ANALELE STIINTIFICE ALE UNIVERSITATII "AL.I.CUZA"IASI
Tomul XXXVIII, s.I.a , Matematica, 1992, f.1

ON A TYPE OF SASAKIAN MANIFOLD

BY

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Introduction: Let (M^n, g) be a contact Riemannian manifold with a contact form η, the associated vector field ξ, (1-1) tensor field ϕ and the associated Riemannian metric g. If ξ is a Killing vector field, then M^n is called a k-contact Riemannian manifold [1], [2]. A k-contact Riemannian manifold is called Sasakian [2] if

$$(D_X\phi)(Y) = g(X,Y)\xi - \eta(Y)X \tag{1}$$

holds, where D denotes the operator of covariant differentiation with respect to g. This paper deals with a type of Sasakian manifold in which

$$R(X,Y) \cdot P = 0 \tag{2}$$

where P is the Weyl-projective curvature tensor, R is the curvature tensor and $R(X,Y)$ is considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X, Y. In this connection we mention the works of K. S e k i g a w a [3], Z.I. S z a b o [4], L. V e r s t r a e l e n [5], M. P e t r o v i c - T o r g a s e v and L. V e r s t r a e l e n [6] who studied Riemannian manifolds or hypersurfaces of such manifolds satisfying the condition $R(X,Y) \cdot R = 0$ or conditions similar to it. It is easy to see that $R(X,Y) \cdot R = 0$ implies $R(X,Y) \cdot P = 0$. But the converse is not necessarily true. So it is meaningful to undertake the study of manifolds satisfying the condition (2). In this paper it is proved that if in a Sasakian manifold (M^n, g) $(n > 2)$ the relation (2) holds then it is of constant curvature.

1.Preliminaries. Let S and r denote respectively the Ricci tensor of type (0,2) and the scalar curvature of M^n. It is known that in a Sasakian manifold M^n, besides the relation (1), the following relations hold [1], [2], [7]

$$(1.1)\quad \phi(\xi) = 0, \qquad (1.2)\quad \eta(\xi) = 1, \qquad (1.3)\quad \phi^2 X = -X + \eta(X)\xi$$

$$(1.4)\quad g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \qquad (1.5)\quad g(\xi, X) = \eta(X)$$

$$(1.6)\quad D_X\xi = -\phi X \qquad (1.7)\quad S(X,\xi) = (n-1)\eta(X)$$

(1.8) $g(R(\xi,X)Y,\xi) = g(X,Y) - \eta(X)\eta(Y)$ (1.9) $R(\xi,X)\xi = -X + \eta(X)\xi$

and

(1.10) $(D_X\phi)(Y) = R(\xi,X)Y$

for any vector fields X, Y.

The above results will be used in the next section.

2. Sasakian manifold satisfying $R(X,Y)\cdot P = 0$. We have

(2.1) $$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]$$

Now using (1), (1.5) and (1.10) it follows

$$\eta(P(X,Y)Z) = g(P(X,Y)Z,\xi) =$$

$$= g(R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y]\xi) =$$

$$= g(R(X,Y)Z,\xi) - \frac{1}{n-1}[S(Y,Z)g(X,\xi) - S(X,Z)g(Y,\xi)] =$$

$$= g(R(Z,\xi)X,Y) - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)] =$$

$$= -g(R(\xi,Z)X,Y) - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)] =$$

$$= -g((D_Z\phi)(X),Y) - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)] =$$

$$= -g(g(Z,X)\xi - \eta(X)Z,Y) - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)] =$$

$$= -g(Z,X)\eta(Y) + \eta(X)g(Z,Y) - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)]$$

or

(2.2) $$\eta(P(X,Y)Z) = -[g(Z,X)\eta(Y) - g(Z,Y)\eta(X)] - \frac{1}{n-1}[S(Y,Z)\eta(X) - S(X,Z)\eta(Y)].$$

Putting $Z = \xi$ in (2.2) we get

(2.3) $$\eta(P(X,Y)\xi) = 0$$

Again , putting $X = \xi$ in (2.2) we have

(2.4) $$\eta(P(\xi,Y)Z) = g(Z,Y) - \frac{1}{n-1}S(Y,Z).$$

In virtue of (2) we get

(2.5) $$R(X,Y)P(U,V)W - P(R(X,Y)U,V)W - P(U,R(X,Y)V)W - P(U,V)R(X,Y)W = 0$$

Therefore

(2.6) $$g[R(\xi,Y)P(U,V)W,\xi] - g[P(R(\xi,Y)U,V)W,\xi] - g[P(U,R(\xi,Y)V)W,\xi] - g[P(U,V)R(\xi,Y)W,\xi] = 0$$

From (2.6) and (2.3) it follows that

(2.7) $$'P(U,V,W,Y) - \eta(Y)\eta(P(U,V)W) + \eta(U)\eta(P(Y,V)W) + \eta(V)\eta(P(U,Y)W) + \eta(W)\eta(P(U,V)Y) - g(Y,U)\eta(P(\xi,V)W) - g(Y,V)\eta(P(U,\xi)W) = 0$$

where $'P(U,V,W,Y) = g(P(U,V)W,Y)$.

Let $\{e_i\}, i = 1,2,...,n$ be an orthonormal basis of the tangent space at any point. Then the sum for $1 \le i \le n$ of the relation (2.7) for $U = Y = e_i$ gives

(2.8) $$\eta(P(\xi,V)W) = \frac{1}{n-1}(\frac{r}{n-1} - n)\eta(V)\eta(W)$$

In virtue of (2.4) and (2.8) we have

(2.9) $$S(V,W) = (n-1)g(V,W) - (\frac{r}{n-1} - n)\eta(V)\eta(W)$$

Putting $W = \xi$ in (2.9) we get

(2.10) $r = n(n-1)$, *since there exists V such that* $\eta(V) \neq 0$.

Using (2.2), (2.4), (2.9) and (2.10) it follows from (2.7) that

$$'P(U,V,W,Y) = 0$$

Hence we can state the following theorem:

Theorem. *If in a Sasakian manifold $M^n (n > 2)$ the relation $R(X,Y)\cdot P = 0$ holds , then it is of constant curvature.*

For a projectively symmetric Riemannian manifold [8] we have $DP = 0$. Hence for such a manifold $R(X,Y) \cdot P = 0$ holds. Thus we have the following corollary of the above theorem.

Corollary. *A projectively symmetric Sasakian manifold $M^n (n > 2)$ is of constant curvature.*

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Received: 23.I.1991
Revised: 2.VII.1991

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Tomul XXXVIII, s.I.a Matematica, 1992, f.1

MINIMAL CENTRED PAVINGS OF HYPERCUBES

BY

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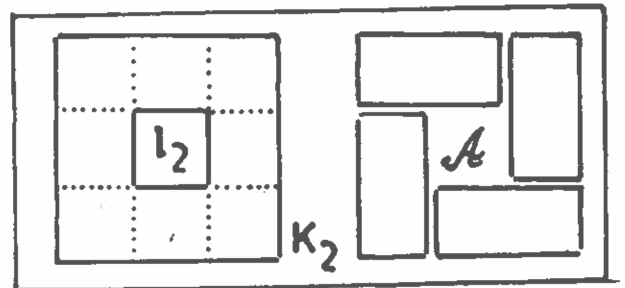
1.Source question. A question from hyperbolic analysis,[1], geometrically formulated has an intrinsic significance but not only.

A square K_2 is divided into 9 squares by parallels to its sides. By excluding the internal one, I_2, one gets a "frame" $F_2 = K_2 - I_2$,

fig.1



divided into 8 squares. By grouping some of these squares in rectangles, one gets pavings $\mathcal{P}$ of F_2 ; let $v(\mathcal{P})$ be the set of vertexes of its rectagles situated on the border of K_2. One proves : a) the minimal number of rectangles of $\mathcal{P}$ is 4; b) if $\mathcal{P}$ is "minimal", card $V(\mathcal{P}) = \forall$.

The question is to extend this planar question for dimensions $n \geq 2$ of the Euclidean space and to solve this extension.

2.General configuration. Let E_n be the n -dimensional Euclidean space reported to a Cartesian system of coordinates. For a point $X \in E_n$ of coordinates $(x_1, ..., x_n)$ we consider the point $p(X) \in E_{n-1}$ of coordinates $(x_1, ..., x_{n-1})$ and the real number $q(X) = x_n$. Let K_n be the " n - cube " containing the points X satisfying the system of inequalities.

$$(1) \qquad (\forall)_{i \in \overline{1,n}} \quad 0 \leq x_i \leq 3.$$

For i in $\overline{1,n}$ and u in $\overline{1,2}$ let $H_{i,u}$ be the hyperplane of equation $x_i = u$. We note by $\mathcal{H}_n$ the set of these 2n hyperplanes.