilde{M}$ is a W_4-manifold and $\tilde{U}$ is analytic and Killing, then (φ, ξ, η, g) is a trans-Sasakian structure on M.*

Proof. We consider a J-basis on $\tilde{M}$ obtained as in Proposition 1.2. Since $\tilde{M}$ is a W_4-manifold, we have

$$(\tilde{\nabla}_{\tilde{X}_j}\tilde{\Phi})(\tilde{X}_j, X)=(\tilde{\nabla}_{J\tilde{X}_j}\tilde{\Phi})(J\tilde{X}_j, X)=-[1/(2n)]\,\tilde{\delta}\tilde{\Phi}(X),$$

for all $X\in\chi(M)$ and $j=r+1, \ldots, n$. Thus, from (1.26) and since $\tilde{U}$ is analytic and Killing, it follows

a) $$\tilde{\delta}\tilde{\Phi}(X)=(n/r)\,\delta\Phi(X).$$

On the other hand,

$$(\tilde{\nabla}_{X_i}\tilde{\Phi})(J\tilde{U}, JX_i)=-(\tilde{\nabla}_{JX_i}\tilde{\Phi})(J\tilde{U}, X_i)=-[1/(2n)]\,\tilde{\delta}\tilde{\Phi}(\tilde{U}),\ \text{for } i=1, \ldots, r.$$

Thus,

$$\delta\eta=-\sum_{i=1}^{r}((\nabla_{X_i}\eta)X_i+(\nabla_{\varphi X_i}\eta)\varphi X_i)=-\sum_{i=1}^{r}((\nabla_{X_i}\Phi)(\xi, \varphi X_i)+$$
$$+(\nabla_{\varphi X_i}\Phi)(\xi, \varphi^2X_i))=-\sum_{i=1}^{r}((\tilde{\nabla}_{X_i}\tilde{\Phi})(J\tilde{U}, JX_i)-(\tilde{\nabla}_{JX_i}\tilde{\Phi})(J\tilde{U}, X_i))=(r/n)\tilde{\delta}\tilde{\Phi}(\tilde{U}),$$

that is,

b) $$\delta\eta=(r/n)\,\tilde{\delta}\tilde{\Phi}(\tilde{U}).$$

Also, from (1.23) and since $\tilde{M}$ is a W_4-manifold, we obtain

$$0=(\tilde{\nabla}_{\tilde{U}}\tilde{\Phi})(\tilde{U}, X)=-[1/(2n)]\,(\tilde{\delta}\tilde{\Phi}(X)+\tilde{g}(\tilde{U}, JX)\,\tilde{\delta}\tilde{\Phi}(\xi)),$$

and thus,

c) $$\tilde{\delta}\tilde{\Phi}(\varphi X)=0,\ X\in\chi(M).$$

Therefore, from a), b), c), (1.12), (1.21) and since $\tilde{M}$ is a W_4-manifold and $\tilde{U}$ analytic and Killing, we have (1.5) and (1.6), and so M is a trans-Sasakian manifold.

2.2. Semiinvariant submanifolds of a $W_1\oplus W_2\oplus W_4$-manifold.

Theorem 2.2. *If $\tilde{M}$ is a $W_1\oplus W_2\oplus W_4$-manifold and $\tilde{U}$ is analytic and Killing, then (φ, ξ, η, g) is a quasi-trans-Sasakian structure on M.*

Proof. In an analogous way to Theorem 2.1, we obtain

a) $\tilde{\delta}\tilde{\Phi}(X)=(n/r)\,\delta\Phi(X)$, $X\in\chi(M)$ b) $\delta\eta=(r/n)\,\tilde{\delta}\tilde{\Phi}(\tilde{U})$.

Thus, from a), b), (1.12), (1.21), (1.24), (1.26), since $\tilde{M}$ is a $W_1\oplus W_2\oplus W_4$-manifold and $\tilde{U}$ analytic and Killing, we have (1.11), and so M is a quasi-trans-Sasakian manifold.

2.3. Semi-invariant Submanifolds of a $W_1\oplus W_4$-Manifold.

Theorem 2.3. If *$\tilde{M}$ is a $W_1\oplus W_4$-manifold and $\tilde{U}$ is analytic and Killing then (φ, ξ, η, g) is a nearly-trans-Sasakian structure on M.*

Proof. Since any $W_1 \oplus W_4$-manifold is a $W_1 \oplus W_2 \oplus W_4$-manifold [5], from Theorem 2.2 it follows

a) $\tilde{\delta}\tilde{\Phi}(X) = (n/r)\, \delta\Phi(X),\ X \in \chi(M)$. b) $\delta\eta = (r/n)\, \tilde{\delta}\tilde{\Phi}(\tilde{U})$.

Now, using (1.23) and since $\tilde{U}$ is analytic and Killing, we deduce $0 = (\tilde{\nabla}_{\tilde{U}}\tilde{\Phi})(\tilde{U}, X) = -[1/(2n)]\, (\tilde{\delta}\tilde{\Phi}(X) - \eta(X)\, \tilde{\delta}\tilde{\Phi}(\xi))$. From this relation and a), we have

c) $\delta\Phi(\varphi X) = 0, \quad U \in \chi(M)$.

Thus, from a), b), c), (1.12), (1.21), since $\tilde{U}$ is Killing and $\tilde{M}$ is a $W_1 \oplus W_4$-manifold, we have (1.7).

On the other hand, since $\tilde{M}$ is a $W_1 \oplus W_2 \oplus W_4$-manifold and a G_1-manifold [5], using (1.12), (1.22), (1.25), a), b), c), we have (1.8). Thus, we obtain that M is a nearly-trans-Sasakian manifold.

2.4. Semi-invariant Submanifolds of a $W_2 \oplus W_4$-Manifold.

Theorem 2.4. *If M is a $W_2 \oplus W_4$-manifold and if $\tilde{U}$ is analytic and Killing, then (φ, ξ, η, g) is an almost-trans-Sasakian structure on M.*

Proof. Since any $W_2 \oplus W_4$-manifold is a $W_1 \oplus W_2 \oplus W_4$-manifold [5], from Theorem 2.2 we have

a) $\tilde{\delta}\tilde{\Phi}(X) = (n/r)\, \delta\Phi(X),\ X \in \chi(M)$. b) $\delta\eta = (r/n)\, \tilde{\delta}\tilde{\Phi}(\tilde{U})$.

Thus, from a), b), (1.12), (1.13), (1.14) and since $\tilde{M}$ is a $W_2 \oplus W_4$-manifold, we have (1.9).

On the other hand, $3d\tilde{\Phi}(\tilde{U}, X, Y) = 2d\eta(X, Y) + (\mathcal{L}_{\tilde{U}}\tilde{\Phi})(X, Y)$ and since $\tilde{U}$ is analytic and Killing, from a), b), (1.12), (1.13), (1.20), we obtain (1.10). Therefore M is an almost-trans-Sasakian manifold.

REFERENCES

1. Blair D.E. — *Contact manifolds in Riemannian geometry*. Lectures Notes in Math. 509 (1976), Springer, Berlin-Heidelberg-New York
2. Blair D.E., Ludden, G.D. and Yano, K. — *Semi-invariant immersions*. Kodai Math. Sem. Rep. 27 (1976) 313—319
3. Chinea D. — *Semi-invariant submanifolds of quasi Kaehlerian manifolds*. To appear
4. Gray, A. — *Some examples of almost Hermitian manifolds*. Illinois J. Math. 10 (1966) 353—366
5. Gray, A. and Hervella, L.M. — *The sixteen classes of almost Hermitian manifolds and their linear invariants*. Ann. Mat. Pura ed Appl. (IV) Vol. CXXIII (1980) 35—58
6. Kobayashi, H. and Nomizu, K. — *Foundations of differential geometry*, vol. I and II, Inters. Pub. (1963), (1969)
7. Oubiña, J.A. — *A classification for almost contact structures*. To appear
8. Sasaki, S. — *On differentiable manifolds with certain structures which are closely related to almost contact structure I*. Tohoku Math. J. 12 (1960), 459—476

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ON GENERALIZED FINSLER SPACES

BY

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Let M be a differentiable manifold and TM its tangent bundle. A coordinate system (x^i) in M induces a canonical coordinate system (x^i, y^i) in TM. Let $L(x, y)$ be a positive-valued differentiable function defined in $TM - \{0\}$, and we assume that L is positively homogeneous of degree 1, and the matrix (a_{ij}) is regular, where $a_{ij} = (1/2)\partial^2 L^2/\partial y^i \partial y^j$. If the length s of a differentiable curve $x^i = x^i(t)$ in M is measured by $s = \int L(x, dx/dt)dt$, we call (M, L) a *Finsler space*. Then a_{ij} is called the *Finsler metric*.

A Finsler metric is sometimes adopted as a basic concept in the theoretical physics. However, the fact that the metric is provided from a function L may be not always desirable for physicists. For example in his paper [2], H. Ishikawa modified the Finsler metric a_{ij} in a Finsler space (M, L), and introduced the metric $g_{ij} = a_{ij} - 2l_i l_j$, where $l_i = \partial L/\partial y^i$. If (a_{ij}) is positive-definite, any vector $y(\neq 0)$ is time-like: $g_{ij}y^i y^j = -a_{ij}y^i y^j < 0$. It seems that this is the reason why he had been interested in such a metric. It has also the another interesting property that the modification is projective. But it is not a Finsler metric.

Another example is given by a conformal change of a Finsler metric $a_{ij}: g_{ij} = e^{2\rho(x,y)}a_{ij}$ (R. Miron - M. Hashiguchi [7]). g_{ij} is not a Finsler metric, if ρ depends on direction. Interestingly, the tensor $2(a_{ij} - 2l_i l_j)/L^2$ is expressed as $\partial^2 \log L^2/\partial y^i \partial y^j$, and appears in the conformal theory of Finsler metrics as an important conformal invariant (M. Hashiguchi [1]).

In his paper [6], R. Miron generalized the notion of Finsler metric to the metric g_{ij} given as a differentiable tensor field of type (0,2) which is symmetric and non-degenerate, and he gave an interesting theory. We shall call this g_{ij} a *generalized Finsler metric* and (M, g_{ij}) a *generalized Finsler space*.

In this note we shall consider such a generalized Finsler space from the standpoint of variational calculus. Then we hope to refer to a significance of axioms for Finsler connections.

The terminology and notations are referred to M. Matsumoto [5].

1. We shall first investigate what conditions should be imposed on a generalized Finsler space (M, g_{ij}). Putting $L = (g_{ij}y^j y^i)^{1/2}$, the length s