For a projectively symmetric Riemannian manifold [8] we have $DP = 0$. Hence for such a manifold $R(X,Y) \cdot P = 0$ holds. Thus we have the following corollary of the above theorem.

Corollary. *A projectively symmetric Sasakian manifold $M^n (n > 2)$ is of constant curvature.*

REFERENCES

1. S a s a k i, S. – *Lecture note on almost contact manifolds* , Part I, Tohoku Universsity, 1965.
2. B l a i r, E.D. – *Contact manifolds in Riemannian Geometry* , Lecture Notes in Mathematics 509, Springer Verlag, 1976.
3. S e k i g a w a, K. – *Almost Hermitian manifolds satisfying some curvature conditions* , Kodai Math. J.2, 1979, 384 - 405.
4. S z a b o, Z.I. – *Structures theorems on Riemannian spaces satisfying $R(X,Y) \cdot R = 0$, I. The local version* , J. Diff. Geom. 17 , 1982, 531 - 582.
5. V e r s t r a e l e n, L. – *A survey of relations between intrinsic and extrinsic properties for hyper-surfaces* , Topics in Differential Geometry Vol.I,II Debrecen, 1984, 1279 - 1296.
6. P e t r o v i c - T o r g a s e v, M. and V e r s t r a e l e n, L. – *On the concircular curvature tensor, the projective tensor and the Einstein curvature tensor of Bochner - Kähler manifolds* , Math. Rep. Toyama Univ. 10, 1987, 37 - 61.
7. S a s a k i, S. – *Lecture note on almost contact manifolds* ,Part II, Tohoku University, 1967.
8. S o o s, Gy. – *Uber die geodatischen Abbildungen Von Riemannschen Raumen auf projectiv symmetrische Riemannsche Raumes* Acta Math. Acad. Sci. Hung. 9 (1958), 359 - 361.

Received: 23.I.1991
Revised: 2.VII.1991

Department of Mathematics
University of Kalyani
Kayani 741235, West Bengal
INDIA.

ANALELE STIINTIFICE ALE UNIVERSITATII "AL.I.CUZA" IASI
Tomul XXXVIII, s.I.a Matematica, 1992, f.1

MINIMAL CENTRED PAVINGS OF HYPERCUBES

BY

DAN BRANZEI, GHEORGHE BANTAS

1.Source question. A question from hyperbolic analysis,[1], geometrically formulated has an intrinsic significance but not only.

A square K_2 is divided into 9 squares by parallels to its sides. By excluding the internal one, I_2, one gets a "frame" $F_2 = K_2 - I_2$,

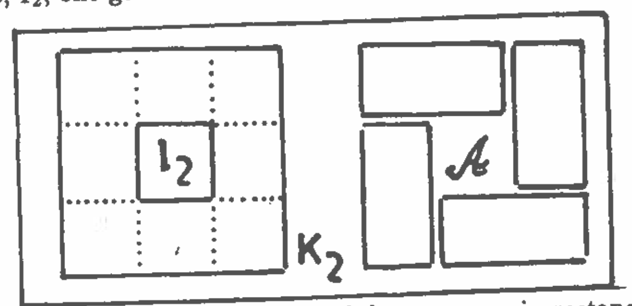

fig.1

divided into 8 squares. By grouping some of these squares in rectangles, one gets pavings $\mathcal{P}$ of F_2 ; let $v(\mathcal{P})$ be the set of vertexes of its rectagles situated on the border of K_2. One proves : a) the minimal number of rectangles of $\mathcal{P}$ is 4; b) if $\mathcal{P}$ is "minimal", card $V(\mathcal{P}) = \forall$.

The question is to extend this planar question for dimensions $n \geq 2$ of the Euclidean space and to solve this extension.

2.General configuration. Let E_n be the n -dimensional Euclidean space reported to a Cartesian system of coordinates. For a point $X \in E_n$ of coordinates $(x_1, ..., x_n)$ we consider the point $p(X) \in E_{n-1}$ of coordinates $(x_1, ..., x_{n-1})$ and the real number $q(X) = x_n$. Let K_n be the " n - cube " containing the points X satisfying the system of inequalities.

$$(1) \qquad (\forall)_{i \in \overline{1,n}} \quad 0 \leq x_i \leq 3.$$

For i in $\overline{1,n}$ and u in $\overline{1,2}$ let $H_{i,u}$ be the hyperplane of equation $x_i = u$. We note by $\mathcal{H}_n$ the set of these 2n hyperplanes.

The hyperplanes of $\mathcal{H}_n$ detach K_n in 3^n n -cubes; the internal one, noted I_n, and the set of the others, noted $\mathcal{F}_n$. The reunion of the elements of $\mathcal{F}_n$ will be noted F_n. Obviously, $F_n = \overline{K_n - I_n} = K_n - \mathrm{Int}I_n$; we appreciate as suggestive the name " n -frame " for F_n. The analytical characterizations

$$X \in I_n \Longleftrightarrow \forall_{i\in\overline{1,n}} \;\; 1 \leq x_i \leq 2 \tag{2}$$

and

$$X \in F_n \Longleftrightarrow \exists_{i\in\overline{1,n}} \;\; (0 \leq x_i \leq 1 \;\; or \;\; 2 \leq x_i \leq 3) \tag{3}$$

will be useful.

Definition 1. An admissible paving *(for short AP) of F_n is a set* $\mathcal{A} = \{A_k | k \in \overline{1,a}\}$ where:
AP.1 - each A_k is a n -parallelipipedon obtained as a reunion of elements of $\mathcal{F}_n$;
AP.2 - the interiors of two distinct A_k, A_h are disjoint.

The set of these AP of F_n will be noted $AP(F_n)$. As $\mathcal{F}_n$ is an AP of F_n (the "finest"), $AP(F_n) \neq \emptyset$.

Remark 1. Adding I_n as element to an AP $\mathcal{A}$ we get "centred admissible paving" $\mathcal{A}'$ of the n - cube K_n. Using the bijection from $\mathcal{A}$ to $\mathcal{A}'$, our results about AP (F_n) may be reformulated about "centred admissible pavings of a hypercube K_n". This possibility justifies the title, reported to the well known notion of hypercubes instead of unfamiliar "hyper-frame" F_n.

3.Minimal admissible pavings. Theorem 1. *For $n \geq 1$ and $\mathcal{A} \in AP(F_n)$:* card$\mathcal{A} \geq 2n$.

Proof. An n -parallelipipedon $A_k \in \mathcal{A}$ cannot cover more than one $(n-1)$-faces of I_n; the number of $(n-1)$-faces of I_n is $2n$.

Theorem 2. *For each $n \geq 1$, there exist $\mathcal{T}$ in $AP(F_n)$ with* card$\mathcal{T} = 2n$.

Proof. (by induction). In the rather trivial case $n = 1, T_1 = [0,1], T_2 = [2,3]$ and $\mathcal{T} = \{T_1, T_2\}$ verifies the enounce.

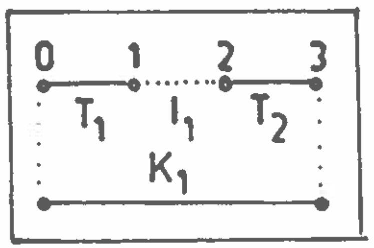

fig.2

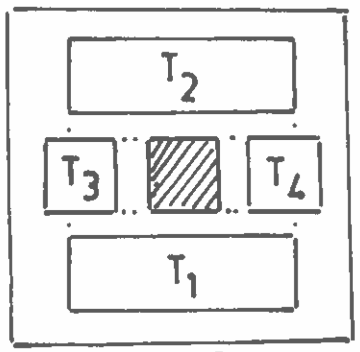

fig 3

For $n = 2$, we consider $\mathcal{T} = \{T_k | k \in \overline{1,4}\}$ where

$$T_1 = [0,3] \times [0,1], \qquad T_2 = [0,3] \times [2,3], \tag{4}$$

$$T_3 = [0,1] \times [1,2] \;\; and \;\; T_4 = [2,3] \times [1,2].$$

(In fig.3 the rectangles T_k are removed off I_2 : for k=1,2 - vertically and for k=3,4 - horizontally).

In the second stage we accept, for the dimension $m = n - 1$ the exsitance of $\mathcal{U} = \{U_h | h \in \overline{1,2m}\}$ in $AP(F_m)$. We define $\mathcal{F} = \{T_k | k \in \overline{1,2n}\}$ taking:

$$T_1 = K_m \times [0,1], \quad T_2 = K_m \times [2,3] \tag{5}$$

$$\forall_{h=\overline{1,2m}} \;\; T_{h+2} = U_h \times [1,2].$$

Obviously $\mathcal{T} \in AP(F_n)$. The theorem is proved by induction.

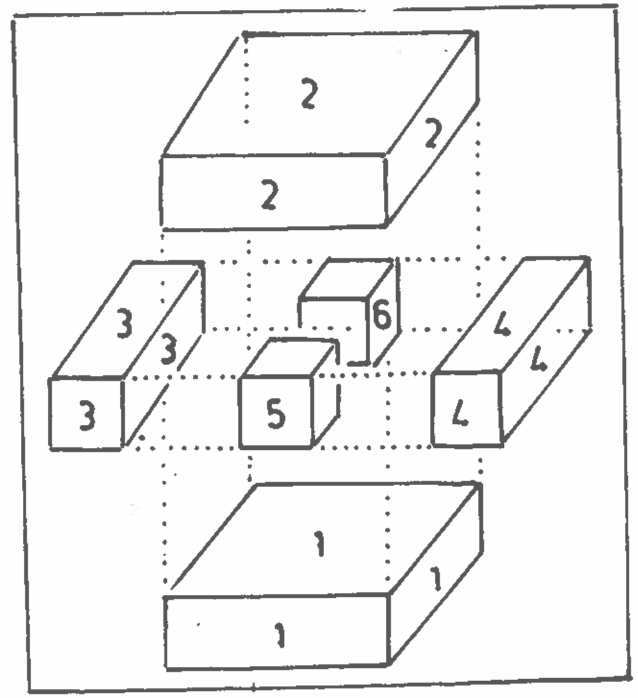

fig. 4

(The idea of this recurrence is illustrated for $n = 3$ by fig.4, with a similar to the previous removing of some T_k).

Notations. For an $\mathcal{A} \in AP(F_n)$ we will add the adjective "minimal" if card$\mathcal{A} = 2n$. The set of minimal admissible pavings (for short MAPs) of F_n will be shortly noted M_n.

For a natural number $d, 0 \leq d < n$, we note by S_d the " n - dimensional skeleton of K_n "; S_0 is the set of vertexes and, for $0 < d < n, S_d$ is the reunion of the interiors of the d - faces of K_n.

For $\mathcal{A} = \{A_k | k \in \overline{1,2n}\} \in M_n$ we will consider the set $V(\mathcal{A})$ of the vertexes of the n - parallelipipedons A_k situated on the border of K_n; its intersections with the skeletons, $V_d(\mathcal{A}) = V(\mathcal{A}) \cap S_d$ will be very usefull.

4. Recurrent approach to MAPs. Let $\mathcal{A} = \{A_k | k \in \overline{1,2n}\}$ be a MAP for $n \geq 2$. As any n - parallelipipedon A_k is in contact with exactly one $(n-1)$ -face Φ_i of I_n, we shall automatically assume $i = k$. Let Φ_1, Φ_2 be given by $x_n = 1$ and $x_n = 2$.

Let us note $n - 1 = m$. For h in $\overline{1,2m}$ let us consider :

- the m - parallelipipedon $B_h = p(A_{h+2}) = \{p(X) | X \in A_{h+2}\}$.
- the interval $q(A_{h+2}) = [a(h), b(h)]$.

We may consider the functions a: $\overline{1,m} \to \{0,1\}$, b: $\overline{1,m} \to \{2,3\}$.

By interpreting E_m in $H_{n,u}$ (for $u = 1$ or $u = 2$ we establish that $\mathcal{B} = \{B_h | h \in \overline{1,2m}\}$ is a MAP for F_m; we will note $\mathcal{B} = p(\mathcal{A})$.

For A_u with $u \in \{1,2\}$, let G_u be the m-face included in $H_{n,u}$. for the subsets P_u of $\overline{1,2m}$ defined by:

(6) $$P_1 = \{h | a(h) = 1\}, \quad P_2 = \{h | b(h) = 2\} \text{ , obviously :}$$

(7) $$G_u = p(A_u) = I_m \cup (\cup_{h \in P_u} B_h)$$

After giving $\mathcal{B}$ in M_m, each P_u ceases to be arbitrary subset of $\overline{1,2m}$; they may be elements of an $U(\mathcal{B})$ which we are going to define. For a subset P of $\overline{1,2m}$ let us note $J(P) = I_m \cup (\cup_{i \in P} B_i)$. Now

(8) $$P \in U(\mathcal{B}) \iff J(P) \text{ is a } m - \text{parallelipipedon.}$$

The improper subsets are, obviously, elements of $U(\mathcal{B})$ but it is possible that some proper subsets be in $U(\mathcal{B})$, too. Let us exemplify. For the MAP $\mathcal{A}$ suggested by fig.1, there are no proper subsets in $U(\mathcal{A})$. For the MAP $\mathcal{T}$ given by fig.3, $U(\mathcal{T})$ contains the following proper subsets of $\overline{1,4}$: $\{3\}, \{4\}, \{3,4\}, \{1,3,4\}$ and $\{2,3,4\}$.

We recapitulate the above consideration by introducing , for the MAP $\mathcal{A}$ of F_n the third copy $t(\mathcal{A}) = (\mathcal{B}, P_1, P_2)$ where $\mathcal{B} = p(\mathcal{A}) \in M_{n-1}$ and P_1, P_2 are in $U(\mathcal{B})$.

Theorem 3. *For each $n \geq 2$, for each $\mathcal{B}$ in M_{n-1}, for each P_1, P_2 in $U(\mathcal{B})$, there exists only one $\mathcal{A}$ in M_n so that $t(\mathcal{A}) = (\mathcal{B}, P_1, P_2)$.*

Proof. Let a, b be the functions

(9) $$a(x) = \begin{cases} 1 & if \quad x \in P_1 \\ 0 & if \quad x \notin P_1 \end{cases} \qquad b(x) = \begin{cases} 2 & if \quad x \in P_2 \\ 3 & if \quad x \notin P_2 \end{cases}$$

Let $\mathcal{A} = \{A_k \mid k = \overline{1,2n}\}$ be defined by

$$A_1 = J(P_1) \times [0,1], \quad A_2 = J(P_2) \times [2,3] \quad and$$

$$\forall_{h \in \overline{1,2(n-1)}} \quad A_{h+2} = B_h[a(h), b(h)].$$

Obviously, $\mathcal{A}$ is the only one element of M_n which satisfies the enounced conditions.

Remark 3. Altought it is easy and expresive to illustrate this theorem for $n = 2$, we will prefer to illustrate the case $n = 3$, where the full generality appears. Let $\mathcal{B} \in M_2$ be given by fig.5; we may take $P_1 = \{2,3\}$ and $P_2 = \{3,4\}$ because BCIG and DFLK are rectangles.

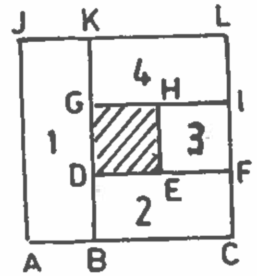

fig 5

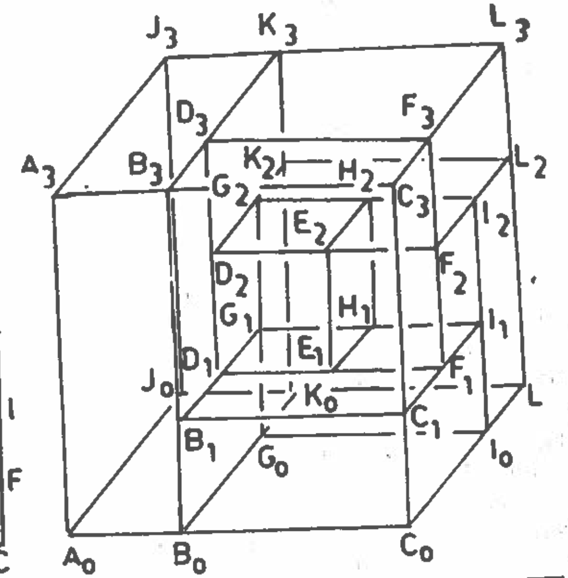

fig 6

On fig.6 the points are indexed by the corresponding values of x_3.

5.Succesive reductions of MAPs . The most general $\mathcal{A}$ of M_n is given by $t(\mathcal{A}) = (\mathcal{B}, P_1, P_2)$. The presence of $\mathcal{B} = p(\mathcal{A})$ seams to be natural, especially in our intention of recurrency. As the presence of P_1 and P_2 seams to be embarrassing , we shall gradually attenuate it.

Lemma 1. *For each $m \geq 1$, for each $\mathcal{B} \in M_m$, if P_1 and P_2 are in $U(\mathcal{B})$, then $Q = P_1 \cap P_2$ is in $U(\mathcal{B})$,too*

Proof. As $J(P_u)$ are m -parallelipipedon, $J(P_1) \cap J(P_2) = J(Q)$ will also be m- parallelipipedon.

Corollary. *If $\mathcal{A} \in M_n$, $t(\mathcal{A}) = (\mathcal{B}, P_1, P_2)$ and $Q = P_1 \cap P_2$, there exsits only one element $\mathcal{C} = r(\mathcal{A})$ of M_n so that $t(\mathcal{C}) = (\mathcal{B}, Q, Q)$.*

We may interpret r as a projection $r : M_n \to M_n$ and so the *first reduction* of the complexity of a MAP.

Theorem 4. *For earh $n \geq 1$, for each $\mathcal{A}$ in M_n, for each d in $\overline{0, n-1}$: if $\mathcal{C} = r(\mathcal{A})$, then $v_d(\mathcal{A}) = v_d(\mathcal{C})$.*

Proof. Let us suppose $t(\mathcal{A}) = (\mathcal{B}, P_1, P_2)$, with $P_1 \neq P_2$ (conversely the conclusion is trivial). We will point out a finite sequence of "elementary modifications" $\{e_i\} i = \overline{1,p}$ so that for $\mathcal{A}_0 = \mathcal{A}$ and $\forall_{i \in \overline{1,p}} e_i(\mathcal{A}_{i-1}) = \mathcal{A}_i$ to result $\mathcal{A}_p = \mathcal{C}$.

To each e_i we will atach a bijection $\hat{e}_i : V(\mathcal{A}_{i-1}) \to V(\mathcal{A}_i)$ conserving the dimension d of the skeletons : $X \in S_d \Rightarrow \hat{e}_i(X) \in S_d$.

In the "lower stratum" $q^{-1}([0,1])$ of K_n we consider $R = P_1 - Q \neq \emptyset$ (conversely there are no modifications in this stratum). We must pass from $A_1 = J(P_1)\times[0,1]$ to $C_1 = J(Q)\times[0,1]$,i.e. to eliminate from A_1 the n- parallelipipedons $B_h \times [0,1]$ with h in R to obtain C_1. Each of these eliminations is compensated by the substitution of $A_{h+2} = B_h \times [1, b(h)]$ by $C_{h+2} = B_h \times [0, b(h)]$. Hence, each h from R indutes an operation e_h: the n- parallelipipedon $B_h \times [0,1]$ passes from A_1 to C_{h+2}.

Let b_h be the $(n-2)$ -face of B_h in contact with I_{n-1}. After an eventual reorder of coordinates and a symetry $x' = 3 - x$,we may accept : $B_h = b_h \times [0,1]$. For the square $S = [0,1]\times[0,1]$, $B_h \times [0,1] = b_h \times S$. In the plane of S, let r_h be the rotation of centre (1,1) maping (0,1) in (1,0). Let R_h be the extension of r_h in E_n keeping constant the coordinates x_i with $i < n-1$. We restrict R_h to $b_h \times [0,1]$ and we get $\hat{e}_h$ by extending it to $V(\mathcal{A}_j)$ for the convenient j. Obviuosly, $\hat{e}_h$ does not change d.

A similar sequence of elementary modifications appears in the "upper stratum" $q^{-1}([2,3])$ and there are no other modifications to pass from $\mathcal{A}$ to $\mathcal{C}$. The theorem is proved.

For illustration we take again $\mathcal{A}$ from fig.6; as the parallelipipedon $A_3ARKJ\times[0,3]$ (on the left hand) does not support modifications, it was not taken in fig.7. The lower stratum imposes only the modification corresponding to $h = 2$; R_2 is the rotation round the axis D_1F_1; $\hat{e}_2$ change B_1 in D_0 (keeping d =2) and C_1 in F_0 (keeping d=1). Similary, in the upper stratum, for $h = 1$, the rotation round axis G_2I_2 change K_2, L_2 in G_3, I_3.

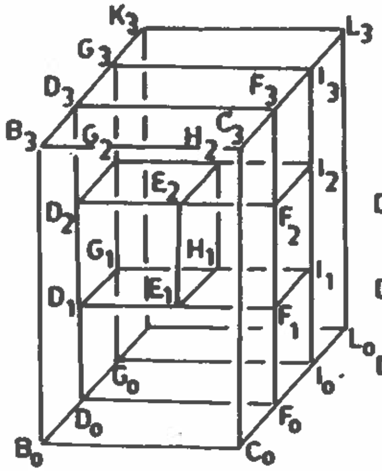

fig.7

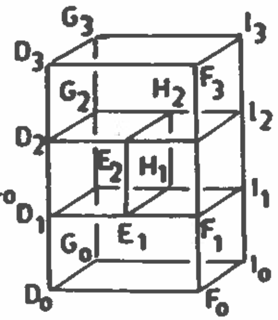

fig.8

To make fig.7 more expressive (and to prepare new reductions for this $\mathcal{A}$, fig.8 takes only the region $1 \leq x_2 \leq 2$.

Theorem 5. *For each $n \geq 2$, for each $\mathcal{A}$ in M_n, there exists $\mathcal{D} \in M_n$ so that $t(\mathcal{D}) = (p(\mathcal{D}), \emptyset, \emptyset)$ and, for each d in $\overline{0, n-1}$, $v_d(\mathcal{D}) = v_d(\mathcal{A})$.*

Proof. According to the previous theorem we may accept $t(\mathcal{A}) = (B, Q, Q)$.

In a *first stage* we adopt the *supplementary hypothesis*: SH: "there exists $q \in Q$ so that $Q' = Q - \{q\} \in U(B)$ " . The above exposed technique of "elementary modifications in the external strata" leads us similary to the intermediate conclusion: for $t^{-1}(B, Q', Q') = \mathcal{A}'_0\ \forall_{d \in \overline{1,n-1}}\ v_d(\mathcal{A}) = v_d(\mathcal{A}')$.

For illustration, to pass from the MAP $\mathcal{C}$ above constructed (with $Q = \{3\}$) to $\mathcal{D}$, (fig.9) we must rotate the rectangles $E_uF_uI_uH_u$ round E_uH_u up to $E_uF_vH_vH_u$ for $(u, v) = (1, 0)$ and $(2, 3)$.

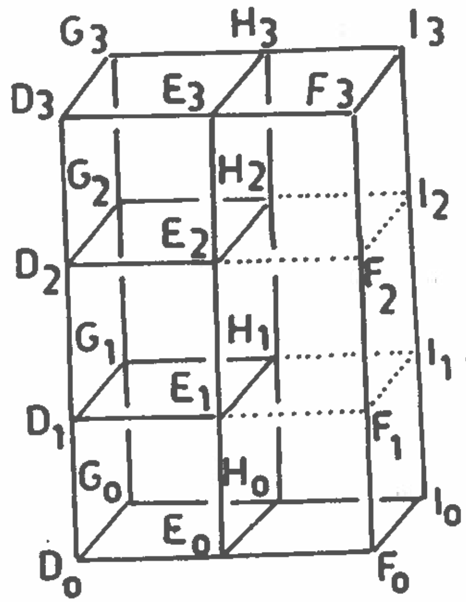

fig.9

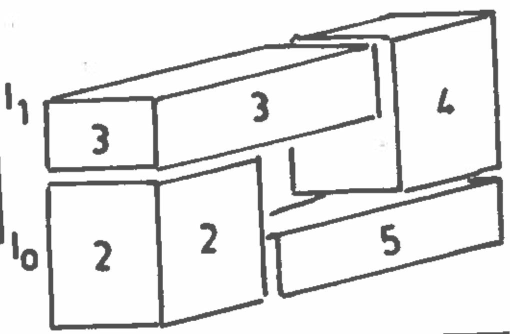

fig.10

Of, course, as long as SH susbsits we may repeat this stage up to void Q. Fig.10 presents $B \in M_3$ in its "central part" $1 \leq x_1 \leq 2$. Here $Q = \overline{2,5}$ is in $U(B)$ but SH does not subsits. In such a situation we cannot eliminate *succesivelly* elements q of Q. It is not impossible to eliminate *smultaneously* more elements of Q but is rather difficult to indicate an algorithm for such an operation. This is the reason to consider the following second stage of the proof.

The central idea of this stage is to temporarily modify $B = p(\mathcal{A})$ to assure SH. Consequently we insert a lemma.

Lemma 2. *For each $m \geq 2$, for each MAP B of F_m, for each unvoid*

$Q \in U(\mathcal{B})$, there exists a MAP $\mathcal{B}'$ of F_m so that $Q \in U(\mathcal{B}')$ and SH be valid for $(\mathcal{B}', Q)$.

Proof of lemma. There exists a $(m-1)$ -face ϕ_k of I_m which is not included in a $(m-1)$ -face of the m -parallelipipedon $J(Q)$. Let H be the $(m-1)$ - plane containing ϕ_k and B'_k the m -parallelipipedon separated by H in $J(Q)$ which does not contains internal points of I_m. We substitute B_k by B'_k and we compensate this extension by diminuating those B_h with $h \neq k$ and $B_h \cap B'_k \neq \emptyset$.

Continuation of the proof of theorem 5. We are able to indicate a finite sequence of elementary modifications to pass from $\mathcal{B}$ to $\mathcal{B}'$; each of these modifications can be lifted to an elementary one on the way from $\mathcal{A}$ to a MAP $\mathcal{A}'$. Each of these modifications in M_n keeps constant the numbers v_d.

We finish the proof by the following indications with respect to its algorithm. The preparatory change of $(\mathcal{A}, \mathcal{B})$ in $(\mathcal{A}', \mathcal{B}')$ makes again SH valid; operating again as in the first stage we continue to dimunuate card Q passing from $(\mathcal{A}, \mathcal{B})$ to $(\mathcal{A}", \mathcal{B}')$ where $Q = \emptyset$; the way from $\mathcal{B}$ to $\mathcal{B}'$ must be inverted and lifted to pass from $(\mathcal{A}", \mathcal{B}')$ to $(\mathcal{D}, \mathcal{B})$; the initial $\mathcal{B}$ is so restored and the proof is over.

6.Calculus of values of v_d . Theorem 6. *For each $n \geq 1$ and for each $\mathcal{A}$ in M_n, $v_0(\mathcal{A}) = 2^n$.*

Proof. Every vertex X of K_n is in $V(\mathcal{A})$ and in S_0, hence in $V_0(\mathcal{A})$. As we defined $V_0(\mathcal{A})$ as an intersection with $S_0, V_0(\mathcal{A}) = S_0 \cdot S_0$ is in bijective correspondence with the set of functions $x : \overline{1,n} \to \{0,3\}$ and its cardinal is, obviously, 2^n, q.e.d.

Theorem 7. *For each $\mathcal{A}$ in $M_2, v_1(\mathcal{A}) = 2^2$.*

Proof. To divide F_2 in $\mathcal{F}_2$ we exclusively use segments starting (horizontally or vertically) from one of the vertexes 1,2,3,4 of I_2 and finishing on a side of K_2. To obtain a MAP $\mathcal{A}$, from each of these vertexes we must use:

- at least one segment - otherwise an A_k is not a rectangle;
- at most one segment - otherwise $\mathcal{A}$ is not minimal.

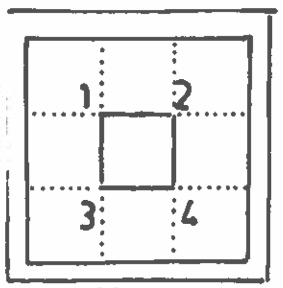

fig.11

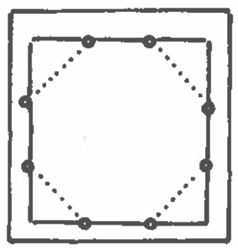
fig.12

Consequently, a MAP $\mathcal{A}$ of F_2 contains exactly an extremity of each of four segments dotted on fig.12; hence $v_1(\mathcal{A}) = 4$, q.e.d.

Theorem 8. *For each $n \geq 1$, for each $\mathcal{A}$ in M_n, for each d in $\overline{0, n-1}$: $v_d(\mathcal{A}) = 2^n$.*

Proof. (by induction about n). The verification stage, for the rather trivial case $n = 1$ is realised by the theorem 6. For $n = 2$ we must also use the theorem 7.

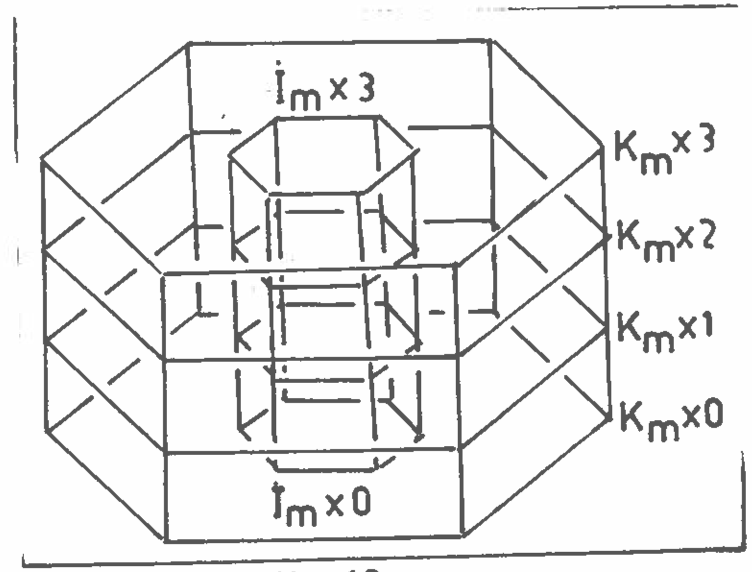

fig.13

In the recursive stage, we accept the conclusion for $n = m$ We suggest by fig.13 a MAP $\mathcal{D} = t^{-1}(\mathcal{B}, \emptyset, \emptyset)$ of M_n with $n = m + 1$. Here K_m and I_m are presented as hexagones thinking their sides as $(m-1)$ -faces (for m =3 there are six faces of a cube). The central column represents $D_1 \cup I_m \cup D_2$. The (missing from figure) n -parallelipipedons $D_{k+2} = B_k \times [0,3]$ assure: for $1 \leq d \leq m$ there are not points X in $V_d(\mathcal{D})$ with $0 < q(X) < 3$.

For $d = m = n-1$, on each of the bases $K_m \times u$ of K_n (here $u = 0$ and $u = 3$) there are N points of $V_m(\mathcal{D})$ where $N = cardS_0(K_m) = 2^m$. Hence : $v_m(\mathcal{D}) = 2 \cdot N = 2^{m+1} = 2^n$.

The case $d = 0$ benefits of the theorem 6.

For $0 < d < m = n-1$, the point X is in $V_d(\mathcal{D})$ iff $q(X) \in \{0,3\}$ and $p(X) \in V_d(\mathcal{B})$. Using (for B) the inductive hypothesis, $V_d(\mathcal{D}) = 2 \cdot 2^m = 2^n$ and the proof is over.

Corollary. *The total number $y_n = cardV(\mathcal{A}$, for each $\mathcal{A}$ in M_n is $y_n = n \cdot 2^n$.*

7.Unsolved problems. As we have precised a configuration, a terminology and some techniques, let us point out some other questions less or more useful for the Hyperbolic Analysis, belonging to the Combinatorics.

a) Which is the number $z_n = cardM_n$?

b) Let us consider $\mathcal{A}$ and $\mathcal{B}$ from M_n "of the same type" iff an Euclidean transformation of K_n on itself maps $\mathcal{A}$ into $\mathcal{B}$. Which is the number u_n of types of

MAPs of F_n ? How those z_n MAPs are distributed into these u_n types ?

c) Let $P_{n,d}$ be the minimal subset of $S_d(K_n)$ including $V_d(A)$ for each A) of M_n ; it is not difficult to state: card $P_{n,d} = 2^n \cdot C_n^d$. For $W \subset P_{n,d}$ with card $W = 2^n$, which are the conditions to exists (at least) a MAP A of F_n so that $W = V_d(A)$?

For $n = 2$ we are able to answer these questions.

a) The above given proof of the theorem 7 allows the estimation of z_2 as the cardinal 2^4 of the set of maps $f : \overline{1,4} \to \{h, v\}$.

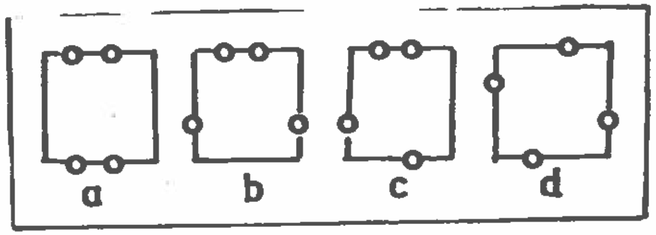

fig. 14

b) and c) The $u_2 = 4$ types of MAPs of F_2 are given in fig.14 by the distribution of $V_1(A)$ on the sides of K_2. In each of these types a,b,c,d there are : 2, 4, 8 respectivelly 2 MAPs. Etc.

Remark 4. For the commodity of the exposition we have considered K_n as n-cubes (not only n- parallelipipepedons) and $\{H_{i,u} \mid u \in \{1,2\}\}$ dividing into three equal parts the transversal edges (not only parallel to the $(n-1)$ -face $x_i =$ const.) ;there are no conceptual difficulties for the general case.

The above exposed problems are advantageously solvable (by different geometrical tecniques) for n -simplexes instead of n -cubes ; this will be the object of a distinct paper.

REFERENCES

B a n t a s, Gh., T u r i n i c i, M. – *Hyperbolic Mean Value Theorems of Non-differential form,* Bol. Unione Matematica Italiana, to appear.

C o x e t e r, H.S.M. – *Regular Complex Polytopes* . Cambridge University Press, 1975.

Received:23.VIII.1991

Department of Mathematics
University "Al.I.Cuza"Iasi
Nr.11, Street Copou, Iasi 6600
ROMANIA