The proof results from Theorem 1 and [1] (Remark 1)

Theorem 2. *Let $f: D \to D^*$ and $h: D^* \to D^{**}$ be diffeomorphisms K (respectively K')-qc according to Gehring's metric definition. Then, the diffeomorphism $u = h \circ f$ is $K \cdot K'$-qc according to the same definition.*

Proof.

$$\delta(x) = \overline{\lim_{r\to 0}} \frac{\sup\limits_{y\in S(x,r)} d(u(x), u(y))}{\inf\limits_{y\in S(x,r)} d(u(x), u(y))} \leqslant \frac{\overline{\lim\limits_{r\to 0}} \dfrac{\sup\limits_{y\in S(x,r)} d(u(x), u(y))}{r}}{\underline{\lim\limits_{r\to 0}} \dfrac{\inf\limits_{y\in S(x,r)} d(u(x), u(y))}{r}} = \frac{\overline{\lim\limits_{y\to x}} \dfrac{d(u(x), u(y))}{d(x, y)}}{\underline{\lim\limits_{y\to x}} \dfrac{d(u(x), u(y))}{d(x, y)}} =$$

$$= \frac{\overline{\lim\limits_{y\to x}} \dfrac{d(u(x), u(y))}{d(f(x), f(y))} \cdot \dfrac{d(f(x), f(y))}{d(x, y)}}{\underline{\lim\limits_{y\to x}} \dfrac{d(u(x), u(y))}{d(f(x), f(y))} \cdot \dfrac{d(f(x), f(y))}{d(x, y)}} \leqslant \frac{\overline{\lim\limits_{f(y)\to f(x)}} \dfrac{d(h(f(x)), h(f(y)))}{d(f(x), f(y))}}{\underline{\lim\limits_{f(y)\to f(x)}} \dfrac{d(h(f(x)), h(f(y)))}{d(f(x), f(y))}} \cdot$$

$$\cdot \frac{\overline{\lim\limits_{y\to x}} \dfrac{d(f(x), f(y))}{d(x, y)}}{\underline{\lim\limits_{y\to x}} \dfrac{d(f(x), f(y))}{d(x, y)}} = \frac{\Lambda_h(f(x))}{\lambda_h(f(x))} \cdot \frac{\Lambda_f(x)}{\lambda_f(x)} = p_1(x) \cdot p_1^*(f(x)) \leqslant K \cdot K'$$

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ON A CERTAIN BOUNDARY VALUE PROBLEM FOR THIRD ORDER DIFFERENTIAL EQUATIONS

BY

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1. Introduction. In this paper we shall be concerned with the study of nonlinear boundary value problem (BVR) for third order differential equations of the form

$$(1) \qquad y''' = g(t, y, y', y'')\, F(t, y, y', y'', y''', \lambda), \qquad a \leqslant t \leqslant c,$$

$$(2) \qquad y(a) = 0, \; y(b) = 0, \; y(c) = 0, \qquad a < b < c,$$

where λ is a real parameter, $g \in C[[a, c] \times R^3, R]$, $F \in C[[a, c] \times R^5, R]$, R denotes the set of real numbers. In the past several years, an enormous amount of attention has been given to the study of special forms of boundary value problems of the type (1)—(2), particularly for second order differential equations, under various types of boundary conditions (see for example [3, 9, 10, 14] and the references given therein). Many sufficient conditions are known for the existence and uniqueness of solutions of BVP (1)—(2) when $F = 1$ under various types of boundary conditions (see, [1, 2, 4, 5, 7]). The main purpose of this paper is to obtain sufficient conditions for the existence, uniqueness and continuous dependence on parameter λ of the solutions of BVP (1)—(2) on $[a, c]$. Our formulation of BVP (1)—(2) is motivated by the study of a very interesting class of boundary value problems by N e h a r i [10], M o r o n e y [9], H o o k e r [3], U l l r i c h [14] and others and a class of boundary value problems studied by K a m e n s k i i and M y s h k i s [6] for second order boundary value problems. The method employed in our analysis is an extension of the method used in [6] which in turn is based on the application of a suitable version of the well known Banach fixed point theorem.

2. Main results. Let $B[a, c]$ be the class of real valued functions $z(t) \in C^3 [a, c]$ with norm

$$(3) \qquad \| z \| = \max \left\{ \frac{81}{2(c-a)^3} \max_{a\leqslant t\leqslant c} | z(t) |, \; \frac{6}{(c-a)^2} \max_{a\leqslant t\leqslant c} | z'(t) |, \; \frac{3}{2(c-a)} \max_{a\leqslant t\leqslant c} | z''(t) |, \; \max_{a\leqslant t\leqslant c} | z'''(t) | \right\}.$$

It is easily seen that $B[a, c]$ is a Banach space with respect to the norm defined in (3). We also use the notations

(4) $$K_1=\max_{a\leqslant t\leqslant c} |g(t, 0, 0, 0)|, \quad K_2=\max_{a\leqslant t\leqslant c} |F(t, 0, 0, 0, 0, \lambda)|,$$

and $B_\rho[a, c]=\{z\in B[a, c] : \|z\|\leqslant \rho\}$, where $\rho>0$ is a constant. In the special case when $\max_{a\leqslant t\leqslant c} |z'''(t)|$ is absent in the definition of norm given in (3), we denote the space $B[a, c]$ by $B^*[a, c]$ and the set $B_\rho[a, c]$ by $B^*_\rho[a, c]$.

It is known that the BVP (1)—(2) is equivalent to the integral equation (see [13, p. 671])

(5) $$y(t)=\int_a^c G(t, s)\, g(s, y(s), y'(s), y''(s))\, F(s, y(s), y'(s), y''(s), y'''(s), \lambda)\, ds,$$

where $G(t, s)$ is Green's function for the BVP

(6) $$y'''(t)=0, \qquad a\leqslant t\leqslant c,$$

(7) $$y(a)=0, \; y(b)=0, \; y(c)=0, \; a<b<c,$$

and its explicit form is given by (see [15, p. 719])

$$G(t,s)=\begin{cases} \dfrac{1}{2}\dfrac{(c-s)^2}{(c-a)(c-b)}(t-a)(t-b)=g_{11}, & b<s<c, \; a\leqslant t\leqslant s,\\[2mm] g_{11}-\dfrac{1}{2}(t-s)^2=g_{12}, & b<s<c, \; s\leqslant t\leqslant c,\\[2mm] g_{11}-\dfrac{1}{2}\dfrac{(b-s)^2}{(c-b)(b-a)}(t-a)(t-c)=g_{21}, & a<s<b, \; a\leqslant t\leqslant s,\\[2mm] g_{21}-\dfrac{1}{2}(t-s)^2=g_{22}, & a<s<b, \; s\leqslant t\leqslant c. \end{cases}$$

In order to establish our main results, we need the following known results :

Lemma 1. *If $G(t, s)$ is Green's function for the BVP (6)—(7), then*

(8) $$\int_a^c |G(t, s)|\, ds\leqslant (2/81)\,(c-a)^3,$$

(9) $$\int_a^c |G_t(t, s)|\, ds\leqslant (1/6)\,(c-a)^2,$$

(10) $$\int_a^c |G_{tt}(t, s)|\, ds\leqslant (2/3)\,(c-a),$$

for $t\in[a, c]$.

Lemma 2. *Let $(E, \|\cdot\|)$ be a Banach space. Let T be a mapping of the sphere $S=\{x\in E : \|x\|\leqslant\rho\}$ into itself satisfying the condition $\|T(x)-T(y)\|\leqslant\beta\|x-y\|$, for $\beta\in(0,1)$. Let $m=\|T(0)\|$ and $m\leqslant\rho(1-\beta)$. Then T has a unique fixed point x_0 which is the limit in the norm of E of the sequence $x_1=T(0)$, $x_2=T(x_1)$, $x_3=T(x_2)$, ...*

Lemma 1 is a suitable version of Lemma given by K o b e l'k o v and K o b y a k o v [8] and Lemma 2 may be found in K a m e n s k i i and M y s h k i s [6, see proof of Theorem 1].

Let T_λ be the operator defined by the relation

(11) $$(T_\lambda y)(t)=\int_a^c G(t, s)\, g(s, y(s), y'(s), y''(s))\, F(s, y(s), y'(s), y''(s), y'''(s), \lambda)\, ds,$$

for $t\in[a, c]$. Define the sequence of functions

(12) $$y_k=T_\lambda y_{k-1}, \quad y_0(t)=\int_a^c G(t, s)\, g(s, 0, 0, 0)\, F(s, 0, 0, 0, 0, \lambda)\, ds,$$

for $t\in[a, c]$ and $k=1, 2, \ldots$.

We use the following hypotheses.

(H_1) There exist nonnegative constants P_1, P_2 such that

$$|g(t, y, y', y'')|\leqslant P_1, \quad |F(t, y, y', y'', y''', \lambda)|\leqslant P_2,$$

for $(t, y, y', y'')\in[a, c]\times R^3$, $(t, y, y', y'', y''', \lambda)\in[a, c]\times R^5$.

(H_2) The function g satisfies the condition

$$|g(t, y_1, y_1', y_1'')-g(t, y_2, y_2', y_2'')|\leqslant L_0|y_1-y_2|+M_0|y_1'-y_2'|+N_0|y_1''-y_2''|,$$

on $B_\rho[a, c]$, where L_0, M_0, N_0 are nonnegative constants and denote

(13) $$\alpha_1=(2/81)\,L_0(c-a)^3+(1/6)\,M_0(c-a)^2+(2/3)\,N_0(c-a).$$

(H_3) For $\lambda\in R$, F satisfies the condition

$$|F(t, y_1, y_1', y_1'', y_1''', \lambda)-F(t, y_2, y_2', y_2'', y_2''', \lambda)|$$
$$\leqslant L|y_1-y_2|+M|y_1'-y_2'|+N|y_1''-y_2''|+Q|y_1'''-y_2'''|,$$

on $B_\rho[a, c]$, where L, M, N, Q are nonnegative constants and denote also

(14) $$\alpha_2=(2/81)\,L(c-a)^3+(1/6)\,M(c-a)^2+(2/3)\,N(c-a)+Q.$$

(H_4) For $\lambda_1, \lambda_2\in R$, F satisfies the condition

$$|F(t, y, y', y'', y''', \lambda_1)-F(t, y, y', y'', y''', \lambda_2)|\leqslant\mu|\lambda_1-\lambda_2|,$$

on $B_\rho[a, c]$, where μ is a nonnegative constant.

We are now ready to establish our main result on the existence of a unique solution of BVP (1)—(2).

Theorem 1. *Assume that the hypotheses (H_1)—(H_3) hold. If $\alpha=P_1\alpha_2+P_2\alpha_1<1$ and*

(15) $$K_1K_2\leqslant\rho(1-\alpha),$$

then for every $\lambda\in R$, *the* BVP (1)—(2), *has a unique solution in* $B_\rho[a,c]$. *This solution is the limit in the* $B[a,c]$ *norm of the sequence of functions defined by* (12).

Proof. It is easy to observe that the operator T_λ defined by (11) maps $B[a,c]$ into itself. From the definition of operator T_λ. the equation (5) becomes the operator equation

$$y(t)=(T_\lambda y)(t),\quad t\in[a,c]. \tag{16}$$

Clearly the solution of the operator equation (16) corresponds to the solution of the BVP (1)—(2). We verify now that the operator T_λ fulfils all the conditions of Lemma 2 on $B_\rho[a,c]$. Let $y,\bar y\in B_\rho[a,c]$, then from (11), hypotheses (H_1)—(H_3) and (3) we have

$$\begin{aligned}(17)\qquad &|(T_\lambda y)'''(t)-(T_\lambda\bar y)'''(t)|\le\\ &\le|g(t,y(t),y'(t),y''(t))\{F(t,y(t),y'(t),y''(t),y'''(t),\lambda)-\\ &\qquad -F(t,\bar y(t),\bar y'(t),\bar y''(t),\bar y'''(t),\lambda)\}|+\\ &+|F(t,y(t),\bar y'(t),\bar y''(t),\bar y'''(t),\lambda)\{g(t,y(t),y'(t),y''(t))-\\ &\qquad -g(t,\bar y(t),\bar y'(t),\bar y''(t))\}|\le\\ &\le P_1[L|y(t)-\bar y(t)|+M|y'(t)-\bar y'(t)|+N|y''(t)-\bar y''(t)|+Q|y'''(t)-\bar y'''(t)|]+\\ &+P_2[L_0|y(t)-\bar y(t)|+M_0|y'(t)-\bar y'(t)|+N_0|y''(t)-\bar y''(t)|]\le\\ &\le(P_1\alpha_2+P_2\alpha_1)\|y-\bar y\|=\alpha\|y-\bar y\|.\end{aligned}$$

From (11), hypotheses (H_1)—(H_3), (3) and (10), we have

$$\begin{aligned}(18)\qquad &|(T_\lambda y)''(t)-(T_\lambda\bar y)''(t)|\le\\ &\le\int_a^c|G_{tt}(t,s)||g(s,y(s),y'(s),y''(s))F(s,y(s),y'(s),y''(s),y'''(s),\lambda)-\\ &\qquad -g(s,\bar y(s),\bar y'(s),\bar y''(s))F(s,\bar y(s),\bar y'(s),\bar y''(s),\bar y'''(s),\lambda)|\,ds\le\\ &\le\alpha\|y-\bar y\|\int_a^c|G_{tt}(t,s)|\,ds\le\alpha(2/3)(c-a)\|y-\bar y\|.\end{aligned}$$

Similarly, from (11), hypotheses (H_1)—(H_3), (3), (9) and (8) we have

$$|(T_\lambda y)'(t)-(T_\lambda\bar y)'(t)|\le\alpha\,.\,(1/6)(c-a)^2\|y-\bar y\|, \tag{19}$$

and

$$|(T_\lambda y)(t)-(T_\lambda\bar y)(t)|\le\alpha\,.\,(2/81)(c-a)^3\|y-\bar y\|. \tag{20}$$

The inequalities (17)—(20) and the definition of norm in $B[a,c]$ imply

$$\|T_\lambda y-T_\lambda\bar y\|\le\alpha\|y-\bar y\|. \tag{21}$$

From (11), (4) and Lemma 1 we obtain

$$|(T_\lambda 0)'''(t)|\le K_1K_2, \tag{22}$$

$$|(T_\lambda 0)''(t)|\le K_1K_2\,.\,(2/3)(c-a), \tag{23}$$

$$|(T_\lambda 0)'(t)|\le K_1K_2\,.\,(1/6)(c-a)^2, \tag{24}$$

$$|(T_\lambda 0)(t)|\le K_1K_2\,.\,(2/81)(c-a)^3. \tag{25}$$

From (22)—(25), (3) and (15) it follows that

$$\|T_\lambda 0\|\le K_1K_2\le\varrho(1-\alpha). \tag{26}$$

Thus all the conditions of Lemma 2 are satisfied, and hence the operator T_λ has a unique fixed point in $B_\rho[a,c]$ which is the limit of the sequence $\{y_k\}$ defined in (12). The fixed point of T_λ is however a solution of BVP (1)—(2). This completes the proof of the theorem.

We next obtain a sufficient conditions for the continuous dependence of solutions of BVP (1)—(2) on the parameter λ. The idea of the proof is based on the result given by the present author in [11, Theorem 2] (see, also [12]).

Theorem 2. *In addition to the hypotheses of Theorem 1, assume that the hypothesis* (H_4) *be fulfilled. Then the solution* $y(t,\lambda)$ *of* BVP (1)—(2), *belonging to* $B_\rho[a,c]$ *is continuous with respect to the variables* (t,λ) *in* $[a,c]\times R$.

Proof. From (11), hypotheses (H_1) and (H_4) and Lemma 1 we have for $y\in B_\rho[a,c]$ and $\lambda_1,\lambda_2\in R$,

$$|(T_{\lambda_1}y)'''(t)-(T_{\lambda_2}y)'''(t)|\le P_1\mu|\lambda_1-\lambda_2|, \tag{27}$$

$$\begin{aligned}(28)\qquad &|(T_{\lambda_1}y)''(t)-(T_{\lambda_2}y)''(t)|\le P_1\mu|\lambda_1-\lambda_2|\int_a^c|G_{tt}(t,s)|\,ds\le\\ &\le P_1\mu|\lambda_1-\lambda_2|\,.\,(2/3)(c-a),\end{aligned}$$

$$\begin{aligned}(29)\qquad &|(T_{\lambda_1}y)'(t)-(T_{\lambda_2}y)'(t)|\le P_1\mu|\lambda_1-\lambda_2|\int_a^c|G_t(t,s)|\,ds\le\\ &\le P_1\mu|\lambda_1-\lambda_2|\,.\,(1/6)(c-a)^2,\end{aligned}$$

$$\begin{aligned}(30)\qquad &|T_{\lambda_1}y)(t)-(T_{\lambda_2}y)(t)|\le P_1\mu|\lambda_1-\lambda_2|\int_a^c|G(t,s)|\,ds\le\\ &\le P_1\mu|\lambda_1-\lambda_2|\,.\,(2/81)(c-a)^3.\end{aligned}$$

From (27)—(30) and (3), it follows that

$$\|T_{\lambda_1}y-T_{\lambda_2}y\|\le P_1\mu|\lambda_1-\lambda_2|. \tag{31}$$

From Theorem 1 there exists a unique function $y(t,\lambda)$ such that $T_{\lambda_1}y(t,\lambda_1)=$ $=y(t,\lambda_1)$ and $T_{\lambda_2}y(t,\lambda_2)=y(t,\lambda_2)$ for $t\in[a,c]$.
From (21) and (31) we have

$$\begin{aligned}(32)\qquad &\|y(t,\lambda_1)-y(t,\lambda_2)\|\le\|T_{\lambda_1}y(t,\lambda_1)-T_{\lambda_1}y(t,\lambda_2)\|+\\ &\qquad+\|T_{\lambda_1}y(t,\lambda_2)-T_{\lambda_2}y(t,\lambda_2)\|\le\\ &\le\alpha\|y(t,\lambda_1)-y(t,\lambda_2)\|+P_1\mu|\lambda_1-\lambda_2|.\end{aligned}$$

From (32) it follows that

$$\|y(t,\lambda_1)-y(t,\lambda_2)\|\le P_1(\mu/(1-\alpha))|\lambda_1-\lambda_2|.$$

This shows that the function $y(t, \lambda)$ is continuous with respect to the variable λ in R uniformly with respect to the variable t in $[a, c]$ and consequently $y(t, \lambda)$ is also continuous with respect to the two variables (t, λ) in $[a, c]\times \times R$ and proof of the theorem is complete.

3. **Some special cases.** In this section we indicate some interesting special cases of BVP (1)—(2) and relate our results to previous works in this area. The BVP (1)—(2), in the special cases when (i) $F=1$ and (ii) $g=1$, reduces to the following boundary value problems

$$y'''=g(t, y, y', y''), \qquad a\leqslant t\leqslant c, \tag{33}$$

$$y(a)=0,\ y(b)=0,\ y(c)=0, \qquad a<b<c, \tag{34}$$

and

$$y'''=F(t, y, y', y'', y''', \lambda), \qquad a\leqslant t\leqslant c, \tag{35}$$

$$y(a)=0,\ y(b)=0,\ y(c)=0, \qquad a<b<c, \tag{36}$$

respectively. Many authors have studied the BVP (33)—(34) under the variant of boundary conditions by using different techniques (see [1, 2, 4, 5,7]). Here we note that the hypothesis (H_1) is not required in order to establish the results on the existence and uniqueness of solutions of BVP (33)—(34) and BVP (35)—(36).

The following result is an immediate consequence of our Theorem 1.

Theorem 3. *Assume that the hypothesis* (H_2) *holds. If* $\alpha_1<1$ *and*

$$K_1\leqslant \varepsilon(1-\alpha_1), \tag{37}$$

then the BVP (33)—(34) *has a unique solution in* $B_\rho^*[a, c]$. *This solution is the limit in the* $B^*[a, c]$ *norm of the sequence of functions defined by* (12), *in case when* $F=1$.

We remark that the result obtained in Theorem 3 can also be found in [1]. For several other results on the existence and uniqueness of the solutions of BVP (33)—(34) with various types of boundary conditions we refer the interested readers to Barr and Sherman [2], Jackson and Schrader [4, 5] and Klassen [7].

We next present the following results as immediate consequences of our Theorems 1 and 2.

Theorem 4. *Assume that the hypothesis* (H_3) *holds. If* $\alpha_2<1$ *and*

$$K_2\leqslant \varepsilon(1-\alpha_2), \tag{38}$$

then for every $\lambda\in R$, *the* BVP (35)—(36) *has a unique solution in* $B_\rho[a, c]$. *This solution is the limit in* $B[a, c]$ *norm of the sequence of functions defined by* (12), *in case when* $g=1$.

Theorem 5. *In addition to the hypotheses of Theorem* 4, *assume that the hypothesis* (H_1) *be fulfilled. Then the solution* $y(t, \lambda)$ *of* BVP (35)—(36) *belonging to* $B_\rho[a, c]$ *is continuous with respect to the variables* (t, λ) *in* $[a, c]\times R$.

Finally, we remark that the existing literature contains a number of results on the theory of BVP (33)—(34) with different boundary conditions by using various methods (see, for example, [4, 5, 7]). The question, whether or not the various methods developed in [4, 5, 7] for boundary value problems of the form (33)—(34) can be extended to the more general BVP (1)—(2) or BVP (35)—(36) remains an open problem. The results obtained in this paper are an essential step towards a general theory of such types of boundary value problems and it is hoped that it would lead to significant progress.

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