$$x_n(t) = x_0 + x_0'(t - t_0) + \int_{t_0}^{t} (t - s)\, F(s, x_{n-1}(s),\ x'_{n-1}(s),\ \max_{\tau \in I} x_{n-1}(\tau),$$

$$\max_{\tau \in I} x'_{n-1}(\tau))\, ds\ ;\ n = 1, 2, 3, \ldots .$$

such that

$$x_n'(t) = x_0' + \int_{t_0}^{t} F(x, x_{n-1}(s),\ x'_{n-1}(s),\ \max_{\tau \in I} x_{n-1}(\tau), \max_{\tau \in I} x'_{n-1}(\tau))\, ds\ ;\ n = 1, 2, 3, \ldots .$$

If in W the sequences $\Sigma x_n(t)$ and $\Sigma x_n'(t)$ are uniformly convergent to the vector functions $x(t)$ and $x'(t)$ respectively then the vector function $f(t, x_n(t), x_n'(t), \max_{\tau \in I} x_n(\tau), \max_{\tau \in I} x_n'(\tau))$ will uniformly converge in J to $f(t, x(t), x'(t), \max_{\tau \in I} x(\tau), \max_{\tau \in I} x'(\tau))$ since $f(t, x(t), x'(t), \max_{\tau \in I} x(\tau), \max_{\tau \in I} x'(\tau))$ is continuous. Consequently $Ax_n \to Ax$ and $Ax_n' \to Ax'$, i.e. the operator A is continuous in W. It is clear, according to (29) that

$$\| A(x(t)) \| \leq a,\ \| A(x'(t)) \| \leq b, \tag{30}$$

which means that A is bounded and mapps W into itself. Also, taking $t_1, t_2 \in J$, $\varepsilon > 0$ sufficiently small and $\delta(\varepsilon) > 0$, we can have

$$\| A(x(t_1)) - A(x(t_2)) \|,\ \| A(x'(t_1)) - A(x'(t_2)) \| < \varepsilon \text{ if } | t_1 - t_2 | < \delta$$

$$\text{where } \delta = \varepsilon / (M\, |\, T + r\, |) \text{ and } \varepsilon / M \text{ respectively.}$$

These results according to Arzela theorem [1] show that the continuous operator A converts the bounded set W into its compact part. So, using Schauder's principle [1] the operator A has at least one fixed point $x(t)$ which is the solution of problem (1), (2).

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EFFECTS OF DISPERSAL ON THE STABILITY OF TWO PREDATORS AND A COMMON PREY

BY

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Introduction. Stability of systems of interacting species has been investigated by various authors, for example Goh and Agenew [7]; Kazarinoff and Driessche [12]; El-Owaidy and Ammar [2, 3, 4, 5]. Effects of dispersal on the stability of the equilibrium state have also been investigated by various authors, for example : Gopalsamy [8]; Hadeler et al [9]; Levin [13]; Nallaswamy and Shukla [10, 11]; El-Owaidy and Ammar [6]. In particular, Nallaswamy and Shukla [10] investigated the effects of dispersal on the linear and nonlinear stability of the equilibrium state of a predator-prey system with functional response.

1. Basic systems. Consider a system modeling the interactions of two competing predator populations living exclusively on a common prey in a linear one dimensional finite habitat $(0 \leq x \leq L)$ with intraspecific interaction of prey and functional response. The system given by :

$$\begin{aligned} \frac{\partial N_1}{\partial t} &= N_1 \left(a_1 - a_{11} N_1 - \frac{a_{12} N_2}{1 + \alpha N_1} - \frac{a_{13} N_3}{1 + \alpha N_1} \right) + \frac{\partial}{\partial x}\left(D_1 \frac{\partial N_1}{\partial x} \right), \\ \frac{\partial N_2}{\partial t} &= N_2 \left(-a_2 + \frac{a_{21} N_1}{1 + \alpha N_1} - a_{22} N_2 \right) + \frac{\partial}{\partial x}\left(D_2 \frac{\partial N_2}{\partial x} \right), \\ \frac{\partial N_3}{\partial t} &= N_3 \left(-a_3 + \frac{a_{31} N_1}{1 + \alpha N_1} - a_{33} N_3 \right) + \frac{\partial}{\partial x}\left(D_3 \frac{\partial N_3}{\partial x} \right), \end{aligned} \tag{1.1}$$

where $N_1(x, t)$ is the population distribution of prey species; $N_2(x, t)$, $N_3(x, t)$ are the population distributions of predator species at time t; and D_1, D_2, D_3 are the variable coefficients of dispersal which are positive. The interaction coefficients a_1, a_2, a_3, a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{21}, a_{31}, and α are positive constants. The factor $(1 + \alpha N_1)^{-1}$ represents the functional response in the model, and α determines the strength of this response. For a positive equilibrium (N_1^*, N_2^*, N_3^*) we have:

$$\begin{gathered} (a_1 - a_{11} N_1^*)(1 + \alpha N_1^*) = a_{12} N_2^* + a_{13} N_3^*, \quad (a_2 + a_{22} N_2^*)(1 + \alpha N_1^*) = a_{21} N_1^*, \\ (a_3 + a_{33} N_3^*)(1 + \alpha N_1^*) = a_{31} N_1^* \end{gathered} \tag{1.2}$$

2. Stability. To investigate the stability of the equilibrium point (N_1^*, N_2^*, N_3^*) of system (1.1.) let $N_i(x, t) = N_i^* + n_i(x, t)$, $i = 1, 2, 3$; system (1.1.) becomes:

(2.1)
$$\frac{\partial n_1}{\partial t} = \frac{N_1^* + n_1}{S}\{[\alpha(a_{12}N_2^* + a_{13}N_3^*) - a_{11}S]n_1 - (1+\alpha N_1^*)(a_{12}n_2 + a_{13}n_3)\} + \frac{\partial}{\partial x}\left(D_1\frac{\partial n_1}{\partial x}\right),$$
$$\frac{\partial n_2}{\partial t} = \frac{N_2^* + n_2}{S}[a_{21}n_1 - a_{22}Sn_2] + \frac{\partial}{\partial x}\left(D_2\frac{\partial n_2}{\partial x}\right),$$
$$\frac{\partial n_3}{\partial t} = \frac{N_3^* + n_3}{S}[a_{31}n_1 - a_{33}Sn_3] + \frac{\partial}{\partial x}\left(D_3\frac{\partial n_3}{\partial x}\right)$$

where $S = (1+\alpha N_1^*)(1+\alpha N_1^* + \alpha n_1)$.

The linearized version of system (2.1) is:

(2.2)
$$\frac{\partial n_1}{\partial t} = -K_{11}n_1 - K_{12}n_2 - K_{13}n_3 + \frac{\partial}{\partial x}\left(D_1\frac{\partial n_1}{\partial x}\right),$$
$$\frac{\partial n_2}{\partial t} = K_{21}n_1 - K_{22}n_2 + \frac{\partial}{\partial x}\left(D_2\frac{\partial n_2}{\partial x}\right),$$
$$\frac{\partial n_3}{\partial t} = K_{31}n_1 - K_{33}n_3 + \frac{\partial}{\partial x}\left(D_3\frac{\partial n_3}{\partial x}\right),$$

where,

(2.3)
$$K_{11} = \frac{2\alpha a_{11}N_1^*}{1+\alpha N_1^*}(N_1^* - a_0); \quad a_0 = \frac{\alpha a_1 - a_{11}}{2\alpha a_{11}},$$

and

$$K_{12} = \frac{a_{12}N_1^*}{1+\alpha N_1^*} > 0, \quad K_{13} = \frac{a_{13}N_1^*}{1+\alpha N_1^*} > 0, \quad K_{21} = \frac{a_{21}N_2^*}{(1+\alpha N_1^*)^2} > 0,$$
$$K_{22} = a_{22}N_2^* > 0, \quad K_{31} = \frac{a_{31}N_3^*}{(1+\alpha N_1^*)^2} > 0, \quad K_{33} = a_{33}N_3^* > 0.$$

We shall assume that $a_0 > 0$.

It is clear that, $K_{11} \geq 0$ for $N_1^* \geq a_0$ and $K_{11} < 0$ for $0 < N_1^* < a_0$.

Systems (2.1) and (2.2) are associated with the following boundary conditions [14]:

(i) *Flux conditions*:

(2.4i)
$$\frac{\partial n_i}{\partial x}(0, t) = \frac{\partial n_i}{\partial x}(L, t) = 0, \; i = 1, 2, 3.$$

These conditions imply that there is no migration of the species across the boundary of the habitate.

(ii) *Reservoir conditions*:

(2.4ii)
$$n_i(0, t) = n_i(L, t) = 0, \; i = 1, 2, 3.$$

These conditions suggest that the species populations remain at equilibrium level on the boundary of the habitat. We have the following theorems:

Theorem 2.1. (a) *If $N_1^* \geq a_0$, the equilibrium point $E(N_1^*, N_2^*, N_3^*)$ of system (1.1) is linear asymptotically stable in the entire positive octant (with and without dispersal).*

b) *If $N_1^* < a_0$, the equilibrium point E is not linear asymptotically stable without dispersal, but with constant dispersal, it is stable under the condition:*

$$D_1 \geq \frac{2\alpha a_{11}L^2}{\pi^2} \cdot \frac{N_1^*(a_0 - N_1^*)}{1+\alpha N_1^*}.$$

Proof. (a) To study the linear stability of the equilibrium state in the absence of dispersal, consider the positive definite function:

$$E = \tfrac{1}{2}(n_1^2 + c_2n_2^2 + c_3n_3^2),$$

where c_2, c_3 are positive constants to be chosen appropriately. Then using (2.2) we have:

$$\frac{dE}{dt} = -K_{11}n_1^2 - c_2K_{22}n_2^2 - c_3K_{33}n_3^2 + (c_2K_{21} - K_{12})n_1n_2 + (c_3K_{31} - K_{13})n_1n_3;$$

chosing,

(2.5)
$$c_2 = K_{12}/K_{21} \text{ and } c_3 = K_{13}/K_{31}$$

then

(2.6)
$$\frac{dE}{dt} = -K_{11}n_1^2 - \frac{K_{12}K_{22}}{K_{21}}n_2^2 - \frac{K_{13}K_{33}}{K_{31}}n_3^2.$$

It is clear that $dE/dt < 0$ for $K_{11} \geq 0$, that is

(2.7)
$$N_1^* \geq a_0$$

and $dE/dt = 0$ only when $n_1 = n_2 = n_3 = 0$.

Thus the equilibrium state is asymptotically stable provided that (2.7) holds.

To see the effects of dispersal on the linear stability of the equilibrium state, consider the following positive definite function:

$$E = \tfrac{1}{2}\int_0^L (n_1^2 + c_2n_2^2 + c_3n_3^2)\,dx,$$

where c_2 and c_3 are given by (2.5), then using (2.2) we have

$$\frac{dE}{dt} = -K_{11}\int_0^L n_1^2\,dx - \frac{K_{12}K_{22}}{K_{21}}\int_0^L n_2^2\,dx - \frac{K_{13}K_{33}}{K_{31}}\int_0^L n_3^2\,dx +$$

$$+\int_0^L\left[n_1\frac{\partial}{\partial x}\left(D_1\frac{\partial n_1}{\partial x}\right)+c_2n_2\frac{\partial}{\partial x}\left(D_2\frac{\partial n_2}{\partial x}\right)+c_3n_3\frac{\partial}{\partial x}\left(D_3\frac{\partial n_3}{\partial x}\right)\right]dx.$$

Integrating and using the boundary conditions (2.4) we have :

$$\frac{dE}{dt}=-K_{11}\int_0^L n_1^2dx-\frac{K_{12}K_{22}}{K_{21}}\int_0^L n_2^2dx-\frac{K_{13}K_{33}}{K_{31}}\int_0^L n_3^2dx-$$
(2.8)
$$-\int_0^L\left[D_1\left(\frac{\partial n_1}{\partial x}\right)^2+c_2D_2\left(\frac{\partial n_2}{\partial x}\right)^2+c_3D_3\left(\frac{\partial n_3}{\partial x}\right)^2\right]dx;$$

we see that, (2.7) is the sufficient condition for asymptotic stability of the equilibrium state.

(b) If $N_1^*<a_0$, i.e. if $K_{11}<0$, we can find certain conditions for asymptotic stability under conditions (2.4ii) provided D_1, D_2 and D_3 are positive constants. In such a case using Poincare's inequality [1] in (2.8) we have

$$\frac{dE}{dt}=-\left(K_{11}+\frac{D_1\pi^2}{L^2}\right)\int_0^L n_1^2dx-\left(\frac{K_{12}K_{22}}{K_{21}}+\frac{K_{12}D_2\pi^2}{K_{21}L^2}\right)\int_0^L n_2^2dx-$$
(2.9)
$$-\left(\frac{K_{13}K_{33}}{K_{31}}+\frac{K_{13}D_3\pi^2}{K_{31}L^2}\right)\int_0^L n_3^2dx.$$

It is clear that $dE/dt<0$ if $K_{11}+\dfrac{D_1\pi^2}{L^2}\geq 0$

or

(2.10) $$D_1\geq\frac{2\alpha a_{11}L^2}{\pi^2}\cdot\frac{N_1^*(a_0-N_1^*)}{1+\alpha N_1^*}.$$

and $dE/dt=0$ only when $n_1=n_2=n_3=0$. Hence stability of the equilibrium state, showing that dispersal has a stabilizing effect.

Theorem 2.2. (a) *If $N_1^*\geq 2a_0$, the equilibrium point $E(N_1^*, N_2^*, N_3^*)$ of system (1.1) is nonlinear asymptotically stable (with and without dispersal) in the entire positive octant.*

(b) *If, $a_0\leq N_1^*<2a_0$, the equilibrium point E is nonlinear asymptotically stable (with and without dispersal) in the region :*

$$A=\{(N_1,\ N_2,\ N_3):\ N_1\geq 2a_0-N_1^*,\ N_2>0,\ N_3>0\}.$$

c) *If $N_1^*<a_0$, the equilibrium point E is not nonlinear asymptotically stable without dispersal, but, with constant dispersal, it is stable in the region :*

$$B=\{(N_1,\ N_2,\ N_3):0<N_1<2a_0-N_1^*,\ 0<N_2,\ 0<N_3\}$$

under the condition :

$$D_1\geq\frac{\alpha a_{11}L^2}{\pi^2}\cdot\frac{(2a_0-N_1^*)^3}{N_1^*(1+\alpha N_1^*)}.$$

Proof.

To study the nonlinear stability of the equilibrium state in the absence of dispersal, consider the following positive definite function :

$$E=n_1-N_1^*\,ln\left(1+\frac{n_1}{N_1^*}\right)+c_2\left[n_2-N_2^*\,ln\left(1+\frac{n_2}{N_2^*}\right)\right]+c_3\left[n_3-N_3^*\,ln\left(1+\frac{n_3}{N_3^*}\right)\right].$$

where c_2 and c_3 are positive constants to be chosen suitably. Then using (2.1) we have

$$\frac{dE}{dt}=\alpha a_{11}(1+\alpha N_1^*)(N_0-N_1)\frac{n_1^2}{s}-c_2a_{22}n_2^2-c_3a_{33}n_3^2+[c_2a_{21}-a_{12}(a+$$
(2.11)
$$+\alpha N_1^*)]\frac{n_1n_2}{s}+[c_3a_{31}-a_{13}(1+\alpha N_1^*)]\frac{n_1n_3}{s},$$

where

(2.12) $$N_0=2a_0-N_1^*$$

Choosing,

(2.13) $$C_2=\frac{a_{12}}{a_{21}}(1+\alpha N_1^*)\quad\text{and}\quad c_3=\frac{a_{13}}{a_{31}}(1+\alpha N_1^*),$$

from (2.11), we have the following two cases :

(i) If $N_0\leq 0$, i.e.

(2.14) $$N_1^*\geq 2a_0;$$

then $dE/dt<0$ in the positive octant $p=\{(N_1,\ N_2,\ N_3):N_i>0,\ i=1,2,3\}$, and the equilibrium state is nonlinear asymptotically stable in the entire positive octant p provided (2.14) is satisfied.

(ii) If $N_0>0$, i.e.

(2.15) $$0<N_1^*<2a_0$$

Consider the subregion $A=\{(N_1,\ N_2,\ N_3):N_1\geq N_0,\ N_2>0,\ N_3>0\}$ of the positive octant. Then $dE/dt<0$ in A, which includes the equilibrium state provided (2.7) is satisfied.

From (2.7) and (2.15) we conclude that the criteria for asymptotic stability of the equilibrium state can be written as

(2.16) $$a_0\leq N_1^*<2a_0$$

since (2.7) is the condition for linear stability it is concluded that if the equilibrium state is linearly stable, it is nonlinearly stable in the region A under (2.16).

To study the effects of dispersal on the nonlinear stability of the equilibrium state, we consider the positive definite function :

$$E=\int_0^L\left\{n_1-N_1^*\,ln\left(1+\frac{n_1}{N_1^*}\right)+C_2\left[n_2-N_2^*\,ln\left(1+\frac{n_2}{N_2^*}\right)\right]+\right.$$
$$\left.+C_3\left[n_3-N_3^*\,ln\left(1+\frac{n_3}{N_3^*}\right)\right]\right\}dx.$$

Then using (2.1) and (2.13), we have

$$\frac{dE}{dt}=\alpha a_{11}(1+\alpha N_1^*)\int_0^L (N_0-N_1)\frac{n_1^2}{s}\,dx-\frac{a_{12}a_{22}}{a_{21}}(1+\alpha N_1^*)\int_0^L n_2^2dx-$$
$$-\frac{a_{13}a_{33}}{a_{31}}(1+\alpha N_1^*)\int_0^L n_3^2dx+\int_0^L \frac{n_1}{N_1^*+n_1}\,\frac{\partial}{\partial x}\left(D_1\frac{\partial n_1}{\partial x}\right)dx+$$
$$+C_2\int_0^L \frac{n_2}{N_2^*+n_2}\,\frac{\partial}{\partial x}\left(D_2\frac{\partial n_2}{\partial x}\right)dx+C_3\int_0^L \frac{n_3}{N_3^*+n_3}\,\frac{\partial}{\partial x}\left(D_3\frac{\partial n_3}{\partial x}\right)dx.$$

Integrating and using (2.4) we have

$$\frac{dE}{dt}=\alpha a_{11}(1+\alpha N_1^*)\int_0^L (N_0-N_1)\frac{n_1^2}{s}\,dx-\frac{a_{12}a_{22}}{a_{21}}(1+\alpha N_1^*)\int_0^L n_2^2dx-$$

(2.17)
$$-\frac{a_{13}a_{33}}{a_{31}}(1+\alpha N_1^*)\int_0^L n_3^2dx-\int_0^L\left[\frac{D_1N_1^*}{(N_3^*-n_3)^2}\left(\frac{\partial n_1}{\partial x}\right)^2+\right.$$
$$\left.+\frac{C_2D_2N_2^*}{(N_2^*+n_2)^2}\left(\frac{\partial n_2}{\partial x}\right)^2+\frac{C_3D_3N_3^*}{(N_3^*+n_3)^2}\left(\frac{\partial n_3}{\partial x}\right)^2\right]dx.$$

From (2.17) the following cases are noted regarding the nonlinear stability of the equilibrium state of system (1.1.):

(i) If $N_0\leqslant 0$, i.e. (2.14) is satisfied, the equilibrium state (N_1^*, N_2^*, N_3^*) which is asymptotically stable in the entire positive octant p remains so with dispersal also.

(ii) If $N_0>0$, $N_1^*\geq N_0$, i.e. (2.16) is satisfied, the equilibrium state (N_1^*, N_2^*, N_3^*) is asymptotically stable in the subregion A of the positive octant with dispersal as well.

(iii) *If* $N_0>0$, $N_1^*<N_0$, i.e. (2.16) does not hold; in the region B, since $N_1<N_0$ then

$$\frac{dE}{dt}\leqslant\alpha a_{11}(1+\alpha N_1^*)\int_0^L (N_0-N_1)\frac{n_1^2}{s}\,dx-\frac{a_{12}a_{22}}{a_{21}}(1+\alpha N_1^*)\int_0^L n_2^2dx-$$

(2.18)
$$-\frac{a_{13}a_{33}}{a_{31}}(1+\alpha N_1^*)\int_0^L n_3^2dx-\frac{D_1N_1^*}{N_0^2}\int_0^L\left(\frac{\partial n_1}{\partial x}\right)^2dx-\int_0^L\left[\frac{C_2D_2N_2^*}{(N_2^*+n_2)^2}\left(\frac{\partial n_2}{\partial x}\right)^2+\right.$$
$$\left.+\frac{C_3D_3N_3^*}{(N_3^*+n_3)^2}\left(\frac{\partial n_3}{\partial x}\right)^2\right]dx.$$

Using Poincare's inequality, we have:

$$\frac{dE}{dt}\leq\alpha a_{11}(1+\alpha N_1^*)\int_0^L (N_0-N_1^*)\frac{n_1^2}{s}\,dx-\frac{a_{12}a_{22}}{a_{21}}(1+\alpha N_1^*)\int_0^L n_2^2dx-$$

(2.19)
$$-\frac{a_{13}a_{33}}{a_{31}}(1+N_1^*)\int_0^L n_3^2dx-\frac{\pi^2D_1N_1^*}{L^2N_0^2}\int_0^L n_1^2dx-\int_0^L\left[\frac{C_2D_2N_2^*}{(N_2^*+n_2)^2}\left(\frac{\partial n_2}{\partial x}\right)^2+\right.$$
$$\left.+\frac{C_3D_3N_3^*}{(N_3^*+n_3)^2}\left(\frac{\partial n_3}{\partial x}\right)^2\right]dx.$$

Then we can show that $dE/dt<0$ if

(2.20)
$$D_1\geq\frac{\alpha a_{11}L^2}{\pi^2}\cdot\frac{(2a_0-N_1^*)^3}{N_1^*(1+\alpha N_1^*)}$$

showing the asymptotic stability of the equilibrium state in the region B provided (2.20) holds. Thus it is concluded that dispersal has stabilizing effect even on the nonlinear stability of the equilibrium state of system (1.1).

3. Examples. We introduce two examples for two predators N_2, N_3 and a common prey N_1, where the first example has an equilibrium point which is linear (and nonlinear) asymptotically stable without dispersal and this stability remains so with dispersal; the second example has an equilibrium point which is not linear asymptotically stable without dispersal but with dispersal it becomes asymptotically stable under the condition (3.5), also this equilibrium point is not nonlinear asymptotically stable without dispersal, but with dispersal this equilibrium becomes nonlinear asymptotically stable in region B under the condition (3.6)

Example (1)

(3.1)
$$\frac{\partial N_1}{\partial t}=N_1\left(10-5N_1-\frac{3N_2}{1+2N_1}-\frac{3N_3}{1+2N_1}\right)+\frac{\partial}{\partial x}\left(D_1\frac{\partial N_1}{\partial x}\right),$$
$$\frac{\partial N_2}{\partial t}=N_2\left(-1+\frac{6N_1}{1+2N_1}-N_2\right)+\frac{\partial}{\partial x}\left(D_2\frac{\partial N_2}{\partial x}\right),$$
$$\frac{\partial N_3}{\partial t}=N_3\left(-1+\frac{15N_1}{1+2N_1}-N_3\right)+\frac{\partial}{\partial x}\left(D_3\frac{\partial N_3}{\partial x}\right).$$

This system has the equilibrium point $(N_1^*, N_2^*, N_3^*)=(1, 1, 4)$. Also we can show that

(3.2)
$$a_0=\frac{3}{4}>0.$$

Since $a_0<N_1^*$, the equilibrium point (1, 1, 4) is linear asymptotically stable (in the positive octant) without dispersal and this stability remains so with dispersal. Also, since $a_0<N_1^*<2a_0$, the equilibrium point (1, 1, 4) is nonlinearly stable, without dispersal, in the region

$$A=\{(N_1,\ N_2,\ N_3): N_1\geq 1/2,\ N_2>0,\ N_3>0\},$$

and this stability remains so with dispersal.

Example (2).

$$\begin{aligned}(3.3)\qquad &\frac{\partial N_1}{\partial t}=N_1\left(5-N_1-\frac{N_3}{1+2N_1}-\frac{2N_3}{1+2N_1}\right)+\frac{\partial}{\partial x}\left(D_1\frac{\partial N_1}{\partial x}\right),\\ &\frac{\partial N_2}{\partial t}=N_2\left(-\frac{3}{4}+\frac{5N_1}{1+2N_1}-\frac{1}{4}N_2\right)+\frac{\partial}{\partial x}\left(D_2\frac{\partial N_2}{\partial x}\right),\\ &\frac{\partial N_3}{\partial x}=N_3\left(-\frac{3}{2}+\frac{10N_1}{1+2N_1}-\frac{1}{2}N_3\right)+\frac{\partial}{\partial x}\left(D_3\frac{\partial N_3}{\partial x}\right),\end{aligned}$$

where $0\leq x\leq L$.

This system has the equilibrium point $(N_1^*,\ N_2^*,\ N_3^*)=(2,\ 5,\ 5)$. And we can show that

$$(3.4)\qquad a_0=9/4$$

since $N_1^*<a_0$, the equilibrium point (2, 5, 5) is not linear asymptotically stable without dispersal, but from (2.10) it is become linear asymptotically stable with dispersal provided that

$$(3.5)\qquad D_1\geq 2L^2/\pi^2$$

since $N_1^*=2<9/4=a_0$, i.e., then the equilibrium point (2, 5, 5) not nonlinearly stable without dispersal, but from (2.20) it become nonlinearly stable in the region

$$B=\left\{(N_1,\ N_2,\ N_3): 0<N_1<\frac{5}{2},0<N_2,0<N_3\right\}$$

provided that

$$(3.6)\qquad D_1\geq 125L^2/8\pi^2.$$

4. Conclusion. Effects of dispersal on the linear and nonlinear stability of the equilibrium state for system of two predators and a common prey have been investigated by using Liapunov's direct method. We have the following conclusions:

(1) Linearly or nonlinearly stable equilibrium state remains so with dispersal.

(2) The dispersal has a stabilizing effect on both linear and nonlinear stability of the equilibrium state.

(3) If the equilibrium state is linearly stable, then a region can be found where it is nonlinearly stable with or without dispersal provided (2.16) holds.

(4) Even if (2.16) is not hold, there is another region where the equilibrium state is asymptotically nonlinearly stable provided (2.20) holds.

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