From (4.14) it is clear that we can choose a local basis of V such that U_1, U_2 and U_3 are principal vectors, that is, we can suppose that

$$AU_k = a_k U_k, \qquad k = 1, 2, 3, \tag{4.15}$$

and so, from (2.9), (4.12) and (4.15), $-2 = g((\nabla_X A)J_k X - (\nabla_{J_k X} A)X, U_k) = 2(\lambda - a_k)\lambda$. Consequently, U_1, U_2 and U_3 are principal vectors with the same principal curvature. Then (see [5]) M is an open subset of a geodesic hypersphere of QP^m.

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Received 18.IX.1986

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Analele ştiinţifice ale Universităţii „Al. I. Cuza" din Iaşi
Tomul XXXIV, s. I a, Matematică, 1988, f. 1

THERMO-ACOUSTICAL WAVES IN LINEAR HOMOGENEOUS ISOTROPIC THERMO-VISCOELASTIC MATERIALS OF INTEGRAL TYPE

BY

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1. Introduction. An unrealistic consequence of the parabolic type equation of heat conduction based upon Fourier's law is that a thermal disturbance at any point in the body is felt instantaneously at all other points. But the experiences made with HeII have shown that the thermal pulses are propagating at finite speed. In order to remedy this so-called „paradox of heat conduction", different theories have been proposed. A first solution has been offered by the equation of heat flux deduced on kinetic-probabilistic grounds by Cattaneo [1], [2] and assumed by Vernotte in [3], [4]. Its use in [5]—[15] has shown that the second sound is possible. Upon the basis of unfounded analogies, the Cattaneo-Vernotte's equation has been deduced by Kaliski [16] within the framework of the Onsager type thermodynamical theory and Fox [17] has derived its objective form in the non-linear case. The entropy inequalities proposed by Müller [18] and Green and Laws [19], as well as the assumption of a constitutive equation for the thermal energy flux by Ghaleb [20] are staying at the basis of some theories which supply finite propagation speeds of thermal disturbances (see [21]—[31]). Also, the thermodynamical theory with internal state variables takes into account the possibility of the second sound (see [32]—[39]).

The idea for giving a constitutive equation for the heat flux independent of the present temperature gradient, analogous with Cattaneo-Vernotte-Kaliski's law, has lied at the basis of the theory of rigid heat conductors with fading memory developed by Curtin and Pipkin [40]. The thermal waves are propagating at finite speed (see [40]—[42]). The theory developed in [40] has been generalized by Mc Carthy [43] in order to take into account the body deformation too. The constitutive equations from [43] have been linearized in [44]. Mc Carthy investigated in [45] the growth and decay of the amplitudes of thermal waves, in the nonlinear rigid heat conductor introduced in [43]. In [46] he examined the thermomechanical waves in the nonlinear one-dimensional material introduced in [43].

In this paper we shall examine the propagation of thermo-acoustical waves in the linear homogeneous isotropic material considered in [44]. It will be shown that two purely mechanical transversal waves and two coupled longitudinal waves are propagating at constant speeds which are depending on the instantaneous material answer. Their amplitudes illustrate the material memory. In the case of rigid heat conductors, it will be studied plane harmonic waves and then, in the same case, a comparison will be made between the models proposed by Fourier, Cattaneo-Vernotte-Kaliski and Gurtin-Pipkin-McCarthy.

When the reference configuration is a natural one with uniform temperature, the constitutive equations of a linear homogeneous isotropic thermoviscoelastic material of integral type written in rectangular Cartesian coordinates and omitting the dependence on the material point are (see [44]) :

$$
\begin{aligned}
T_{ij}(t) &= \lambda_0 u_{k,k}(t)\delta_{ij} + \mu_0(u_{i,j}(t)+u_{j,i}(t)) + \int_0^\infty \dot{\lambda}(s)u_{k,k}(t-s)\,ds\,\delta_{ij} + \\
&+ \int_0^\infty \dot{\mu}(s)(u_{i,j}(t-s)+u_{j,i}(t-s))\,ds - n_0 T(t)\delta_{ij} - \int_0^\infty \dot{n}(s)T(t-s)\,ds,
\end{aligned} \tag{1.1}
$$

$$
\rho_0\eta(t) = n_0 u_{k,k}(t) + \int_0^\infty \dot{p}(s)u_{k,k}(t-s)\,ds + \frac{\rho_0 c_0}{\theta_0}T(t) + \frac{\rho_0}{\theta_0}\int_0^\infty \dot{c}(s)T(t-s)\,ds,
$$

$$
q_i(t) = -\frac{1}{\tau}\int_0^\infty \varkappa(s)T_{,i}(t-s)\,ds, \quad \tau = \text{constant} > 0,
$$

where $T_{ij}(t)$ is the Cauchy stress tensor, $\eta(t)$ — the specific entropy and $q_i(t)$ — the heat flux at t; $u_i(t)$ is the displacement; a comma followed by a suffix denotes the spatial derivative with respect to a coordinate; ρ_0 θ_0 are, respectively, the density and the absolute temperature in the reference configuration; $T(t)=\theta(t)-\theta_0$, with $\theta(t)$ the absolute temperature; $\lambda(s)$, $\mu(s)$, $n(s)$, $p(s)$, $c(s)$, $\varkappa(s)$ are scalar functions defined on $R_+=[0,\infty)$, λ_0, μ_0, n_0, p_0, c_0, $\varkappa_0$ being their values for $s=0$; we call $c(s)$ *the specific heat relaxation function* (per mass unity), c_0 — the *instantaneous specific heat*, $\varkappa(s)$ — the *conductivity relaxation function*, $\varkappa_0$ — the *instantaneous conductivity* and τ — the *heat flux relaxation time*.

In [44] we denoted $g(s)\equiv\rho_0 c(s)/\theta_0$, $k(s)\equiv\varkappa(s)/\tau\theta_0$. With the notations from this paper, in the case of a rigid heat conductor, the internal energy ε becomes $\varepsilon(t)=\theta_0\,\eta(t)=c_0T(t)+\int_0^\infty \dot{c}(s)T(t-s)\,ds$, which justifies the name of the relaxation function $c(s)$; also $\varkappa(s)$ has the dimension of the thermal conductivity coefficient from the Fourier's or Cattaneo's law.

The functions $\dot{\lambda}(s)$, $\dot{\mu}(s)$, $\dot{n}(s)$, $\dot{p}(s)$, $\dot{c}(s)$, $\varkappa(s)$, where the superposed dot denotes the derivative with respect to the time argument s, are integrable on R_+ and $\lim_{s\to\infty}[\lambda(s), \mu(s), n(s), p(s), c(s), \varkappa(s)]$ exist and are finite, with $\lim_{s\to\infty}\varkappa(s)=0$ (see [44]). We call $c_\infty=\lim_{s\to\infty}c(s)$ the *equilibrium specific heat*. When $\lambda(s)=\lambda_0$, $\mu(s)=\mu_0$, $n(s)=p(s)=n_0$, $c(s)=c_0$, $\varkappa(s)=\varkappa_0 e^{-s/\tau}$ $\forall s\in R_+$, the material defined by (1.1) is reduced at the thermo-elastic material of Cattaneo-Vernotte-Kaliski type analised by Tokuoka in [14] and [15].

In [44] we showed that the Clausius-Duhem inequality and the Coleman's principle of fading memory impose

$$
p_0 = n_0, \quad \varkappa_0 \geqslant 0 \tag{1.2}
$$

and if additional conditions (of Day type) are admitted (see [44]), then

$$
p(s) = n(s) \quad \forall s \in \mathbf{R}_+, \quad c_\infty \geqslant c_0, \quad \dot{c}(0) \equiv \dot{c}_0 \geqslant 0. \tag{1.3}
$$

In the following, we assume

$$
\varkappa_0 \neq 0, \quad c_0 > 0. \tag{1.4}
$$

The framework considered in [44] did not give informations about the other constitutive functions which appear in (1.1). In this paper we admit that in the material characterized by (1.1) it is possible to propagate thermomechanical disturbances whose amplitudes are not growing at the same time with their propagation in space. This will impose new restrictions on the relaxation functions, namely on the instantaneous material answer.

2. Thermo-acoustical waves; finite speeds of propagation. In this paragraph we determine the speeds with which the Hadamard singular surfaces cross the material and we establish relations between the acceleration jump and the temperature time derivative jump.

In the linear theory, the equations of balance of momentum and energy are

$$
\rho_0\ddot{u}_i = T_{ij,j} + \rho_0 b_i, \quad \rho_0\dot{\varepsilon} = -q_{i,i} + \rho_0 r, \tag{2.1}
$$

where b_i is the specific body force and r is the heat supply function per unit mass. Since $\varepsilon=\theta_0\eta$, $(2.1)_2$ becomes

$$
\rho_0\dot{\eta} = -q_{i,i}/\theta_0 + \rho_0 r/\theta_0. \tag{2.2}
$$

We assume that the material was held in the reference configuration until the time $t=0$. Then

$$
u_i(\mathbf{x},t)=0, \quad T(\mathbf{x},t)=0 \quad \forall \mathbf{x}=(x_1,x_2,x_3)\in\mathbf{B}, \ t<0, \tag{2.3}
$$

where $\mathbf{B}$ is the region of an Euclidian space occupied by the body that lies in the reference configuration.

Let $\mathbf{S}_t(0\leqslant t<\infty)$ be a one-parameter family of surfaces with $\mathbf{S}_t\subset\mathbf{B}$; Σ_x is the hypersurface

$$
\Sigma_x = \{(\mathbf{x},t)\,|\,\mathbf{x}\in\mathbf{S}_t,\ 0\leqslant t<\infty\};
$$

ω_x, for each $\mathbf{x}$, is the point set $\omega_x=\{t\,|\,\mathbf{x}\in\mathbf{S}_t\}$.

Definition 2.1. (see [17]). *By a thermo-acoustical wave we mean a solution* $\{u_i, T, T_{ij}, \eta, q_i\}$ *of (2.1) and (2.3) such that*:

(A. 1) The displacement u_i is continuously differentiable and the temperature T is continuous on $\mathbf{B}\times(-\infty,\infty)$; however the second and the first order derivatives of displacement and temperature have finite jump discontinuities across Σ_x, but are continuous everywhere else on $\mathbf{B}\times(-\infty,\infty)$. Moreover, each of these second and first order derivatives are bounded on every compact subset of $\mathbf{B}\times(-\infty,\infty)$.

(A. 2) The functions $\dot{\lambda}(s)$, $\dot{\mu}(s)$, $\dot{n}(s)$, $\dot{p}(s)$, $\dot{c}(s)$ $\varkappa(s)$ are continuous on $\mathbf{R}_+$ while the body force b_i and the heat source r are continuous on $\mathbf{B}\times(-\infty,\infty)$.

(A. 3) The hypersurface Σ_s is smooth and orientable.

(**A**. 4) For each $\mathbf{x}$, the set $\omega_{\mathbf{x}}$ has zero measure.

It results from (**A**. 3) that there exists at every $(\mathbf{x}, t) \in \Sigma_s$ a normal ν to Σ_s. Such a vector is

$$\nu \equiv (\mathbf{n}, -U) \equiv (n_1, n_2, n_3, -U), \quad U \geqslant 0,$$

where $\mathbf{n} = (n_1, n_2, n_3)$ is a unit normal to S_t at the point $\mathbf{x}$. The vector $\mathbf{n}$ is called the *direction of propagation* and the number U is called the *speed of propagation*. We assume $U > 0$. The following lemma will be used.

Lemma 2.1. (Fisher and Gurtin [47]). *Let g and k be functions on $\mathbf{B} \times (-\infty, \infty)$ with the following properties:*

(a) g is continous;

(b) k is continous everywhere except for a possible jump discontinuity across Σ_s;

(c) k is bounded on every compact subset of $\mathbf{B} \times (-\infty, \infty)$. Moreover assume (**A**. 4) *holds. Then the function Λ defined by*

$$\Lambda(\mathbf{x}, t) = \int_0^t g(\mathbf{x}, t-s) k(\mathbf{x}, s) ds$$

is continous on $\mathbf{B} \times (-\infty, \infty)$.

We denote by $[f]$ the jump of the function f across Σ_s, that is $[f(\mathbf{x}, t)] = \lim_{(\mathbf{x}^+, t^+) \to (\mathbf{x}, t)} f(\mathbf{x}^+, t^+) - \lim_{(\mathbf{x}^-, t^-) \to (\mathbf{x}, t)} f(\mathbf{x}^-, t^-)$, where $(\mathbf{x}^+, t^+)$ and $(\mathbf{x}^-, t^-)$ lie on opposite sides of Σ_s and $(\mathbf{x}, t) \in \Sigma_s$.

If f is a function that has zero jump across Σ_s, i.e. $[f] = 0$, then the geometrical-kinematical conditions of compatibility have the form

$$[f_{,i}] = -[\dot{f}] n_i U^{-1} \tag{2.4}$$

and the dynamical-energetical conditions of compatibility in the case of thermo-acoustical waves (and in the linear case) are

$$\rho_0 [\ddot{u}_i] = [T_{ij,j}] + \rho_0 [b_i], \quad \rho_0 [\dot{\eta}] = -\theta_0^{-1} [q_{i,i}] + \rho_0 \theta_0^{-1} [r]. \tag{2.5}$$

From (**A**. 1) we have

$$[u_i] = [u_{i,j}] = [\dot{u}_i] = [T] = [b_i] = [r] = 0, \quad [\ddot{\mathbf{u}}] \equiv \mathbf{a} \neq 0, \quad [\dot{T}] \equiv b \neq 0;$$

$\mathbf{a}$ is called the *mechanical amplitude* and b – the *thermal amplitude* of the wave. Using the equations (2.5) we shall deduce relations between these amplitudes.

From definition 2.1, lemma 2.1 and the constitutive equations (1.1) we obtain

$$[T_{ij}] = [\eta] = [\varepsilon] = [q_i] = 0. \tag{2.6}$$

Then the geometrical-kinematical conditions of compatibility (2.4) give us

$$[u_{i,jk}] = a_i n_j n_k U^{-2}, \quad [\dot{u}_{i,j}] = -a_i n_j U^{-1}, \quad [T_{,i}] = -b n_i U^{-1}, \tag{2.7}$$

$$[T_{ij,k}] = -[\dot{T}_{ij}] n_k U^{-1}, \quad [q_{i,j}] = -[\dot{q}_i] n_j U^{-1}. \tag{2.8}$$

Taking into account (**A**. 2), the conditions (2.5) become

$$\rho_0 [\ddot{u}_i] = [T_{ij,j}], \quad \rho_0 [\dot{\eta}] = -\theta_0^{-1} [q_{i,i}]. \tag{2.9}$$

The jump $[T_{ij,j}]$ is calculated using the relations (2.8), $(1.1)_1$, (2.3), (2.7), definition 2.1 and lemma 2.1. We find

$$[T_{ij,j}] = (\lambda_0 + \mu_0) U^{-2} a n_i + \mu_0 U^{-2} a_i + n_0 U^{-1} b n_i, \tag{2.10}$$

where $a = a_i n_i$. Analogously is calculated the jump $[q_{i,i}]$, by using an integration by parts in the expression for $q_i(t)$. We get

$$[q_{i,i}] = -\varkappa_0 \tau^{-1} U^{-2} b. \tag{2.11}$$

If we differentiate the constitutive equation $(1.1)_2$ with respect to time and if we apply lemma 2.1 and definition 2.1, we obtain

$$\rho_0 [\dot{\eta}] = -n_0 U^{-1} a + \rho_0 c_0 \theta_0^{-1} b. \tag{2.12}$$

The introduction of the relations (2.10)–(2.12) into (2.9) leads to the following homogeneous system with four equations and five unknowns (a_i, b, U)

$$\begin{gathered} ((\lambda_0 + \mu_0) U^{-2} n_k n_i + (\mu_0 U^{-2} - \rho_0) \delta_{ki}) a_k + n_0 U^{-1} n_i b = 0, \\ n_0 U^{-1} n_k a_k - (\rho_0 c_0 \theta_0^{-1} - \varkappa_0 (\tau \theta_0 U^2)^{-1}) b = 0. \end{gathered} \tag{2.13}$$

The system (2.13) admits nontrivial solutions (a_i, b) if and only if $\det (R_{\alpha\beta}) = 0$, where

$$\| R_{\alpha\beta} \| = \left\| \begin{matrix} (\lambda_0 + \mu_0) U^{-2} n_k n_i + (\mu_0 U^{-2} - \rho_0) \delta_{ki} & n_0 U^{-1} n_k \\ n_0 U^{-1} n_k & -\rho_0 c_0 \theta_0^{-1} + \varkappa_0 (\tau \theta_0 U^2)^{-1} \end{matrix} \right\|$$

is called the *thermo-acoustical tensor*. We denote

$$\begin{gathered} U_\perp^2 = \mu_0 \rho_0^{-1}, \quad U_L^2 = (\lambda_0 + 2\mu_0) \rho_0^{-1}, \quad U_T^2 = \varkappa_0 (\rho_0 c_0 \tau)^{-1} > 0, \\ \alpha = n_0 \rho_0^{-1}, \quad \sigma = n_0 \theta_0 (\rho_0 c_0)^{-1} \end{gathered} \tag{2.14}$$

and introduce the dimensionless quantities

$$\varepsilon = \alpha \sigma U_T^{-2} > 0, \quad \delta = U_L^2 U_T^{-2},$$

where ε is called the *coupling constant*. The characteristic equation $\det (R_{\alpha\beta}) = 0$ becomes

$$(U^2 - U_\perp^2)^2 (U^4 - U_T^2 (1 + \varepsilon + \delta) U^2 + \delta U_T^4) = 0. \tag{2.15}$$

From (2.15) and (2.13) it results

Theorem 2.1. *In the material defined by (1.1) it is possible to propagate two transversal purely mechanical waves at the constant speed $U_\perp$ if and only if*

$$\mu_0 > 0. \tag{2.16}$$

Theorem 2.2. *If we have $\varepsilon \neq 0$ (i.e. $n_0 \neq 0$) and*

$$\lambda_0 + 2\mu_0 > 0, \tag{2.17}$$

then in the material defined by (1.1) it is possible to propagate two coupled longitudinal waves with the constant speeds $U_+>0$, $U_->0$, $U_+>U_-$, given by

$$U_{+,-}^2=(U_T^2/2)(1+\delta+\varepsilon\pm\sqrt{(1-\delta)^2+2\varepsilon(1+\delta)+\varepsilon^2})>0$$

and with the following relations between amplitudes

$$a_+=b_+(U_+^2-U_T^2)(\sigma U_+)^{-1},\quad b_-=a_-(U_-^2-U_L^2)(\alpha U_-)^{-1}, \tag{2.18}$$

where $a_i=an_i$.

We note that (2.17) is only a sufficient condition, which is not necessary one.

Theorem 2.3. *In the ideal case of uncoupling (i.e. $\varepsilon=0$) and assuming $\delta<1$ (see [6]), in the material defined by (1.1) are propagating a longitudinal purely thermal wave with the speed $U_+=U_T$ and a longitudinal purely mechanical wave with the speed $U_-=U_L$, $U_+>U_-$ if and only if $\lambda_0+2\mu_0>0$.*

Consequently, we call the coupled wave corresponding to U_+ — *thermomechanical* wave and the coupled wave corresponding to U_- — *mechanicthermal* wave. In what follows we shall assume that (2.16) and (2.17) hold.

The same propagation velocities $U_\perp$, U_+, U_- and the same relations between amplitudes (2.18) were derived by Tokuoka in [15]. Consequently we have

Remark. Concerning the speeds of propagation and the relations between amplitudes, the linear homogeneous isotropic thermoviscoelastic material of integral type cannot be distinguished from a linear homogeneous isotropic thermo-elastic material characterized by Cattaneo-Vernotte-Kaliski's law.

In the case of weak coupling (see [6]) we have $\varepsilon\ll 1$, such that

$$U_+^2\simeq U_T^2(1+(1-\delta)^{-1}\varepsilon),\quad U_-^2\simeq U_L^2(1-(1-\delta)^{-1}\varepsilon).$$

If in addition $\delta\ll 1$, then

$$U_+^2\simeq U_T^2(1+\varepsilon),\quad U_-^2\simeq U_L^2(1-\varepsilon),$$

which implies $U_+\gg U_-$. Also

$$a_+\simeq\varepsilon U_T\sigma^{-1}b_+=\alpha U_+^{-1}b_+\ \text{ i.e. }\ a_+U_T^{-1}\simeq(\alpha\theta_0U_T^{-2})b_+\theta_0^{-1}.$$

Since $\varepsilon\ll 1$ it results $\alpha\theta_0U_T^{-2}\ll 1$, such that

$$|a_+U_T^{-1}|\ll|b_+\theta_0^{-1}|,$$

hence the mechanical component of the *thermomechanical* wave is weak in comparison with its thermal component. Analogously

$$|b_-\theta_0^{-1}|\ll|a_-U_L^{-1}|,$$

hence the thermal component of the *mechanicthermal* wave is weak in comparison with its mechanical component.

Thus, the experimental determination of the speeds of propagation U_+, U_- furnishes informations about only the initial values of the relaxation functions. These informations can be enriched if we study the behaviour of the amplitudes while the wave front advances in space and time.

3. Thermo-acoustical waves; determination of the amplitudes. We suppose in addition that the functions $\ddot\lambda(s)$, $\ddot\mu(s)$, $\ddot n(s)$, $\ddot p(s)$, $\ddot c(s)$, $\ddot\varkappa(s)$ are integrable on $\mathbf{R}_+$, $\lim\limits_{s\to\infty}\varkappa(s)<\infty$ and that the following strengthened hypothesis holds:

(A. 2)*. The functions $\dddot\lambda(s)$, $\dddot\mu(s)$, $\dddot n(s)$, $\dddot p(s)$, $\dddot c(s)$, $\dddot\varkappa(s)$ are continuous on $\mathbf{R}_+$, while the body forces and heat sources are of class C^1 on $\mathbf{B}\times(-\infty,\infty)$.

In this case the equations of balance give

$$\rho_0[\dddot u_i]=[\ddot T_{ij,j}],\quad \rho_0[\ddot\gamma]=-\theta_0^{-1}[\ddot q_{i,i}]. \tag{3.1}$$

The jumps from (3.1) can be calculated from the lemma 2.1 and the iterated geometrical-kinematical conditions of compatibility (see [18], II): in the writing of these conditions it was taken into account that U is constant, hence that the succesive wave fronts are *parallel surfaces*. Thus, denoting

$$[\dddot u_i]\equiv A_i,\quad [\dddot T]\equiv B,$$

it is not difficult to show that the equations (3.1) give

$$\begin{aligned}\rho_0A_i=&(\lambda_0+\mu_0)U^{-2}An_i+\mu_0U^{-2}A_i+n_0U^{-1}Bn_i+\\&+(\lambda_0+\mu_0)\Big\{-2U^{-2}\frac{\delta a}{\delta t}n_i-U^{-1}a^{\Gamma\Delta}(n_ix_{j,\Delta}+n_jx_{i,\Delta})a_{j,\Gamma}+\\&+U^{-1}a^{\Gamma\Delta}a^{\Delta\Phi}b_{\Gamma\Delta}x_{i,\Phi}x_{j,\Delta}a_j\Big\}+\mu_0\Big\{-2U^{-2}\frac{\delta a_i}{\delta t}+2U^{-1}\Omega a_i\Big\}+\\&+(\dot\lambda_0+\dot\mu_0)U^{-2}an_i+\dot\mu_0U^{-2}a_i-n_0\Big\{U^{-1}\frac{\delta b}{\delta t}n_i+a^{\Delta\Gamma}x_{i,\Delta}b_{,\Gamma}\Big\}+\dot n_0U^{-1}bn_i,\end{aligned} \tag{3.2}$$

$$\begin{aligned}&-n_0U^{-1}A+\rho_0c_0\theta_0^{-1}B+n_0U^{-1}\frac{\delta a}{\delta t}+n_0a^{\Delta\Gamma}x_{i,\Delta}a_{i,\Gamma}-\dot p_0U^{-1}a+\rho_0\dot c_0\theta_0^{-1}b=\\&=\varkappa_0(\tau\theta_0U^2)^{-1}B-2\varkappa_0(\tau\theta_0U^2)^{-1}\frac{\delta b}{\delta t}+2\varkappa_0(\tau\theta_0U)^{-1}\Omega b+\dot\varkappa_0(\tau\theta_0U^2)^{-1}b,\end{aligned} \tag{3.3}$$

where $\dot\lambda_0=\dot\lambda(0)$, $\dot\mu_0=\dot\mu(0)$, $\dot n_0=\dot n(0)$, $\dot p_0=\dot p(0)$, $\dot\varkappa_0=\dot\varkappa(0)$, $A=A_in_i$; $a^{\Delta\Gamma}$ and $b_{\Gamma\Delta}$ denote, respectively, the first and second fundamental form of the singular surface, $\dfrac{\delta}{\delta t}$ is the time derivative introduced by Thomas [49] and

$\Omega=\dfrac{1}{2}a^{\Gamma\Delta}b_{\Gamma\Delta}$ is the mean curvature of the surface.

Discussion. In the case of purely mechanical transversal waves, because $a_in_i=0$, we have

$$n_ja_{j,\Gamma}=a^{\Phi\Delta}b_{\Gamma\Delta}x_{j,\Phi}a_j. \tag{3.4}$$

Multiplying the equation (3.2) by a_i, taking into account (3.4), (2.17) and denoting $a^2=a_ia_i$, we obtain the following

Theorem 3.1. *The time variation of the magnitude of transversal wave is governed by the differential equation*

$$\frac{\delta a_\perp^2}{\delta t}=\frac{\dot\mu_0}{\mu_0}a_\perp^2+2U_\perp\Omega a_\perp^2\,. \tag{3.5}$$

The comparison of the expressions $(2.14)_1$ and (3.5) with (22) and (60) from [50], respectively, leads to

Remark. Concerning the speed of propagation and the amplitude of transversal wave, the linear homogeneous isotropic thermo-viscoelastic material of integral type cannot be distinguished from a linear homogeneous isotropic viscoelastic material of Boltzmann-Volterra type.

The integration of the equation (3.5) is made using the expression of Ω given in [51], § 25. We get

$$a_\perp^2(t)=a_\perp^2(0)e^{\dot\mu_0 t/\mu_0}(1-2\Omega_0U_\perp t+K_0U_\perp^2t^2). \tag{3.6}$$

where Ω_0 and K_0 denote the mean curvature and the Gaussian curvature of the surface S_t at the time $t=0$. From (3.6) and (2.16) it results

Theorem 3.2. *The (mechanical) amplitude of the transversal wave is attenuated in time (and space) if and only if $\dot\mu_0\leq 0$.*

When $\dot\mu_0<0$, the first term from (3.5) characterizes the attenuation due to viscous effects and the second that due to the wave form. The amplitude $a_\perp^2$, as well as the propagation velocity, are unaffected by thermal influences.

Thus, the experimental determination of the speed of propagation μ_0/ρ_0 and of the attenution coefficient $-\dot\mu_0/\mu_0$ in the case of the transversal wave can offer informations about the relaxation function $\mu(s)$ concerning only the values μ_0 and $\dot\mu_0$. However, if $\mu(s)=\mu_0e^{-s/\tau}$, μ_0 and τ being positive constants, the experimental study determines completely $\mu(s)$, because $\dot\mu_0=-\mu_0/\tau$.

In the case of coupled (longitudinal) waves we have $a_i=an_i$, which gives

$$a_{j,\Gamma}=a_{,\Gamma}n_j+an_{j,\Gamma}=a_{,\Gamma}n_j-aa^{\Phi\Delta}b_{\Gamma\Delta}x_{j,\Phi} \tag{3.7}$$

and the system (2.13) becomes

$$\begin{aligned}&(U^2-U_L^2)a-\alpha Ub=0\\ &-\sigma Ua+(U^2-U_T^2)b=0.\end{aligned} \tag{3.8}$$

Multiplying (3.7) by $a^{\Delta\Gamma}x_{j,\Delta}$, we obtain

$$a^{\Delta\Gamma}x_{j,\Delta}a_{j,\Gamma}=-2\Omega a. \tag{3.9}$$

Multiplying (3.2) by n_i and taking into account $\dfrac{\delta n_i}{\delta t}=0$, we find

$$\begin{aligned}\{\rho_0U^2-(\lambda_0+2\mu_0)\}A-n_0UB=&-2(\lambda_0+2\mu_0)\frac{\delta a}{\delta t}+\\ &+2(\lambda_0+2\mu_0)U\Omega a+(\dot\lambda_0+2\dot\mu_0)a-n_0U\frac{\delta b}{\delta t}+\dot n_0Ub.\end{aligned} \tag{3.10}$$

From (3.3) and (3.9) we get

$$\begin{aligned}-n_0UA+(\rho_0c_0\theta_0^{-1}U^2-\varkappa_0\tau^{-1}\theta_0^{-1})B=&-n_0U\frac{\delta a}{\delta t}+2n_0U^2\Omega a+\\ &+\dot p_0Ua-2\varkappa_0\tau^{-1}\theta_0^{-1}\frac{\delta b}{\delta t}+2\varkappa_0\tau^{-1}\theta_0^{-1}U\Omega b-\rho_0\dot c_0\theta_0^{-1}U^2b+\dot\varkappa_0\tau^{-1}\theta_0^{-1}b.\end{aligned} \tag{3.11}$$

Since the eigenvalues and eigenvectors problem (3.8) admits simple eigenvalues ($U_+>U_-$), it results that every eigenvector $(a(t), b(t))$ has the form $(a_0f(t), b_0f(t))$, where (a_0, b_0) denotes a solution of (3.8) for U_+ or U_- at a particular time t_0. We require $f(0)=1$ i.e. $t_0=0$. Multiplying (3.10) and (3.11) by a_0 and b_0 respectively, summing and taking into account that (a_0, b_0) verifies the system (3.8), we obtain

Theorem 3.3. *The differential equation which governs the behaviour of the function f is*

$$\frac{\delta f}{\delta t}=\frac{(\dot\lambda_0+2\dot\mu_0)a_0^2+(\dot n_0+\dot p_0)Ua_0b_0+(\dot\varkappa_0\tau^{-1}\theta_0^{-1}-\rho_0\dot c_0\theta_0^{-1}U^2)b_0^2}{2(\rho_0U_L^2a_0^2+n_0Ua_0b_0+\rho_0c_0\theta_0^{-1}U_T^2b_0^2)}f+U\Omega f. \tag{3.12}$$

We note that the denominator in (3.12) is strictely positive. This results by writting the system (3.8) for (a_0, b_0), multiplying $(3.8)_1$ by a_0, $(3.8)_2$ by b_0, summing the results and taking into account $(2.18)_{2,3}$. The equation (3.12) is of the same type as (3.5). Thus we have

Theorem 3.4. *The thermomechanical amplitude* (a, *b*) *of the longitudinal wave corresponding to U_+ or U_- is attenuated in time (and space) if and only if*

$$(\dot\lambda_0+2\dot\mu_0)a_0^2+(\dot n_0+\dot p_0)Ua_0b_0+(\dot\varkappa_0\tau^{-1}\theta_0^{-1}-\rho_0\dot c_0\theta_0^{-1}U^2)b_0^2\leq 0, \tag{3.13}$$

where (a_0, b_0) satisfies $(2.18)_1$ or $(2.18)_2$ for $U=U_+$ or $U=U_-$, respectively.

Even if $(1.3)_1$ holds, so that $p_0=n_0$, the condition (3.13) remains a restriction on the initial values of the relaxation functions and of their derivatives. When (3.13) holds, the first term from (3.12) characterizes the attenuation due to viscous and thermal effects and the second that due to the wave form. From the theorem 3.2, it results

Corollary 3.1. *In the case of an isothermal process ($b_0=0$), (3.13) is reduced at $\dot\lambda_0+2\dot\mu_0<0$ and for a rigid heat conductor ($a_0=0$, $U=U_T$), we obtain*

$$\dot c_0/c_0-\dot\varkappa_0/\varkappa_0>0. \tag{3.14}$$

If we admit $(1.3)_3$ and if we apply $(2.18)_{2,3}$ and $(1.2)_2$, it results that a *sufficient* condition for (3.14) to be valid is $\dot\varkappa_0<0$. If the thermal wave in the rigid heat conductor is plane, then the solution of (3.12) is

$$f(t)=e^{-\frac{1}{2}(\dot c_0/c_0-\dot\varkappa_0/\varkappa_0)t}. \tag{3.15}$$

The amplitudes determined by Tokuoka in [15] represent a particular case of those determined in this paper.

Conclusion. The accomplished study shows that if the relaxation functions have the properties specified by the theorems 2.1, 2.2, 3.2, 3.4 and corollary 3.1, then in the material defined by (1.1) are possible real transversal and longitudinal thermo-acoustical waves whose amplitudes are not growing at the same time with their propagation in space.

4. Plane harmonic waves in rigid heat conductors. It appears reasonable to expect that the method of plane harmonic waves gives the same results as those obtained with the method of the singular surfaces when the frequency is growing to infinity. We shall see that this expectation proves to be true in the case of the rigid heat conductor specified by the constitutive equation

$$\varepsilon(t) = 0_0\eta(t) = c_0 T(t) + \int_0^\infty \dot c(s)\, T(t-s)\,ds,$$

$$(4.1)\qquad q_i(t) = -\frac{1}{\tau}\int_0^\infty \varkappa(s)\, T_{,i}(t-s)\,ds,\quad \tau = \text{constant} > 0.$$

We choose a solution of the equations (4.1) and $(2.1)_2$, in which $r=0$, having the form

$$(4.2)\qquad T = be^{i(\omega t - l x\cdot n)},\quad l = \omega/v - i\alpha,\quad b = b_1 + ib_2,$$

with ω, v, α, b_1, b_2 real, ω — a given *frequency*, v — the *phase velocity*, α — the *attenuation coefficient*, physical meaning having the real part of T. From (4.2), (4.1) and $(2.1)_2$ we find

$$v^2(\omega) = 2\omega\,\frac{K_1^2 + K_2^2}{(C_1K_1 + C_2K_2)^2}\,\{C_2K_1 - C_1K_2 + \sqrt{(C_1^2 + C_2^2)(K_1^2 + K_2^2)}\},$$

$$(4.3)$$

$$\alpha^2(\omega) = \frac{\omega}{2(K_1^2 + K_2^2)}\,\{C_2K_1 - C_1K_2 + \sqrt{(C_1^2 + C_2^2)(K_1^2 + K_2^2)}\},$$

where

$$C_1 = C_1(\omega) = c_0 + \int_0^\infty \dot c(s)\cos\omega s\,ds,\quad C_2 = C_2(\omega) = \int_0^\infty \dot c(s)\sin\omega s\,ds,$$

$$K_1 = K_1(\omega) = \frac{1}{\rho_0\tau}\int_0^\infty \varkappa(s)\cos\omega s\,ds,\quad K_2 = K_2(\omega) = \frac{1}{\rho_0\tau}\int_0^\infty \varkappa(s)\sin\omega s\,ds.$$

It is easy to show that $v^2(\omega) > 0$, $\alpha^2(\omega) > 0$ and we shall take $v(\omega) > 0$, $\alpha(\omega) > 0$, the sense of propagation being determined by n. We are interested by the behaviour of $v(\omega)$ and $\alpha(\omega)$ when $\omega\to\infty$. To continue the study, additional assumptions are necessary. Also the following theorem is needed.

Theorem 4.1. (S n e d d o n [52], I, § 3). *If the function f satisfies the Dirichlet condition* (see [52]) *on* R_+, *then*

$$\lim_{\omega\to\infty}\int_0^\infty f(x)\cos\omega x\,dx = 0,\quad \lim_{\omega\to\infty}\int_0^\infty f(x)\sin\omega x\,dx = 0.$$

We suppose that the relaxation functions $\dot c(s)$, $\varkappa(s)$, $\ddot c(s)$, $\dot\varkappa(s)$, $\ddot\varkappa(s)$ satisfy the Dirichlet condition on R_+ and are integrable on R_+. It results that $\lim\limits_{s\to\infty}[\dot c(s), \varkappa(s), \ddot c(s), \dot\varkappa(s), \ddot\varkappa(s)]$ exist and are finite. In these conditions, after

an integration by part, we have the following evaluations :

$$C_1(\omega) = c_0 + \varepsilon_1(\omega)/\omega,\quad C_2(\omega) = \dot c_0/\omega + \varepsilon_2(\omega)/\omega,$$

$$(4.4)\qquad K_1(\omega) = -\dot\varkappa_0/\rho_0\tau\omega^2 + \varepsilon_3(\omega)/\omega^2,\quad K_2(\omega) = \varkappa_0/\rho_0\tau\omega + \varepsilon_4(\omega)/\omega^2,$$

where, by the theorem 4.1, $\lim\limits_{\omega\to\infty}\varepsilon_i(\omega) = 0$, $i = 1, 2, 3, 4$. Substituting (4.4) in (4.3), making $\omega\to\infty$ and taking into account (3.14) and (3.15), we obtain

Theorem 4.2. *When the frequency ω tends to infinity the phase velocity and the attenuation coefficient tend to the values v_∞, α_∞ obtained by using the method of singular surfaces :*

$$(4.5)\qquad v_\infty = U_T,\quad \alpha_\infty U_T = \dot c_0/c_\infty - \dot\varkappa_0/\varkappa_0 > 0.$$

In the remainder of this paper we shall make a comparative analysis of the models given by Fourier (I), Cattaneo-Vernotte-Kaliski (II) and Gurtin-Pipkin-McCarthy (III) for a rigid heat conductor in the case of plane harmonic waves. The constitutive equations which define these three models are :

$$(4.6)\ (\mathrm{I})\qquad \varepsilon(t) = c_0 T(t),\quad q_i(t) = -\varkappa_0 T_{,i}(t),$$

$$(4.7)\ (\mathrm{II})\qquad \varepsilon(t) = c_0 T(t),\quad \tau\dot q_i(t) + q_i(t) = -\varkappa_0 T_{,i}(t),\quad q_i(0) = 0,$$

$$\varepsilon(t) = c_0 T(t) + \int_0^\infty \dot c(s)\, T(t-s)\,ds,\quad c(s) = c_\infty - (c_\infty - c_0)e^{-s/\tau},$$

$$(4.8)\ (\mathrm{III})\qquad q_i(t) = -\frac{1}{\tau}\int_0^\infty \varkappa(s)\, T_{,i}(t-s)\,ds,\quad \varkappa(s) = \varkappa_0 e^{-s/\tau},$$

where c_0, c_∞, $\varkappa_0$, τ are positive constants with $c_\infty > c_0$ and τ — a relaxation time. For metals, $\tau \simeq 10^{-11}$ sec. (see [53]). The solution of $(4.7)_{2,3}$ is given by $(4.8)_2$. A similar calculus to that one which has given the expressions (4.3) shows that the phase velocities and the attenuation coefficients in the case of the three models are, respectively,

$$v_I(\omega) = \sqrt{2\varkappa_0\omega/\rho_0 c_0} = U_T\sqrt{2\omega\tau} = 2\tau U_T^2\alpha_I(\omega),$$

$$v_{II}(\omega) = 2\tau U_T^2\alpha_{II}(\omega) = U_T\sqrt{2\omega\tau}/\sqrt{\omega\tau + \sqrt{1 + \omega^2\tau^2}},$$

$$v_{III}(\omega) = M^{-1}2\tau U_T^2\omega\alpha_{III}(\omega) = U_T\sqrt{2\omega\tau}/\sqrt{\omega\tau + \sqrt{M^2 + \omega^2\tau^2}},$$

where $U_T^2 = \varkappa_0/\rho_0 c_0\tau$ (as in (2.14)) and $M = c_\infty/c_0 > 1$ is a measure of the memory of specific heat of the body. When $\tau\to 0$, (II) becomes (I) and when simultaneously $\tau\to 0$, $M = 1$, (III) becomes (I). The graphs from Figures 1 and 2 represent the variation of dimensionless phase speeds and dimensionless attenuation coefficients obtained by dividing $v(\omega)$ and multiplying $\alpha(\omega)$ to U_T and τU_T, respectively.

It follows from Figures 1 and 2 that

(i) When $\omega\to\infty$, we have $v_I\to\infty$, $v_{II}\to U_T$, $v_{III}\to U_T$ and $\alpha_I\to\infty$, $\alpha_{II}\to(\tau U_T)^{-1}$, $\alpha_{III}\to M(2\tau U_T)^{-1}$, according with the theory of singular surfaces. We see that when M varies, v_{III} tends to the same limit, while $\lim\limits_{\omega\to\infty}\alpha_{III}(\omega)$

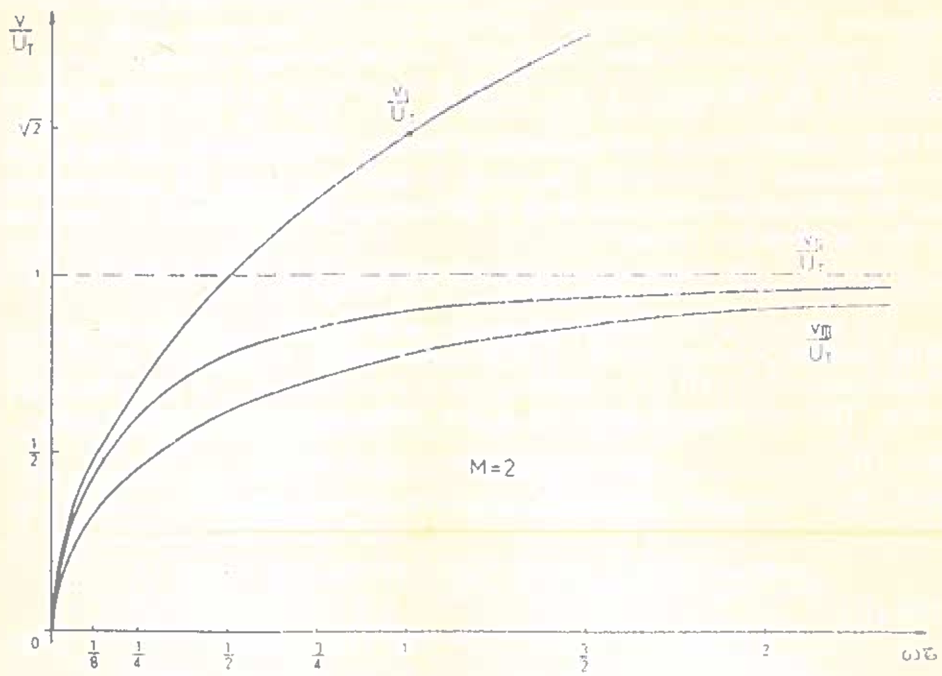

Fig. 1 — Variation of the three dimensionless phase velocities versus $\omega\tau$

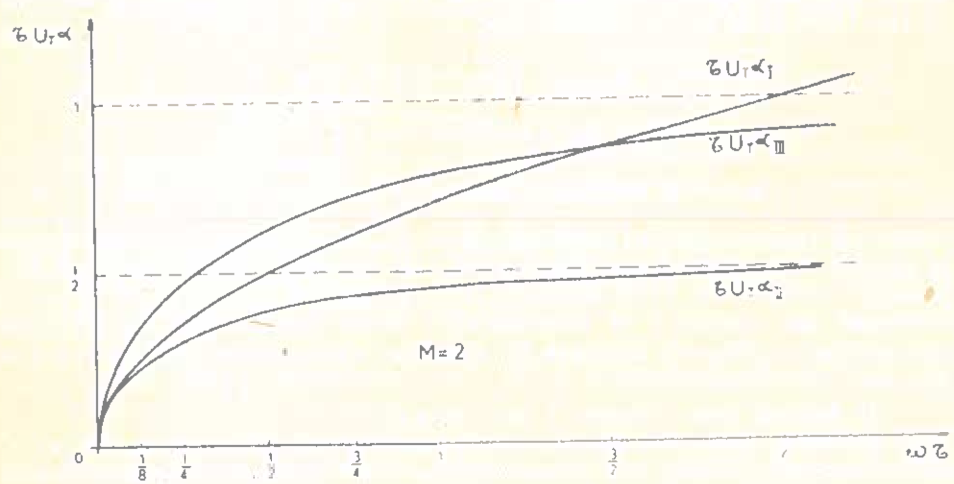

Fig. 2 — Variation of the three dimensionless attenuation coefficients versus $\omega\tau$

grows together with M. Hence, the memory effects for the model (III) when $\omega\to\infty$ are felt only in the attenuation coefficient.

(ii) For $\omega\to 0$, hence $\omega\tau$ in the neighbourhood of the origin, the model (I) estimates very well the other two. Since the frequencies in nature are usually such that $\omega\tau << 1$ (see [54]), this may constitue an explanation for the success of the Fourier's law.

(iii) If $M\simeq 1$, the models (II) and (III) can be used equally for every. If $M\geqslant 1$, they give nearing values for the phase velocities only when $\omega\to 0$ and $\omega\to\infty$ and for the attenuation coefficient only when $\omega\to 0$.

(iv) The phase velocities are in the relations $v_{III}<v_{II}<v_{I}$ for every ω, according to the dissipative nature of the models.

(v) When $\omega<(M^2-1)/2\tau$, α_{III} does not differ too much from α_I. When $\omega=(M^2-1)/2\tau$, $\alpha_{III}=\alpha_I$.

(vi) $\omega^*=\tau^{-1}$ has the role of a characteristic frequency for models (II) and (III). For $\omega>\omega^*$ i.e. $\omega\tau>1$, the differences between (I) on the one hand and (II), (III) on the other hand become signifiant qualitatively as well as quantitatively.

Conclusion. The models (I), (II) and (III) can be used equally if $\omega\to 0$. At the same time with the growth of ω are growing the differences between them. It depends upon the studied problem if these differences are important or not.

In a next paper I shall investigate the propagation of strong thermo-mechanical waves in the material defined by (1.1).

Aknowledgement. I am grateful to Mr. E. Soós for his encouragements and criticisms of a previous draft of this manuscript.

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Received 18.X.1986

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Analele științifice ale Universității „Al. I. Cuza" din Iași
Tomul XXXIV, s. I a, Matematică, 1988, f. 1

RECENZII

J. V. FIELD, J. J. GRAY (Editors): *The Geometrical Work of Girard Desargues*, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1987, X + 237 p., DM 158

În această carte este prezentată în principal o traducere în engleză a broșurii lui G. Desargues „Brouillon project d'une atteinte aux événements des rencontres du cone avec un plan", apărută în 1639 și considerată cea mai importantă cercetare a lui G. Desargues în domeniul matematicilor pure. Pe lîngă această traducere mai sînt prezentate alte cercetări de geometrie practică, cum ar fi cele asupra perspectivei și cele în domeniul ingineriei militare și arhitecturii. Editorii încadrează aceste lucrări într-o perspectivă istorică, contribuind cu comentarii importante la înțelegerea dezvoltării geometriei în secolul al XVII-lea. Acest ultim aspect este discutat chiar în primul capitol al cărții. În capitolul al doilea sînt prezentate diversele aspecte ale geometriei aplicate așa cum rezultă din lucrările lui Desargues și lucrările altor contemporani. În continuare în capitolele III—V, sînt comentate diverse aspecte matematice din broșura mai sus citată a lui Desargues. Traducerea propriu-zisă în engleza este prezentată în capitolul al VI-lea, iar în capitolele VII—IX sînt prezentate traducerile altor lucrări cu caracter aplicativ ale lui Desargues. Cartea mai conține cinci appendixuri în care sînt reproduse scrisori ale lui R. Descartes și De Beaugrand către Desargues. Eseul asupra conicelor al lui Pascal, introducerea punctelor de la infinit de către Kepler și textul francez, fotocopiat al lucrării „Perspectiva" a lui Desargues. Cartea mai conține o bogată bibliografie de istoria matematicii, precum și un index de termeni tehnici în broșura lui Desargues.

V. Oproiu

H. LEOFELL: *Blaise Pascal. 1623—1662.* (Vita Mathematica, Bd. 2), Birkhäuser Verlag, Basel, Boston, Stutgart, 1987, 175 p., sFr. 40.00

Prezentarea vieții lui Blaise Pascal, matematician și filozof francez, constituie subiectul celui de-al doilea volum din această excelentă colecție a editurii Birkhäuser, care se cheamă Vita Mathematica. Concepută după un plan original, cartea prezintă contribuția lui Pascal în planul matematicii (geometria proiectivă în calculul aritmetic, în calculul probabilităților, în calculul infinitezimal) și în planul filozofic (în special, reflecții asupra metodelor matematice, cosmogonici, fizicii). Este de prisos să insistăm asupra utilității citirii cărții de către orice cercetător de istoria matematicii, de orice filozof și, în general, de orice om de cultură.

Mircea Ștefănescu

W. PURKERT, H. J. ILGAUDS: *Georg Cantor. 1845—1918.* (Vita Mathematica, Bd. 1), Birkhäuser Verlag, Basel, Boston, Stuttgart, 1987, 262 p. sFr. 48,00

Monografia prezintă viața matematicianului Georg Cantor, cunoscut ca inițiatorul studierii mulțimilor. Scrisă într-un stil elegant și concis, cartea reușește să reînvie personalitatea lui Cantor, ca matematician și filozof, utilizîndu-se, în acest scop, atît documente din epocă, lucrărilor lui Cantor, dar și cărții de istoria matematicii în care sînt prezentate rezultatele matematice obținute de matematicianul german și discuțiile pe care le-au generat antinomiile din teoria mulțimi-