Proof. Let M' be a leaf of the disiribution $D^{\perp}\oplus\{\xi\}$ and denote by ∇' the Levi-Civita connection on M'. Then the equation of Gauss for the immersion of M' in M is given by

$$\nabla_X Y=\nabla'_X Y+h'(X, Y), \text{ for any } X, Y\in\Gamma(D^{\perp}\oplus\{\xi\}), \tag{3.4}$$

where h' is the second fundamental form of the immersion of M' in M. Now, we take $Y\in\Gamma(D_a)$. Then there exists $V\in\Gamma(\bar{D})$ such that $Y=\varphi_a V$ and by using (1.3), (1.4), (2.7), (2.9) and (3.4) we obtain

$$g(h'(X, Y), Z)=g(h(X, \varphi_a Z), V)=g(B_a h(X, \varphi_a Z), Y) \tag{3.5}$$

for all $X\in\Gamma(D^{\perp})$ and $Z\in\Gamma(D)$. Taking account that $D^{\perp}\oplus\{\xi\}$ is supposed integrable, by (2.34) and (3.5) we get

$$g(h'(X, Y), Z)=0, \tag{3.6}$$

for any $X\in\Gamma(D^{\perp})$, $Y\in\Gamma(D_a)$ and $Z\in\Gamma(D)$.
On the other hand, by (1.5), (2.7) and (3.4) we obtain

$$g(h'(X, \xi_a), Z)=0, \tag{3.7}$$

for any $X\in\Gamma(D^{\perp}\oplus\{\xi\})$ and $Z\in\Gamma(D)$. Thus by (3.6) and (3.7) we infer that M' is totally geodesic immersed in M.
Next, we denote by h^* the second fundamental form for the immersion of M' in N. Then we have

$$h^*(X, Y)=h(X, Y)+h'(X, Y), \tag{3.8}$$

for any $X, Y\in\Gamma(D^{\perp}\oplus\{\xi\})$. We take a nonzero vector field $X\in\Gamma(D_a)$ and $Y=\xi_a$ in (3.8). Then, taking acoount that M' is totally geodesic immersed in M, by (2.15) we get $h^*(X, \xi_a)=h(X, \xi_a)=-\varphi_a X\neq 0$. Hence M' is not totally geodesic immersed in N and the proof of Theorem 3.2 is complete.

REFERENCES

1. B e j a n c u, A. — *Generic submanifolds of a manifold with a Sasakian 3-structure*, to appear in Math. Rep. Toyama Univ., 1985, vol. 8.
2. B l a i r, D.E. — *Contact manifolds in Riemannian geometry*, Lecture Notes in Mathematics, 509 (1976), Springer-Verlag.
3. K u o, Y.Y. — *On almost contact 3-structure*, Tohoku Math. J., 22 (1970), 325—332.

Received 20. IV. 1984

Polytechnic Institute of Iasi
Department of Mathematics,
R.S. Romania

Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXXI, s. I-a. Matematică, fasc. 1, 1985

ON NORMAL CURVATURE OF A HYPERSURFACE OF FINSLER SPACES

BY

CHŌKŌ HIBATA, SHIDEO SHIMADA and MASAYOSHI AZUMA

Introduction. A systematic study of geometry of hypersurfaces in Finsler spaces was recently given by M a t s u m o t o [5]. He gave perfect proofs of H a i m o v i c i - K i k u c h i - R a p c s a k's results and, in particular, concluded: There exists a hyperplane of the 1-st kind (with respect to $C\Gamma$) which is characterized by the equation $H_\alpha=0$ at each hypersurface element of a Finsler space $F^n(n>2)$, if and only if F^n is projectively flat.

The main purpose of this paper is to give an answer of Professor Dr. Aurel B e j a n c u 's conjecture. A totally geodesic hypersurface of a projectively flat Finsler space is also projectively flat*. This will be proved in §2. §1 is devoted to a study of a second fundamental tensor $\Omega_{\alpha\beta}$ defined by B e r w a l d, and we consider the umbilical hypersurface with respect to the Cartan connection.

The terminologies and notations in the present paper are referred to M. M a t s u m o t o's paper [5] unless otherwise stated.

We wish to express our sincere gratitude to Professor Dr. M. M a t s u m o t o for the invaluable suggestions and encouragement.

§1. **Berwald's second fundamental tensor**. We shall consider an n-dimensional Finsler space $F^n=(M^n, L(x, y))$, reffered to a local coordinate system (x^i), whose fundamental function $L(x, y)$ is (1) p-homogeneous in $y=(y^i)$. The metric tensor of the F^n is defined by

$$g_{ij}(x, y)=\partial^2(L^2/2)/\partial y^i\partial y^j.$$

A hypersurface M^{n-1} of the underlying differentiable manifold M^n may be parametrically represented by the equations

$$x^i=x^i(u^\alpha), \quad \alpha=1,\dots, n-1,$$

where u^α are Gaussian coordinates on M^{n-1}. We assume the matrix whose elements are

$$B^i_\alpha(u)=\partial x^i/\partial u^\alpha$$

This research is supported by Grant-in Aid for Scientific Research (No. 59540003), Ministry of Education of Japan, 1984.

* This conjecture was proposed during the Romanian-Japanese Colloquium on Finsler Geometry in Romania, 1984.

is of rank $n-1$. The following notations are employed

$$B^i_{\alpha\beta}(u)=\partial^2 x^i/\partial u^\alpha \partial u^\beta, \quad B^{ij}_{\alpha\beta}=B^i_\alpha B^j_\beta.$$

To introduce a Finsler structure in M^{n-1}, the supporting element y^i at a point (u^α) of M^{n-1} is assumed to be tangential to M^{n-1}, so that we may write $y^i=B^i_\alpha v^\alpha$, and then v^α is thought of as the supporting element of M^{n-1} at the point (u^α). Denoting y^i by $y^i(u, v)$, the function $L(u, v):=L(x(u), y(u, v))$ gives rise to a Finsler metric of M^{n-1}. Thus we get an $(n-1)$-dimensional Finsler space $F^{n-1}=(M^{n-1}, L(u, v))$. At each point (u^α) of F^{n-1}, a unit normal vector $B^i(u, v)$ is defined by

$$g_{ij}B^i_\alpha B^j=0, \qquad g_{ij}B^i B^j=1.$$

Let (B^α_i, B_i) be the inverse matrix of (B^i_α, B^i) ; we have

$$B^i_\alpha B^\beta_i=\delta^\beta_\alpha, \quad B^i B^\alpha_i=0, \quad B^i_\alpha B^\alpha_j+B^i B_j=\delta^i_j.$$

We now introduce important tensors from Cartan's C-tensor g_{ijk}

$$M_{\alpha\beta}=g_{ijk}B^{ij}_{\alpha\beta}B^k, \quad M_\alpha=g_{ijk}B^i_\alpha B^j B^k, \quad M=g_{ijk}B^i B^j B^k. \tag{1.1}$$

Let F^n be a Finsler space equipped with a Finsler connection $F\Gamma=(F^i_{jk}, N^i_j, C^i_{jk})$. Putting the induced Finsler connection $IF\Gamma=(F^\alpha_{\beta\gamma}, N^\alpha_\beta, C^\alpha_{\beta\gamma})$ on a hypersurface F^{n-1} of F^n, we have

$$B^i_{\beta\gamma}+B^j_\beta(F^i_{jk}B^k_\gamma+C^i_{jk}B^k H_\gamma)=F^\alpha_{\beta\gamma}B^i_\alpha+H_{\beta\gamma}B^i, \tag{1.2}$$

where $H_{\beta\gamma}$ (resp. H_α) is called *the second fundamental h-tensor* (resp. *the normal curvature vector*) [5], and are respectively given by

$$\begin{aligned} &H_{\beta\gamma}:=B_i\{B^i_{\beta\gamma}+B^j_\beta(F^i_{jk}B^k_\gamma+C^i_{jk}B^k H_\gamma)\}, \\ &H_\alpha:=B_i(B^i_{0\alpha}+N^i_j B^j_\alpha), \quad B^i_{0\alpha}:=B^i_{\beta\alpha}v^\beta. \end{aligned} \tag{1.3}$$

We now treat the Cartan connection $C\Gamma=(F^i_{jk}, N^i_j, C^i_{jk})$ of F^n and indicate the quantities with respect to the induced Cartan connection by putting „c" on them. Then the second fundamental h-tensor $\overset{c}{H}_{\beta\gamma}$ of (1.3), given by

$$\overset{c}{H}_{\beta\gamma}=B_i(B^i_{\beta\gamma}+B^{jk}_{\beta\gamma}F^i_{jk})+M_\beta H_\gamma \tag{1.4}$$

is generally not symmetric and we have

$$\overset{c}{H}_{\alpha\beta}-\overset{c}{H}_{\beta\alpha}=M_\alpha H_\beta-M_\beta H_\alpha, \quad \overset{c}{H}_{0\alpha}=H_\alpha, \quad \overset{c}{H}_{\alpha 0}=H_\alpha+M_\alpha H_0. \tag{1.5}$$

It is well-known [5] that the normal curvature vector H_α vanishes if and only if H_0 vanishes.

Thus from (1.5) we have

Proposition 1. *The second fundamental h-tensor $\overset{c}{H}_{\alpha\beta}$ is symmetric if and only if the vector M_α vanishes or the normal curvature vector H_α vanishes.*

The quantity N defined by

$$N(u, v):=H_0(u, v)/L^2(u, v) \tag{1.6}$$

is called *the normal curvature of F^{n-1} at (u, v)* ([5]).

If we put

$$\Omega_{\alpha\beta}=(\partial^2 H_0/\partial v^\alpha \partial v^\beta)/2, \tag{1.7}$$

the above, together with (1.6), yields

$$\Omega_{\alpha\beta}=N_{\alpha\beta}L^2/2+N_\alpha v_\beta+N_\beta v_\alpha+N g_{\alpha\beta}, \tag{1.7'}$$

where $N_\alpha=\dot\partial_\alpha N$, $N_{\alpha\beta}=\dot\partial_\beta N_\alpha$, $\dot\partial_\alpha=\partial/\partial v^\alpha$.

The Finsler tensor field $\Omega_{\alpha\beta}$, defined by (1.7), is called *Berwald's second fundamental tensor*. Since $\dot\partial_\beta H_0=2H_\beta+H_0 M_\beta$ and $\dot\partial_\beta H_\gamma=\overset{c}{H}_{\beta\gamma}+Q_{\beta\gamma}$, the above $\Omega_{\alpha\beta}$ can be also expressed as

$$\Omega_{\alpha\beta}=\overset{c}{H}_{\alpha\beta}+Q_{\alpha\beta}+H_\alpha M_\beta+(\dot\partial_\alpha M_\beta+M_\alpha M_\beta)H_0/2, \tag{1.8}$$

where $Q_{\alpha\beta}=P_{ijk}B^{ij}_{\alpha\beta}B^k$, P^i_{jk} is the (v) hv-torsion tensor of $F\Gamma$. This shows the relation among $\Omega_{\alpha\beta}$ and $\overset{c}{H}_{\alpha\beta}$. And from (1.7') and (1.8) we have

$$\begin{aligned} \overset{c}{H}_{\alpha\beta}=N g_{\alpha\beta}+N_{\alpha\beta}L^2/2+N_\alpha v_\beta+N_\beta v_\alpha-Q_{\alpha\beta}-H_\alpha M_\beta-(\dot\partial_\alpha M_\beta+ \\ +M_\alpha M_\beta)H_0/2. \end{aligned} \tag{1.9}$$

Referring to homogeneity properties, it follows that $\Omega_{00}=H_{00}=H_0$, so we obtain

$$\Omega_{\alpha\beta}v^\alpha v^\beta/L^2=N(u, v). \tag{1.10}$$

We assume that the normal curvature N of F^{n-1} is constant c $(\neq 0)$. Then from (1.7') we obtain $\Omega_{\alpha\beta}=c g_{\alpha\beta}$. On the other hand, (1.10) shows that the converse is true.

Thus we get

Theorem 1. *The normal curvature N is constant c $(\neq 0)$ if and only if Berwald's second fundamental tensor $\Omega_{\alpha\beta}$ defined by (1.7) is given by $\Omega_{\alpha\beta}=c g_{\alpha\beta}$.*

In [5], Matsumoto showed the important results for surviving of quantities which are derived from Cartan's C-tensor.

Lemma ([5]). *If there exists a hypersurface with $Q_{\alpha\beta}(=P_{ijk}B^{ij}_{\alpha\beta}B^k)=0$ at each hypersurface element of a Finsler space F^n, then F^n is a Landsberg space.*

From this and Proposition 1 we have

Theorem 2. *If there exists a hypersurface such that Berwald's second fundamental tensor $\Omega_{\alpha\beta}$ coincides with the second fundamental h-tensor $\overset{c}{H}_{\alpha\beta}$ at each hypersurface element, then, the Finsler space F^n is a Landsberg space.*

Proof. Assume that $\Omega_{\alpha\beta}=\overset{c}{H}_{\alpha\beta}$ holds good at each hypersurface element of a Finsler space F^n. Since $\overset{c}{H}_{\alpha\beta}$ is symmetric, from Proposition 1 we have $M_\alpha=0$ or $H_\alpha=0$, so (1.8) implies $Q_{\alpha\beta}=0$. Consequently, by Lemma, the Finsler space F^n is Landsberg.

Further we have

Theorem 3. *If there exists an umbilical hypersurface F^{n-1} (with respect to the Cartan connection $C\Gamma$), i.e., $\overset{c}{H}_{\alpha\beta}=c g_{\alpha\beta}$ $(c\neq 0$: constant) at each hypersurface element, then, the Finsler space F^n is Landsberg.*

Proof. Assume that $\overset{c}{H}_{\alpha\beta}=cg_{\alpha\beta}$, ($c\neq 0$: constant) holds good at each hypersurface element. Then we have $\overset{c}{H}_{00}=H_0=cL^2$, so that (1.6) implies $N=c$. Therefore, from Theorem 1, we get $\Omega_{\alpha\beta}=cg_{\alpha\beta}$, so it follows that $\Omega_{\alpha\beta}=\overset{c}{H}_{\alpha\beta}$. Consequently, from Theorem 2, F^n is Landsberg.

§2. **Bejancu's conjecture.** We are concerned with the Gauss equation in case of $C\Gamma$ which is given by

$$R_{\alpha\beta\gamma\delta}=R_{hijk}B^{hijk}_{\alpha\beta\gamma\delta}+P_{hijk}B^{hi}_{\alpha\beta}(B^j_\gamma H_\delta-B^j_\delta H_\gamma)B^k+(\overset{c}{H}_{\alpha\gamma}\overset{c}{H}_{\beta\delta}-\overset{c}{H}_{\alpha\delta}\overset{c}{H}_{\beta\gamma}), \tag{2.1}$$

where $R_{\alpha\beta\gamma\delta}$ is the h-curvature tensor of the Finsler hypersurface F^{n-1}.

We treat the Finsler space of scalar curvature K which is characterized by the equation

$$R_{0h0k}=KL^2h_{hk}.$$

and if the scalar K is constant, F^n is called to be *of constant curvature* K. Then (1.5) and (2.1) show

$$R_{\alpha\beta\gamma\delta}v^\beta v^\delta=KL^2h_{\alpha\gamma}+(\overset{c}{H}_{\alpha\gamma}-\Omega_{\alpha\gamma})H_0-\overset{c}{H}_{\alpha 0}\overset{c}{H}_{0\gamma}. \tag{2.2}$$

An umbilical hypersurface of Riemannian space R^n ($n\geqslant 3$) of constant curvature, as is well-known, is also of constant curvature. O. Varga considered some generalizations of this property to Finsler geometry. He proved the following.

Theorem (Varga [10]). *In a Finsler space of constant curvature K, if $\overset{c}{H}_{\alpha\beta}=cg_{\alpha\beta}$ holds good in F^{n-1}, then F^{n-1} has the constant curvature $K/2-c^2$.*

It was pointed out by Matsumoto that this constant curvature should be $K+c^2$.

Now, in above Theorem if $\overset{c}{H}_{\alpha\beta}=cg_{\alpha\beta}$ holds good in F^{n-1} at each hypersurface element, then, by Theorem 3, F^n is Landsberg. Therefore by Numata's theorem [6], the Landsberg space F^n of constant curvature $K(\neq 0)$ is Riemannian. Consequently we have.

Proposition 2. *In a Finsler space of constant curvature K ($\neq 0$), if there exists an umbilical hypersurface F^{n-1} (with respect to the Cartan connection) at each hypersurface element, then the Finsler space F^n is Riemannian and the hypersurface F^{n-1} is also the Riemannian space of constant curvature $K+c^2$.*

Next, we shall consider A. Bejancu's conjecture. It is said that his conjecture is affirmative.

Theorem 4. *If the Finsler space $F^n(n>3)$ is projectively flat, then the totally geodesic hypersurface F^{n-1} is also projectively flat.*

Proof. It is well-known [4] that a Finsler space $F^n(n\geqslant 3)$ is Projectively flat if and only if the Weyl tensor and the Douglas tensor both vanish, and then F^n is of scalar curvature [8]. Suppose F^n ($n>3$) be projectively flat. Then from Rapcsák-Matsumoto's theorem [5], the normal curvature vector H_α vanishes. Therefore, from (1.5) we have $\overset{c}{H}_{0\alpha}=\overset{c}{H}_{\alpha 0}=0$ and $H_0=0$. The equation (2.2) shows that the hypersurface F^{n-1} is also of scalar curvature, so the Weyl tensor $W^\alpha_{\beta\gamma\delta}$ of F^{n-1} vanishes.

Next we are in a position to show vanishing of the Douglas tensor $D^\alpha_{\beta\gamma\kappa}$ of F^{n-1}. Since F^n is projectively flat, we have a rectilinear coordinate system (x^i) such that $G^i(x, y)$ appearing in the differential equations of a geodesic curvre of F^n is written as $G^i(x, y)=-p(x, y)y^i$, which implies

$$G^i_j :=\dot\partial_j G^i(=N^i_j)=-p_j y^i-p\delta^i_j, \quad p_j=\dot\partial_j p. \tag{2.3}$$

Taking account of the conditions $H_\alpha=0$ and $B_iy^i=0$, (1.3) and (2.3) show $B_iB^i_{0\alpha}=0$. Thus there exists some Finsler tensor field A^α_β (u, v) such that

$$B^i_{0\beta}=A^\gamma_\beta B^i_\gamma\,. \tag{2.4}$$

Differentiating this by $v^\alpha v^\lambda$, we get $(\dot\partial_\lambda\dot\partial_\alpha A^\gamma_\beta)B^i_\gamma=0$. Since $B_\alpha(u)=(B^i_\alpha(u))$ are $n-1$ linealy independent vectors tangent to M^{n-1} at a point (u^α), it follows that $\dot\partial_\lambda\dot\partial_\alpha A^\gamma_\beta=0$. so the quantities $\dot\partial_\beta A^\alpha_\gamma$ depend on only u^α, i.e., $\dot\partial_\beta(A^\alpha_\gamma(u, v))=(\partial_\beta A^\alpha_\gamma)$ (u). Let denote by $B\Gamma=(G^\alpha_{\beta\gamma}, G^\alpha_\beta, 0)$ the intrinsic Berwald connection determined from the induced metric $L(u, v)$. Then we have ([5])

$$N^\alpha_\beta=(B^i_{0\beta}+G^i_jB^j_\beta)B^\alpha_i, \quad N^\alpha_0=2G^\alpha,$$

therefore the quantities G^α of a hypersurface F^{n-1} are given by

$$G^\alpha=(G^i+B^i_{00}/2)B^\alpha_i. \tag{2.5}$$

Substituting $G^i=-py^i$ and (2.4) into this, we have

$$G^\alpha=-pv^\alpha+X^\alpha, \tag{2.6}$$

where we put $X^\alpha=A^\alpha_0/2$. Further from (2.6) we get the connection coefficients $G^\alpha_\beta=\dot\partial_\beta G^\alpha$ and $G^\alpha_{\beta\gamma}=\dot\partial_\gamma G^\alpha_\beta$ of $B\Gamma$ as follows :

$$G^\alpha_\beta=-p_\beta v^\alpha-p\delta^\alpha_\beta+X^\alpha_\beta, \quad X^\alpha_\beta=\dot\partial_\beta X^\alpha, \quad p_\beta=\dot\partial_\beta p, \tag{2.7}$$

$$G^\alpha_{\beta\gamma}=-p_{\beta\gamma}v^\alpha-p_\beta\delta^\alpha_\gamma-p_\gamma\delta^\alpha_\beta+X^\alpha_{\beta\gamma}, \quad p_{\beta\gamma}=\dot\partial_\gamma p_\beta, \quad X^\alpha_{\beta\gamma}=\dot\partial_\gamma X^\alpha_\beta\,. \tag{2.8}$$

For the vector X^α in (2.6), we take the partial derivative for v^β to get

$$2X^\alpha_\beta=(\dot\partial_\beta A^\alpha_\varepsilon)v^\varepsilon+A^\alpha_\beta.$$

Differentiating this by v^γ and referring to $(\dot\partial_\beta A^\alpha_\gamma)$ (u), we get

$$2X^\alpha_{\beta\gamma}=2\dot\partial_\gamma X^\alpha_\beta=\dot\partial_\beta A^\alpha_\gamma+\dot\partial_\gamma A^\alpha_\beta\,.$$

Thus we observe $X^\alpha_{\beta\gamma}=X^\alpha_{\beta\gamma}(u)$. Taking account of this fact, we get $X^\alpha_{\beta\gamma\delta}:=\dot\partial_\delta X^\alpha_{\beta\gamma}=0$. Differentiating (2.8) by v^δ, we have the hv-curvature tensor $G^\alpha_{\beta\gamma\delta}=\dot\partial_\delta G^\alpha_{\beta\gamma}$ of $B\Gamma$ as

$$G^\alpha_{\beta\gamma\varepsilon}=-p_{\beta\gamma\varepsilon}v^\alpha-p_{\beta\varepsilon}\delta^\alpha_\gamma-p_{\gamma\varepsilon}\delta^\alpha_\beta-p_{\beta\gamma}\delta^\alpha_\varepsilon, \tag{2.9}$$

where we put $p_{\beta\gamma\varepsilon}=\dot\partial_\varepsilon p_{\beta\gamma}$. From this the hv-Ricci tensor $G_{\beta\gamma}=G^\varepsilon_{\beta\gamma\varepsilon}$ is written by

$$G_{\beta\gamma}=-np_{\beta\gamma}. \tag{2.10}$$

Therefore (2.9) can be rewritten as

(2.9′) $$G^{\alpha}_{\beta\gamma\varepsilon}=(v^{\alpha}\dot{\partial}_{\varepsilon}G_{\beta\gamma}+\delta^{\alpha}_{\varepsilon}G_{\beta\gamma}+\delta^{\alpha}_{\beta}G_{\varepsilon\gamma}+\delta^{\alpha}_{\gamma}G_{\beta\varepsilon})/n,$$

which shows that the Douglas tensor $D^{\alpha}_{\beta\gamma\delta}$ od F^{n-1} vanishes. Consequently the totally geodesic hypersurface F^{n-1} is projectively flat.

Department of Mathematics
Kushiro College
Hokkaido University of
Education, Kushiro,
Japan (C. S.)

and

Institute of Mathematics
Kushiro Technical
College, Kushiro,
Japan (H. S., M. A.)

REFERENCES

1. Brown, G. — *A study of tensors which characterize a hypersurface of Finsler space*, Canad. J. Math. 20 (1968), 1025—1036.
2. Haimovici, M. — *Variétés totalment extrémales et variétés totalment géodésiques dans les espaces de Finsler*, Ann. Sci. Univ. Iași 25 (1939), 559—644.
3. Matsumoto, M. — *Foundations of Finsler geometry and special Finsler spaces*, Kyoto, 1977 (unpublished).
4. Matsumoto, M. — *Projectively flat Finsler spaces of constant curvature*, J. Nat. Acad. Math. India 1 (1983), 142—164.
5. Matsumoto M. — *The induced and intrinsic Finsler connections of hypersurface and Finslerien projective geometry*, J. Math. Kyoto Univ. 25 (1985), 107—144.
6. Numata, N. — *On Landsberg spaces of scalar curvature*, J. Korean Math. Soc. 12 (1975), 97—100.
7. Rund, H. — *Curvature properties of hypersurfaces of Finsler and Minkowskian spaces*, Tensor, N.S. 14 (1963), 226—244.
8. Szabó, Z. I. — *Ein Finslerschen Räum ist gerade dann von skalarer Krümmung, wenn seine Weylsche Projectivkrümmung verschwindet*, Act. Sci. Math. Szged, 39 (1977), 163—168.
9. Varga, O. — *Uber hyperflächen Konstanter Normalkrümmung in Minkowskischen Räumen*, Tensor, N. S. 13 (1963) 246—250.
10. Varga, O. — *Hyperflächen Konstanter Normal Krümmung in Finslersches Räumen*, Math. Nacher. 38 (1968), 47—52.

Analele științifice ale Universității „Al. I. Cuza" din Iași
Tomul XXXI, s. I-a. Matematică, fasc. 1, 1985

C. T. CHONG: *Techniques of Admissible Recursion Theory*, Lecture Notes in Mathematics Vol. 1106, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1984, IX+214 p., DM 31,50

Teoria generalizată a recursivității este teoria recursivității pe alte ordinale decît Σ_0, deci este partea „efectivă" a teoriei mulțimilor a lui Zermelo-Fraenckel. Teoria recursivității admisibile este teoria generalizată a recursivității pe ordinale admisibile (un număr ordinal este admisibil dacă este o limită și satisface axioma înlocuirii pentru Σ_1). Cartea conține cele mai importante rezultate obținute în domeniu, începînd cu anul 1972. Accentul este pus pe prezentarea diferitelor tehnici specifice domeniului, fiecare asemenea metodă însoțind un rezultat a cărui demonstrare a condus la transformarea metodei într-o tehnică cu o arie largă de aplicabilitate.

Cristian Masalagiu

Models and Sets. Proceedings of the Logic Colloquium held in Aachen, July 18—23, 1983. Part I. G. H. MÜLLER, M. M. RICHTER, (Editors) Lecture Notes in Mathematics, vol. 1103, Springer Verlag, Berlin, Heidelberg, New York, Tokyo, 1984; VIII + 484 p.; DM 64,00

Part II. *Computation and Proof Theory*, M. M. RICHTER, E. BÖRGER, (Editors), W. OBERSCHELP, B. SCHINZEL, W. THOMAS, Lecture Notes in Mathematics, vol. 1104, 1984; VIII + 475 p., DM 64,00

Lucrările Colocviului de Logică din iulie 1983 de la Aachen apar publicate în două volume, unul centrat pe teoria modelelor și a mulțimilor, algebrele Boole și analizei non-standard, iar celălalt consacrat teoriei demonstrației, teoriei recursiei, matematicii constructive și chestiunilor de logică din știința calculatoarelor. Vom exemplifica prin unele dintre titlurile și autorii articolelor din cele două volume: R. Bonnet — Asupra tipurilor de omomorfisme ale algebrelor Boole de interval superatomice; G. L. Cherlin — Teorii decidabile ale cîmpurilor pseudo-algebric inchise; W. Hodges — Extinderi finite ale grupurilor finite; S. C. Liu — O abordare prin teoria demonstrației a analizei non-standard; D. Mundici — Teoria modelelor abstracte și rețele de C-algebre; S. Feferman — Între matematica constructivă și matematica clasică; P. G. Hinman — Mulțimi aproximabile finit; Y. N. Moschovakis — Recursia abstractă ca fundament al teoriei algoritmilor.

Mirela Ștefănescu

Matematiceskaja logyka. Mathematical Logic. Proceedings of the Conference on Mathematical Logic. Dedicated to the Memory of A. A. Markov (1903—1979), Sofia, September 22—23, 1980; Izdat. Bolgar. Akad. Nauk, Sofia, 1984; 172 p.

Cel mai sigur mod de a recomanda această culegere de articole este acela de a cita titlurile și autorii contribuțiilor publicate în ea: A. O. Buda — O caracterizare simplă a arborilor de limbaje coregulate; G. K. Gargov — O notă asupra logicii demonstrabilității unor extinderi ale aritmeticii Heyting; N. V. Georgieva — O teoremă asupra reprezentabilității funcțiilor general recursive; A. B. Dicev — Asupra calculabilității în sens Moschovakis și legătura sa cu recursia parțială prin numerație; L. L. Ivanov — Recursivitate Kleene și spații operatoriale iterative; M. I. Kanovici — Cuantificatori mărginiți în sisteme semantice (rezumat); K. A. Kirov — Un analog intuiționist al logicii modale a demonstrabilității în