$$+[(\bar{D}_U\lambda)(Y, Z)]X-[(\bar{D}_U\lambda)(X, Z)]Y+g(Y, Z)(\bar{D}_UL)(X)-$$
$$-g(X, Z)(\bar{D}_UL)(Y)\}+P(U, V)[K(X, Y)Z+\lambda(Y, Z)X-$$
$$-\lambda(X, Z)Y+g(Y, Z)LX-g(X, Z)LX]$$

which gives by virtue of (2.7) and (2.4)

$$(\bar{D}_V\bar{D}_UC)(X, Y)Z=B(V)(\bar{D}_UC)(X, Y)Z+P(U, V)C(X, Y)Z \tag{2.18}$$

From (2.18) we can state the following theorem

Theorem 1. *If a Riemannian manifold (M^n, g) $(n>3)$ admits a semi-symmetric metric connection whose curvature tensor is a generalised 2-recurrent tensor with respect to the Levi-Civita connection having B as its associated 1-form and P as its tensor of recurrence, then the manifold (M^n, g) is generalised conformally 2-recurrent with B as its 1-form and P as its tensor of recurrence.*

If, in particular, B is zero, then the curvature tensor of D becomes 2-recurrent [3] with respect to the Levi-Civita connection and consequently (M^n, g) becomes conformally 2-recurrent [4]. Hence we have the following Corollary

Corollary. *If a Riemannian manifold (M^n, g) $(n>3)$ admits a semi-symmetric metric connection whose curvature tensor is 2-recurrent with respect to the Levi-Civita connection, then (M^n, g) is conformally 2-recurent [4].*

The case considered in the above Corollary has been treated by the author [5].

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Department of Mathematics
University of Kalyani
Kalyani — 741235
West Bengal, India

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PARAMETER DEPENDENT VECTOR FIELDS

BY

CĂTĂLIN ŢIGĂERU

In [3] the notion of smooth distribution is introduced as a function associating to each point $x\in M$, the linear subspace $\Delta(x)=\mathrm{sp}\{X_\lambda(x),\ \lambda\in\Lambda\}$, where $\{X_\lambda,\ \lambda\in\Lambda\}$ is an everywhere defined family of vector fields (i.e. $\bigcup_{\lambda\in\Lambda}\mathrm{dom}\,X_\lambda=M$). In [3] the maximal integral manifold property is also defined.

In this paper we shall study the dependence on λ of this distribution and its implications to the intgrabillity of Δ.

1. Definition (1.1). *Let M be a smooth and finite dimensional manifold and $\Lambda\subseteq \mathrm{R}$ a set of parameters; a map $X: M\times\Lambda\to TM$ is a parameter dependent vector field if:*

(i) $X(x,\lambda)\in T_xM$, for any $\lambda\in\Lambda$ such that $X(x,\lambda)$ is defined, and $U_\lambda=\{x\in M;\ X(x,\lambda)\in T_xM\}$, where λ is fixed, is an open set of M.

(ii) $X_\lambda: U_\lambda\to TM$, $X_\lambda(x)=X(x,\lambda)$ is a C^∞ — vector field.

(iii) $\Lambda(x)=\{\lambda\in\Lambda;\ X(x,\lambda)\in T_xM\}$ is an open set in R.

The family $\{X_\lambda;\ \lambda\in\Lambda\}$ has the following property

$$\bigcup_{\lambda\in\Lambda}U_\lambda=M,\quad U_\lambda=\mathrm{dom}\,X_\lambda.$$

Example. For $M=\mathbf{R}^2$, let $X: M\times\mathbf{R}\to TM$ be defined by

$$X((x, y),\lambda)=\frac{1}{\lambda-x}\cdot\frac{\partial}{\partial x}(x, y)=\left(\frac{1}{\lambda-x},\ 0\right).$$

Here we have $U_\lambda=\{(x, y)\in M;\ x\neq\lambda\}$ and $\Lambda(x)=\{\lambda\in\mathrm{R};\ \lambda\neq x\}$.
Then X is a parameter dependent vector field.
It is clear that every parameter dependent vector field spans a smooth distribution in the above sense

$$\Delta: x\to\Delta(x),\quad \Delta(x)=\mathrm{sp}\{X_\lambda(x);\ \lambda\in\Lambda(x)\}.$$

Next we impose that

(iv) X has continuous partial derivative with respect to λ, for every $x\in M$.

Then there exists a parameter dependent vector field

$\partial X: M\times\Lambda\to TM$ given by $\partial X(x,\lambda)=\dfrac{\partial X}{\partial\lambda}(x,\lambda)$, $(x,\lambda)\in\mathrm{dom}\,X$.

We denote by $\partial\Delta$ the distribution $\partial\Delta(x)=\mathrm{sp}\left\{\dfrac{\partial X}{\partial\lambda}(x,\lambda);\ \lambda\in\Lambda(x)\right\}$.

In local coordinates, these definitions become: if $\frac{\partial}{\partial x^1}, \dots, \frac{\partial}{\partial x^m}$ is a local basis for $TM|_U$, where U is a coordinate neighbourhood of a point x, then

$$X(x, \lambda) = X_1(x, \lambda)\frac{\partial}{\partial x^1} + \dots + X_m(x, \lambda)\frac{\partial}{\partial x^m} \quad \text{and} \tag{1.2}$$

$$\frac{\partial X}{\partial \lambda}(x, \lambda) = \frac{\partial X_1}{\partial \lambda}(x, \lambda) + \dots + \frac{\partial X_m}{\partial \lambda}(x, \lambda), \tag{1.3}$$

Proposition (1.4). *The following relation holds:* $\partial\Delta \subseteq \Delta$.

Proof. Let $x \in M$, $\lambda \in \Lambda(x)$ and $X_\lambda(x)$, $X_{\lambda+\Delta\lambda}(x)$ be two vectors, such that $\lambda + \Delta\lambda \in \Lambda(x)$; then $X_{\lambda+\Delta\lambda}(x) - X_\lambda(x)$ belongs to $\Delta(x)$ and $\frac{1}{\Delta\lambda}(X_{\lambda+\Delta\lambda}(x) - X_\lambda(x))$, too. Because $\Delta(x)$ is a closed set of T_xM it follows that $\lim\limits_{\Delta\lambda\to 0}\frac{1}{\Delta\lambda}(X_{\lambda+\Delta\lambda}(x) - X_\lambda(x)) = \frac{\partial X}{\partial\lambda}(x, \lambda))$ belongs to $\Delta(x)$.

Corollary (1.5). *If X is k times differentiable with respect to λ, then $\partial^k\Delta \subseteq \subseteq \dots \subseteq \partial\Delta \subseteq \Delta$.*

2. First of all, we remind some definitions and main results ([2], [3], [4]).

Definition. (2.1). *A submanifold $S \subseteq M$ is said to be integral submanifold of Δ if, for any $x \in S$, the tangent space T_xS is equal with $\Delta(x)$.*

Definition (2.2). *A maximal integral submanifold of Δ is a connected integral submanifold of Δ with the property that every other connected integral submanifold of Δ, which contains S, coincides with S.*

Definition (2.3). *A distribution Δ has the maximal integral manifold property if through every $x \in M$, passes a maximal integral submanifold of Δ.*

Hector Sussman established in [3] (theorem 4.2) a necessary and sufficient condition for a distribution to have maximal manifold property. Another aspects of these problems can be found in [2] and [4].

Sussman's theorem. *A distribution Δ has the maximal integral manifold property if and only if, for every vector field $X \in \Delta$ and every pair $(t, x) \in R \times M$ such that the flow $X_t(x)$ is defined, the following relation holds:*

$$(^*)\ dX_t(\Delta(x)) \subseteq \Delta(X_t(x)),$$

where dX_t is the differential of X_t and $t \to X_t(x)$ is the integral curve of X through x

Using Sussman's theorem, we give now a new condition for Δ to hav. the integral manifold property.

We also assume hence forward that

(v) Δ has the property that the set $\bigcap\limits_{\lambda\in\Lambda(x)} U_\lambda$ is a neighbourhood of x.

Theorem (2.4). *The distribution Δ has the maximal integral manifold property if and only if, for any $x \in M$, $\lambda \in \Delta(x)$, it results that $dX_{\lambda,t}(\partial\Delta(x)) \subseteq \subseteq \Delta(X_{\lambda,t}(x))$.*

For the proof we need the lemma:

Lemma (2.5.) *If X is a vector field and X_t its flow, then*

$$dX_t(X(x)) = X(X_t(x)).$$

Proof. Let $x \in M$ be a fixed point, and $V(t) = dX_{-t}(X(X_t(x)))$. Then $V(t) \in T_xM$ and, by Lemma 6.2 from [3], we have

$$\frac{dV}{dt}(t) = dX_{-t}([X, X](X_t(x))) = 0.$$

Because $V(0) = X(x)$ it results that $V(t) = X(x)$. Then $dX_t(X_{(x)}) = X(X_{t(x)})$.

Proof of Theorem (2.4). Our main argument is the following: if a curve $c: \Lambda \subseteq R \to V$, where V is a linear space, has the property that its derivative with respect to λ, belongs to Δ, where Δ is a linear subspace of V, and there is a point λ_0 such that $c(\lambda_0) \in \Delta$ then $c(\Lambda) \subseteq \Delta$.

For each $\lambda \in \Delta(x)$ and $t \in (-\varepsilon, \varepsilon)$, we define the curve

$$c: \Lambda(x) \to T_{X_{\lambda,t}(x)}M, \quad c(\mu) = dX_{\lambda,t}(X_\mu(x)).$$

We shall prove that the image of this curve is entirely contained in $\Delta(X_{\lambda,t}(x))$. It follows that

$$\frac{dc}{d\mu}(\mu) = \lim_{\Delta\mu\to 0}\frac{1}{\Delta\mu}[dX_{\lambda,t}(X_{\mu+\Delta\mu}(x)) - dX_{\lambda,t}(X_\mu(x))] =$$

$$= dX_{\lambda,t}\left(\frac{\partial X}{\partial\lambda}(x, \mu) \in \Delta(X_{\lambda,t}(x))\right) \text{ (from hypotesis)}$$

and for $\mu = \lambda$ we find $dX_{\lambda,t}(X_\lambda(x)) = X_\lambda(X_{\lambda,t}(x)) \in \Delta(X_{\lambda,t}(x))$, by Lemma (2.5). Consequently, the curve c satisfies the properties $\frac{dc}{d\mu} \in \Delta(X_{\lambda,t}(x))$ and there is $\lambda \in \Delta(x)$, such that $c(\lambda) \subseteq \Delta(X_{\lambda,t}(x))$.

Because the familly $\{X_\mu(x);\ \mu \in \Lambda\}$ spans $\Delta(x)$, it follows that

$$dX_{\lambda,t}(\Delta(x)) \subseteq \Delta(X_{\lambda,t}(x)).$$

Conversely, if Δ satisfies $(^*)$, then $\partial\Delta \subseteq \Delta$ implies that

$$dX_{\lambda,t}(\partial\Delta(x)) \subseteq \Delta(X_{\lambda,t}(x)), \text{ for any } \lambda \in \Lambda(x).$$

Example (2.5). Let $M = R^2$, $p = (x, y)$ and

$$X(p, \lambda) = \frac{\partial}{\partial x}(p) + xa(\lambda)\frac{\partial}{\partial y}(p), \quad \text{where} \quad a \in C^1(\mathbf{R}).$$

The integral curve of X_λ through p is

$$p_\lambda(t) = X_{\lambda,t}(p) = \begin{cases} x(t) = x + t \\ y(t) = \dfrac{a(\lambda)t^2}{2} + ta(\lambda)x + y \end{cases} \text{ and}$$

the flow is $X_{\lambda,t}: M \to M$, $X_{\lambda,t}(p) = p_\lambda(t)$, and the differential $dX_{\lambda,t}: T_pM \to \to T_{X_{\lambda,t}(p)}M$ becames

$$\begin{Vmatrix} 1 & 0 \\ ta(\lambda) & 1 \end{Vmatrix}.$$

We conclude that

$$\Delta(p)=\begin{cases} \left.\begin{array}{ll} \mathrm{sp}\left\{\dfrac{\partial}{\partial x}(p)\right\}, & \text{when } x=0 \\ T_pM, & \text{when } x\neq 0 \end{array}\right\} & \text{if } a'(\lambda)\not\equiv 0 \\ \mathrm{sp}\{X(p,\lambda_0)\}, & \text{if } a'(\lambda)\equiv 0 \text{ (i.e.} a(\lambda)=\lambda_0) \end{cases}$$

and

$$\partial\Delta(p)=\begin{cases} \mathrm{sp}\left\{\dfrac{\partial}{\partial y}(p)\right\}, & \text{if } a'\not\equiv 0 \\ 0 \quad , & \text{if } a'\equiv 0 \text{ or if } x=0. \end{cases}$$

We observe that the distribution Δ has the maximal integral manifold property if and only if $a'\equiv 0$ (then it is equal with the distribution spanned by only a vector field).

Indeed, if $a'\not\equiv 0$ and $x=0$, then it does not exist a maximal integral manifold which passes through p.

Now we shall verify that Δ has maximal integral manifold property if and only if $dX_{\lambda,t}(\partial\Delta(p))\subseteq\Delta(X_{\lambda,t}(p))$ (see Theorem 2.4). Let $p_1=\left(-1,\ y+\dfrac{a(\lambda)}{2}\right)$; the integral curve which passes through p_1 is

$$\begin{cases} x(t)=t-1, \\ y(t)=\dfrac{a(\lambda)t^2}{2}-a(\lambda)t+y+\dfrac{a(\lambda)}{2}, \text{ and} \end{cases}$$

$$\partial\Delta(p_1)=\begin{cases} \mathrm{sp}\left\{\dfrac{\partial}{\partial y}(p)\right\}, & \text{if } a'\not\equiv 0 \\ 0 \quad , & \text{if } a'\equiv 0 \end{cases}$$

Let $t=1$ and $p_2=p_\lambda(1)=(0,y)$. It follows that

$$dX_{\lambda,1}(\partial\Delta(p_1))=\mathrm{sp}\left\{\left\|\begin{matrix} 1 & 0 \\ a(\lambda) & 1 \end{matrix}\right\|\left\|\begin{matrix} 0 \\ \alpha \end{matrix}\right\|;\ \text{where } \left\|\begin{matrix} 0 \\ \alpha \end{matrix}\right\|\in\partial\Delta(p_1)\right\}p_2=$$

$$=\begin{cases} \mathrm{sp}\left\{\dfrac{\partial}{\partial y}(p_2)\right\} & \text{if } \alpha\neq 0 \\ 0 & \text{if } \alpha=0 \end{cases}$$

We conclude that if $a'\not\equiv 0$, then $dX_{\lambda,1}(\partial\Delta(p_1)\not\subset\Delta(p_2)$ and if $a'\equiv 0$, then $dX_{\lambda,1}(\partial\Delta(p_1))=0\in\Delta(p_2)$. We can argue in a similar manner for any other point $p\in M$.

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Universitatea „Ştefan cel Mare"
Suceava, România

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SOME THEOREMS IN THE THEORY OF NONSIMPLE ELASTIC BODIES

BY

D. IEŞAN and R. QUINTANILLA

1. Introduction. Many papers have been devoted to the study of the theory of nonsimple elastic solids. For typical results in this direction we refer the reader to [1]—[5]. In this paper we establish some results in the dynamic theory. First, we derive a reciprocal theorem and prove that it leads to a uniqueness result of Brun type [6] and a minimum principle. We also obtain the classical reciprocity relation avoiding both the use of the Laplace transform and the incorporation of the initial conditions into the field equations. The uniqueness result is established with no definiteness assumption on the constitutive coefficients. Finally, we use results of the semigroup theory of linear operators to obtain an existence theorem. We also obtain a continuous dependence result.

2. Basic equations. We consider a body that at time $t=0$ occupies the region B of Euclidean three-dimensional space and is bounded by the piecewise smooth surface ∂B. The motion of the body is referred to a fixed system of rectangular Cartesian axes Ox_i $(i=1,2,3)$. We designate by $\mathbf{n}$ the outward normal of ∂B. Letters in boldface stand for tensors of an order $p\geq 1$, and if $\mathbf{v}$ has the order p, we write $v_{ij\ldots k}$ (p subscripts) for the components of $\mathbf{v}$ in the Cartesian coordinate frame. We shall employ the usual summation and differentiation conventions: Latin subscripts are understood to range over the integers (1, 2, 3), summation over repeated subscripts is implied and subscripts preced by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. In all that follows, we use a superposed dot to denote partial differentiation with respect to the time.

We consider the linear theory of nonsimple elastic bodies. Let ϑ be an arbitrary material volume in the continuum bounded by a surface A at time t; we suppose that V is the corresponding volume in the initial undeformed state of the continuum, bounded by a surface A. We postulate (cf. [1]—[3]) an energy balance for an arbitrary material volume, in the form

$$\int_V \rho(v_i\dot{v}_i+\dot{U})dv=\int_V F_iv_idv+\int_A(t_iv_i+\mu_{ji}v_{i,j})da, \qquad (2.1)$$

where u_i is the displacement vector, $v_i=\dot{u}_i$, ρ is the density in the reference configuration, U is the internal energy per unit mass, F_i is the body force per unit volume in the reference configuration, t_i is a stress vector, μ_{ji} is the dipolar surface force.