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ON A TYPE OF SEMI–SYMMETRIC METRIC CONNECTION ON A RIEMANNIAN MANIFOLD

BY

U. C. DE

Introduction. The present paper deals with a type of semi-symmetric metric connection D [1] on a Riemannian manifold (M^n, g) $(n>3)$ such that the curvature tensor R of D satisfies the relation

$$(\bar{D}_V \bar{D}_U R)(X, Y)Z = B(V)(\bar{D}_U R)(X, Y)Z + P(U, V)R(X, Y)Z, \tag{1}$$

where B is a i-form, P is a (0,2) tensor and $\bar{D}$ denotes the Levi-Civita connection. It is shown that if (1) is satisfied i.e., if the curvature tensor R of D is generalised 2-recurrent [2], then the manifold (M^n, g) is generalised conformally 2-recurrent [2].

1. Preliminaries. Let (M^n, g) be an n-dimensional $(n>3)$ Riemannian manifold with Levi-Civita connection $\bar{D}$. A linear connection D on (M^n, g) is said to be semi-symmetric if its torsion tensor T satisfies the relation

$$T(X, Y) = \omega(Y)X - \omega(X)Y \tag{1.1}$$

where ω is a 1-form.
It is known [1] that if D is a semi-symmetric metric connection, then

$$D_X Y = \bar{D}_X Y + \omega(Y)X - g(X, Y)\rho \tag{1.2}$$

where

$$g(X, \rho) = \omega(X) \tag{1.3}$$

for every vector field X. Further, it is also known [1] that if R and K denote the curvature tensor of D and $\bar{D}$ respectively, then

$$R(X, Y)Z = K(X, Y)Z - \alpha(Y, Z)X + \alpha(X, Z)Y - g(Y, Z)AX + g(X, Z)AY \tag{1.4}$$

where α is a tensor field of type (0,2) defined by

$$\alpha(X, Y) = (\bar{D}_X\omega)(Y) - \omega(X)\omega(Y) + \tfrac{1}{2}\omega(\rho)\, g(X, Y) \tag{1.5}$$

and A is a tensor field of type (1, 1) defined by

$$g(AX, Y) = \alpha(X, Y) \tag{1.6}$$

for any vector fields X, Y.
We shall use the above results in the next section.

2. A special type of semi-symmetric connection. In this section we consider a semi-symmetric metric connection D on (M^n, g) whose curvature tensor satisfies the condition

$$(\bar{D}_V\bar{D}_U R)(X, Y)Z = B(V)(\bar{D}_U R)(X, Y)Z + P(U, V)R(X, Y)Z \tag{2.1}$$

where B is a 1-form and P is a (0,2) tensor. From (1.4) we get

$$\begin{aligned}(\bar{D}_V\bar{D}_U R)(X, Y)Z &= (\bar{D}_V\bar{D}_U K)(X, Y)Z - [(\bar{D}_V\bar{D}_U\alpha)(Y, Z)]X + \\ &+ [(\bar{D}_V\bar{D}_U\alpha)(X, Z)]Y - g(Y, Z)(\bar{D}_V\bar{D}_U A)(X) + g(X, Z)(\bar{D}_V\bar{D}_U A)(Y)\end{aligned} \tag{2.2}$$

In virtue of (2.1) we can express (2.2) as follows

$$\begin{aligned}&[(\bar{D}_V\bar{D}_U K)(X, Y)Z - B(V)(\bar{D}_U K)(X, Y)Z - P(U, V)K(X, Y)Z] - \\ &-[(\bar{D}_V\bar{D}_U\alpha)(Y, Z) - B(V)(\bar{D}_U\alpha)(Y, Z) - P(U, V)\alpha(Y, Z)]X + \\ &+[(\bar{D}_V\bar{D}_U\alpha)(X, Z) - B(V)(\bar{D}_U\alpha)(X, Z) - P(U, V)\alpha(X, Z)]Y - \\ &-g(Y, Z)[(\bar{D}_V\bar{D}_U A)(X) - B(V)(\bar{D}_U A)(X) - P(U, V)A(X)] + \\ &+g(X, Z)[(\bar{D}_V\bar{D}_U A)(Y) - B(V)(\bar{D}_U A)(Y) - P(U, V)A(Y)] = 0\end{aligned} \tag{2.3}$$

Let C denote Weyl's conformal curvature tensor of (M^n, g). Then

$$C(X, Y)Z = K(X, Y)Z + \lambda(Y, Z)X - \lambda(X, Z)Y + g(Y, Z)LX - g(X, Z)LY \tag{2.4}$$

where

$$\lambda(Y, Z) = -\frac{1}{n-2}S(Y, Z) + \frac{r}{2(n-1)(n-2)}g(Y, Z) \tag{2.5}$$

and L is a (1, 1) tensor field such that

$$g(LY, Z) = \lambda(Y, Z), \tag{2.6}$$

S and r being the Ricci tensor and the scalar curvature of (M^n, g) respectively. From (2.4) we get

$$\begin{aligned}(\bar{D}_V\bar{D}_U C)(X, Y)Z &= (\bar{D}_V\bar{D}_U K)(X, Y)Z + [(\bar{D}_V\bar{D}_U\lambda)(Y, Z)]X - \\ &-[(\bar{D}_V\bar{D}_U\lambda)(X, Z)]Y + g(Y, Z)(\bar{D}_V\bar{D}_U L)(X) - g(X, Z)(\bar{D}_V\bar{D}_U L)(Y)\end{aligned} \tag{2.7}$$

Since $\bar{D}g = 0$ we have $(\bar{D}_V\bar{D}_U g)(LY, Z) = 0$, from which we get, in virtue of (2.6),

$$(\bar{D}_V\bar{D}_U\lambda)(Y, Z) = g[(\bar{D}_V\bar{D}_U L)(Y), Z] \tag{2.8}$$

Again, since $(\bar{D}_V\bar{D}_U g)(AY, Z) = 0$ from this we get by using (1.6)

$$(\bar{D}_V\bar{D}_U\alpha)(Y, Z) = g[(\bar{D}_V\bar{D}_U A)(Y), Z] \tag{2.9}$$

Now from (2.1) we get

$$(\bar{D}_V\bar{D}_U S')(Y, Z) = B(V)(\bar{D}_U S')(Y, Z) + P(U, V)S'(Y, Z) \tag{2.10}$$

and

$$\bar{D}_V\bar{D}_U r' = B(V)\bar{D}_U r' + P(U, V)r' \tag{2.11}$$

where S' and r' are the Ricci tensor and the scalar curvature of D respectively. Again, from (1.4) it follows that

$$S'(Y, Z) = S(Y, Z) - (n-2)\alpha(Y, Z) - ag(Y, Z) \tag{2.12}$$

and

$$r' = r - 2(n-1)a \tag{2.13}$$

where a is the trace of A. From (2.12) we obtain

$$\begin{aligned}(\bar{D}_V\bar{D}_U S')(Y, Z) &= (\bar{D}_V\bar{D}_U S)(Y, Z) - (n-2)(\bar{D}_V\bar{D}_U\alpha)(Y, Z) - \\ &-(\bar{D}_V\bar{D}_U a)g(Y, Z).\end{aligned} \tag{2.14}$$

Again, from (2.13) we have

$$\bar{D}_V\bar{D}_U r' = \bar{D}_V\bar{D}_U r - 2(n-1)\bar{D}_V\bar{D}_U a$$

which gives, in virtue of (2.11) and (2.13),

$$\begin{aligned}\bar{D}_V\bar{D}_U a - B(V)\bar{D}_U a - P(U, V)a = \frac{1}{2(n-1)}[\bar{D}_V\bar{D}_U r - B(V)\bar{D}_U r - \\ -P(U, V)r]\end{aligned} \tag{2.15}$$

Using (2.10) we can express (2.14) as

$$\begin{aligned}B(V)(\bar{D}_U S')(Y, Z) + P(U, V)S'(Y, Z) &= (\bar{D}_V\bar{D}_U S)(Y, Z) - \\ &-(n-2)(\bar{D}_V\bar{D}_U\alpha)(Y, Z) - (\bar{D}_V\bar{D}_U a)g(Y, Z)\end{aligned}$$

which gives in virtue of (2.12) and (2.15)

$$\begin{aligned}&(n-2)[(\bar{D}_V\bar{D}_U\alpha)(Y, Z) - B(V)(\bar{D}_U\alpha)(Y, Z) - P(U, V)\alpha(Y, Z)] = \\ &= [(\bar{D}_V\bar{D}_U S)(Y, Z) - B(V)(\bar{D}_U S)(Y, Z) - P(U, Y)S(Y, Z)] - \\ &-\frac{1}{2(n-1)}g(Y, Z)[\bar{D}_V\bar{D}_U r - B(V)\bar{D}_U r - P(U, V)r]\end{aligned}$$

Now using (2.5) we obtain

$$\begin{aligned}(\bar{D}_V\bar{D}_U\alpha)(Y, Z) - B(V)(\bar{D}_U\alpha)(Y, Z) - P(U, V)\alpha(Y, Z) = \\ = -(\bar{D}_V\bar{D}_U\lambda)(Y, Z) + B(V)(\bar{D}_U\lambda)(Y, Z) + P(U, V)\lambda(Y, Z)\end{aligned} \tag{2.16}$$

In virtue of (2.9), (2.16), (2.8) and (2.6) we obtain

$$\begin{aligned}(\bar{D}_V\bar{D}_U A)(Y) - B(V)(\bar{D}_U A)(Y) - P(U, V)AY = -(\bar{D}_V\bar{D}_U L)(Y) + \\ + B(V)(\bar{D}_U L)(Y) + P(U, V)LY\end{aligned} \tag{2.17}$$

Using (2.16) and (2.17) we can express (2.3) as follows

$$\begin{aligned}&(\bar{D}_V\bar{D}_U K)(X, Y)Z + [(\bar{D}_V\bar{D}_U\lambda)(Y, Z)]X - [(\bar{D}_V\bar{D}_U\lambda)(X, Z)]Y + \\ &+ g(Y, Z)(\bar{D}_V\bar{D}_U L)(X) - g(X, Z)(\bar{D}_V\bar{D}_U L)(Y) = B(V)\{(\bar{D}_U K)(X, Y)Z +\end{aligned}$$

$$+[(\bar{D}_U\lambda)(Y,Z)]X-[(\bar{D}_U\lambda)(X,Z)]Y+g(Y,Z)(\bar{D}_UL)(X)-$$
$$-g(X,Z)(\bar{D}_UL)(Y)\}+P(U,V)[K(X,Y)Z+\lambda(Y,Z)X-$$
$$-\lambda(X,Z)Y+g(Y,Z)LX-g(X,Z)LX]$$

which gives by virtue of (2.7) and (2.4)

$$(\bar{D}_V\bar{D}_UC)(X,Y)Z=B(V)(\bar{D}_UC)(X,Y)Z+P(U,V)C(X,Y)Z \tag{2.18}$$

From (2.18) we can state the following theorem

Theorem 1. *If a Riemannian manifold (M^n, g) $(n>3)$ admits a semi-symmetric metric connection whose curvature tensor is a generalised 2-recurrent tensor with respect to the Levi-Civita connection having B as its associated 1-form and P as its tensor of recurrence, then the manifold (M^n, g) is generalised conformally 2-recurrent with B as its 1-form and P as its tensor of recurrence.*

If, in particular, B is zero, then the curvature tensor of D becomes 2-recurrent [3] with respect to the Levi-Civita connection and consequently (M^n, g) becomes conformally 2-recurrent [4]. Hence we have the following Corollary

Corollary. *If a Riemannian manifold (M^n, g) $(n>3)$ admits a semi-symmetric metric connection whose curvature tensor is 2-recurrent with respect to the Levi-Civita connection, then (M^n, g) is conformally 2-recurent [4].*

The case considered in the above Corollary has been treated by the author [5].

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PARAMETER DEPENDENT VECTOR FIELDS

BY

CĂTĂLIN ŢIGĂERU

In [3] the notion of smooth distribution is introduced as a function associating to each point $x\in M$, the linear subspace $\Delta(x)=\mathrm{sp}\{X_\lambda(x),\ \lambda\in\Lambda\}$, where $\{X_\lambda,\ \lambda\in\Lambda\}$ is an everywhere defined family of vector fields (i.e. $\bigcup_{\lambda\in\Lambda}\mathrm{dom}\,X_\lambda=M$). In [3] the maximal integral manifold property is also defined.

In this paper we shall study the dependence on λ of this distribution and its implications to the intgrabillity of Δ.

1. Definition (1.1). *Let M be a smooth and finite dimensional manifold and $\Lambda\subseteq\mathbf{R}$ a set of parameters; a map $X: M\times\Lambda\to TM$ is a parameter dependent vector field if:*

(i) $X(x,\lambda)\in T_xM$, for any $\lambda\in\Lambda$ such that $X(x,\lambda)$ is defined, and $U_\lambda=\{x\in M;\ X(x,\lambda)\in T_xM\}$, where λ is fixed, is an open set of M.

(ii) $X_\lambda: U_\lambda\to TM$, $X_\lambda(x)=X(x,\lambda)$ is a C^∞ — vector field.

(iii) $\Lambda(x)=\{\lambda\in\Lambda;\ X(x,\lambda)\in T_xM\}$ is an open set in $\mathbf{R}$.

The family $\{X_\lambda;\ \lambda\in\Lambda\}$ has the following property

$$\bigcup_{\lambda\in\Lambda}U_\lambda=M,\quad U_\lambda=\mathrm{dom}\,X_\lambda.$$

Example. For $M=\mathbf{R}^2$, let $X: M\times\mathbf{R}\to TM$ be defined by

$$X((x,y),\lambda)=\frac{1}{\lambda-x}\cdot\frac{\partial}{\partial x}(x,y)=\left(\frac{1}{\lambda-x},\ 0\right).$$

Here we have $U_\lambda=\{(x,y)\in M;\ x\neq\lambda\}$ and $\Lambda(x)=\{\lambda\in\mathbf{R};\ \lambda\neq x\}$.

Then X is a parameter dependent vector field.

It is clear that every parameter dependent vector field spans a smooth distribution in the above sense

$$\Delta: x\to\Delta(x),\quad \Delta(x)=\mathrm{sp}\{X_\lambda(x);\ \lambda\in\Lambda(x)\}.$$

Next we impose that

(iv) X has continuous partial derivative with respect to λ, for every $x\in M$.

Then there exists a parameter dependent vector field

$$\partial X: M\times\Lambda\to TM \text{ given by } \partial X(x,\lambda)=\frac{\partial X}{\partial\lambda}(x,\lambda),\ (x,\lambda)\in\mathrm{dom}\,X.$$

We denote by $\partial\Delta$ the distribution $\partial\Delta(x)=\mathrm{sp}\left\{\frac{\partial X}{\partial\lambda}(x,\lambda);\ \lambda\in\Lambda(x)\right\}$.