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SOME CONDITIONS ON REAL HYPERSURFACES OF QUATERNIONIC PROJECTIVE SPACE

BY

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1.Introduction. Let M be a connected real hypersurface of the quaternionic projective space $QP^m, m \geq 2$, endowed with the metric g of constant quaternionic sectional curvature 4. Let N be a unit local normal vector field on M and $U_k = -J_k N, k = 1, 2, 3$, where $\{J_k\}_{k=1,2,3}$ is a local basis of the quaternionic structure of QP^m ,[3]. Let us denote by $D^\perp = \text{Span}\{U_1, U_2, U_3\}$ and by $\mathbf{D}$ its orthogonal complement in TM. $\mathbf{D}$ can be considered as a subbundle of TM , [11].

Recently, [1], J.B e r n d t , generalizing previous results of A. M a r t i n e z and the author, [8], has proved that any real hypersurface of $QP^m, m \geq 2$ such that $g(A\mathbf{D}, \mathbf{D}^\perp) = \{0\}$, where A is the Weingarten endomorphism of M is congruent to an open subset of either a tube of radius r, $0 < r < \pi/2$, over the canonically (totally geodesic) embedded quaternonic projective space $QP^k, k \in \{0, .., m-1\}$, or of a tube of radius $r, 0 < r < \pi/4$, over the canonically (totally geodesic) embedded complex projective space CP^m.

In [7], A.M a r t i n e z introduced ruled real hypersurfaces of QP^m, obtaining several examples, as real hypersurfaces for which the distribution $\mathbf{D}$ is integrable. This is equivalent to the fact that $g(A\mathbf{D}, \mathbf{D}) = \{0\}$, [9].

The purpose of the present paper is to prove the following

Theorem 1. *Let M be a areal hypersurface of $QP^m, m > 3$. Then if $\mathbf{D}$ is a curvature inariant subspace of TM and $g(A\mathbf{D}, \mathbf{D}^\perp) \neq \{0\}$, then M is congruent to an open subset of a ruled real hypersurface of QP^m.*

Theorem 2. *Let M be a hypersurface of $QP^m, m \geq 2$, satisfying $L_{U_i}\Phi_i = 0$, $i = 1, 2, 3$ where L denotes the Lie derivative and Φ_i is the tangent component of J_i Then M is congruent to an open subset of a tube of radius r , $0 < r < \pi/2$, over the canonically (totally geodesic) embedded quaternionic projective space $QP^k, k \in \{0, ..., m-1\}$.*

It is known, [10, Lemma 4.2] that there do not exist parallel real hypersurfaces in quaternionic projective space. Thus we study weaker conditions in the following Theorems.

Theorem 3. *There do not exist real hypersurfaces M of $QP^m, m \geq 2$, satisfying $\sigma(R(X, Y)AZ) = 0$, for any X, Y, Z tangent to M, where σ denotes the* *cyclic sum and R is the curvature tensor of M.*

We now can consider the action of the derivation $R(X,Y)$ on the algebra of tensor fields of a real hypersurface M of quaternionic projective space QP^m, [13]. If T is a tensor field of type (r,s), then $R(X,Y)\cdot T=\nabla_X\nabla_Y T-\nabla_Y\nabla_X T-\nabla_{[X,Y]}T$ for vector fields X and Y tangent to M, where ∇ denotes the covariant derivative of M. We shall denote by $R\cdot T$ the tensor field of type $(r,s+2)$ defined by $(R.T)(X_1,...,X_s,X,Y)=(R(X,Y)\cdot T)(X_1,...,X_s)$.

Then we obtain from Theorem 3 the following

Corollary 4. *There do not exist real hypersurfaces M of $QP^m, m\geq 2$, satisfying $R\cdot A=0$.*

Corollary 4 in the case of real hypersurfaces of CP^m has been proved by S. M a e d a, [5]. Similar results to the ones appearing in Theorem 1 and 2 when the ambient space is CP^m have been proved by S. M a e d a and S. U d a g a w a, [6].

We shall say that M has pseudo-parallel second fundamental form if for any X,Y tangent to M the following expression is satisfied

$$(1.1)\qquad (\nabla_X A)Y=a\sum_{i=1}^{3}\{2f_i(X)(A\Phi_i-\Phi_i A)Y+$$

$$+f_i(Y)(A\Phi_i-3\Phi_i A)X+g((A\Phi_i-3\Phi_i A)X,Y)U_i\}$$

for some $a\in R$. If $a=0$ M is parallel. Therefore we suppose that a is nonnull. We have

Theorem 5. *The unique pseudo-parallel real hypersurface of $QP^m, m\geq 2$, for any $a\in R-\{0\}$, are open subsets of geodesic hyperspheres of radius r, $0<r<\pi/2$, such that $\cot r=1/(2a)$.*

2.Preliminaries. Let M be a connected real hypersurface of $QP^m, m\geq 2$, with metric g of constant quaternionic sectional curvature 4. If X is a vector field tangent to M we write $J_iX=\Phi_iX+f_i(X)N, i=1,2,3$, where Φ_iX denotes the tangential component of J_iX and $f_i(X)=g(X,U_i)$. As $J_i^2=-\mathrm{Id}, i=1,2,3$, where Id denotes the identity endomorphism on TQP^m, we get

$$(2.1)\qquad \Phi_i^2X=-X+f_i(X)U_i,\quad f_i(\Phi_iX)=0,\quad \Phi_iU_i=0,\quad i=1,2,3.$$

for any X tangent to M. As $J_iJ_j=-J_jJ_i=J_k$, where (i,j,k) is a cyclic permutation of $(1,2,3)$, we obtain

$$(2.2)\qquad \Phi_iX=\Phi_j\Phi_kX-f_k(X)U_j=-\Phi_k\Phi_jX+f_j(X)U_k,$$

$$f_i(X)=f_j(\Phi_kX)=-f_k(\Phi_jX),$$

for any X tangent to M, where (i,j,k) is a cyclic permutation of $(1,2,3)$. It is also easy to see that for any X,Y tangent to M and $i=1,2,3$,

$$(2.3)\qquad g(\Phi_iX,Y)+g(X,\Phi_iY)=0,\quad g(\Phi_iX,\Phi_iY)=g(X,Y)-f_i(X)f_i(Y)$$

and

$$(2.4)\qquad \Phi_iU_j=U_k=-\Phi_jU_i,\quad (i,j,k)\ \textit{being a cyclic permutation of}\ (1,2,3).$$

From the expression of the curvature tensor of $QP^m, m\geq 2$, we have that the equations of Gauss and Codazzi of M are given respectively by

$$(2.5)\qquad R(X,Y)Z=g(Y,Z)X-g(X,Z)Y+\sum_{i=1}^{3}\{g(\Phi_iY,Z)\Phi_iX-$$

$$-g(\Phi_iX,Z)\Phi_iY+2g(X,\Phi_iY)\Phi_iZ\}+g(AY,Z)AX-g(AX,Z)AY$$

and

$$(2.6)\qquad (\nabla_XA)Y-(\nabla_YA)X=\sum_{i=1}^{3}\{f_i(X)\Phi_iY-f_i(Y)\Phi_iX+2g(X,\Phi_iY)U_i\},$$

for any X,Y,Z tangent to M.

Finally from the expression of the covariant derivatives of $J_i, i=1,2,3$, see [3], it is easy to see that

$$(2.7)\qquad \nabla_XU_i=-p_j(X)U_k+p_k(X)U_j+\Phi_iAX$$

and

$$(2.8)\qquad (\nabla_X\Phi_i)Y=-p_j(X)\Phi_kY+p_k(X)\Phi_jY+f_i(Y)AX-g(AX,Y)U_i$$

for any X,Y tangent to M, (i,j,k) being a cyclic permutation of $(1,2,3)$ and $p_i, i=1,2,3$, local 1-forms on QP^m.

3.Proof of Theorem 1. As $R(\mathbf{D},\mathbf{D})\mathbf{D}\subset\mathbf{D}$, from (2.5), $R(X,Y,Z,U_i)=0$ for any $X,Y,Z\in\mathbf{D}, i=1,2,3$, gives

$$(3.1)\qquad 0=g(AY,Z)g(AX,U_i)-g(AX,Z)g(AY,U_i).$$

We shall distinguish the following cases:

CASE 1. $(AU_1)^{\mathbf{D}}\neq 0$, $(AU_2)^{\mathbf{D}}=(AU_3)^{\mathbf{D}}=0$, where $(AU_i)^{\mathbf{D}}$ denotes the $\mathbf{D}$-component of $AU_i, i=1,2,3$. We write $AU_1=\gamma_1X_1+\sum_{i=1}^{3}\mu_iU_i$, where $X_1\in\mathbf{D}$ is a unit vector field. Taking $X=X_1$ and $Y\in\mathbf{D}$ and orthogonal to X_1 in (3.1), for any $Z\in\mathbf{D}$ we have $\gamma_1g(AY,Z)=0$, that is, $AY=0$ for any $Y\in\mathbf{D}$ and orthogonal to X_1. Thus $AX_1=g(AX_1,X_1)X_1+\gamma U_1$ and from (2.1) Φ_1A is an endomorphism of $\mathbf{D}$ whose rank is ≤ 1. For any unit $X\in TQP^m$, let us call $Q(X)=\mathrm{Span}\{X,\Phi_1X,\Phi_2X,\Phi_3X\}$. Then from (2.5) for any unit $Y\in\mathbf{D}$ such that $Q(X_1)\perp Q(Y)$, the sectional curvatures for planes contained into $Q(Y)$ are equal to 4. But the fact that $\mathrm{rank}(\Phi_1A)\leq 1$ from (2.5) also gives that the sectional curvatures of planes contained into $Q(X_1)$ are equal to 4. Then from [7] the result follows in this case.

CASE 2. $(AU_1)^{\mathbf{D}}$ and $(AU_2)^{\mathbf{D}}$ are nonnull, $(AU_3)^{\mathbf{D}} = 0$ and $AU_1 = \mu_1 X_1 + \gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 U_3$, $AU_2 = \mu_2 X_2 + \gamma_2 U_1 + \delta_2 U_2 + \delta_3 U_3$, for unit $X_1, X_2 \in \mathbf{D}$ with $g(X_1, X_2) = 0$. Then from (3.1) we obtain that for any $Y \in \mathbf{D}$ orthogonal to Span$\{X_1, X_2\}$, $AY = 0$. And also $g(AX_1, X_1) = g(AX_2, X_2) = 0$. Thus we can write $AX_1 = g(AX_1, X_2)X_2 + \mu_1 U_1$ and $AX_2 = g(AX_1, X_2)X_1 + \mu_2 U_2$. Taking $X = Z = X_2$ and $Y = X_1$ in (3.1) when $i = 2$ we obtain $0 = \mu_2 g(AX_1, X_2)$. This implies that $g(AX_1, X_2) = 0$ and finishes the proof in this case.

CASE 3. $(AU_1)^{\mathbf{D}}, (AU_2)^{\mathbf{D}}$ are nonnull and linearly dependent and $(AU_3)^{\mathbf{D}} = 0$. Thus we can write $AU_1 = \mu_1 X_1 + \delta_1 U_1 + \delta_2 U_2 + \delta_3 U_3$, $AU_2 = \mu_2 X_1 + \delta_2 U_1 + \beta_2 U_2 + \beta_3 U_3$, for a unit $X_1 \in \mathbf{D}$. From (3.1) it is easy to see that $AZ = 0$ for any $Z \in \mathbf{D}$ such that $Z \perp X_1$. Then for any $Y \in \mathbf{D}$ such that $Q(Y) \perp Q(X_1)$, the sectional curvature for planes of $Q(Y)$ is equal to 4. Analogously, the sectional curvatures for sections of $Q(X_1)$ are equal to 4 and the result follows from [7].

CASE 4. $(AU_i)^{\mathbf{D}} \neq 0$, $i = 1, 2, 3$.

Subcase 4-1. $(AU_i)^{\mathbf{D}} = \mu_i X_1$, $i = 1, 2, 3$. From (3.1) it follows that for any $Z \in D$, orthogonal to X_1, $AZ = 0$. Thus $AX_1 = g(AX_1, X_1)X_1 + \mu_1 U_1 + \mu_2 U_2 + \mu_3 U_3$ and the result follows as in Case 3.

Subcase 4-2. $(AU_1)^{\mathbf{D}} = \mu_1 X_1, (AU_2)^{\mathbf{D}} = \mu_2 X_2, (AU_3)^{\mathbf{D}} = \delta_1 X_1 + \delta_2 X_2$, for unit $X_1, X_2 \in \mathbf{D}$ such that $g(X_1, X_2) = 0$. From (3.1) we have that for any $Y, Z \in \mathbf{D}$ orthogonal to $X_1, g(AY, Z) = g(AY, X_1) = 0$ and for any $Y, Z \in \mathbf{D}$ orthogonal to X_2, $g(AY, Z) = g(AY, X_2) = 0$. Thus $AY = 0$ for any $Y \in \mathbf{D}$ orthogonal to Span$\{X_1, X_2\}$ and $g(AX_1, X_1) = g(AX_1, X_2) = g(AX_2, X_2) = 0$. This finishes the proof in this subcase.

Subcase 4-3. $(AU_i)^{\mathbf{D}} = \mu_i X_i, i = 1, 2, 3$, with $g(X_i, X_j) = \delta_{ij}, i, j = 1, 2, 3$. The result follows as in Subcase 4-2.

This concludes the proof.

4.Proof of Theorem 2. Let $X \in TM$. Then $0 = (L_{U_1}\Phi_1)X = [U_1, \Phi_1 X] - \Phi_1([U_1, X]) = (\nabla_{U_1}\Phi_1)X - \nabla_{\Phi_1 X}U_1 + \Phi_1 \nabla_X U_1$. From (2.7) and (2.8) this gives

$$(4.1) \qquad 0 = p_3(U_1)\Phi_2 X - p_2(U_1)\Phi_3 X + f_1(X)AU_1 - p_3(\Phi_1 X)U_2 + p_2(\Phi_1 X)U_3 - \Phi_1 A \Phi_1 X + p_3(X)U_3 + p_2(X)U_2 - AX.$$

Let $X \in \mathbf{D}$ and take the scalar product of U_1 with (4.1). Then we have $g(AX, U_1) = 0$. The similar expressions to (4.1) corresponding to $0 = (L_{U_j}\Phi_j)\, X, j = 2, 3$, give

$$(4.2) \qquad g(AX, U_1) = 0, \quad i = 1, 2, 3$$

for any $X \in \mathbf{D}$. Thus $g(A\mathbf{D}, \mathbf{D}^{\perp}) = \{0\}$. This implies, [1], that we can suppose $AU_i = \alpha_i U_i$, $i = 1, 2, 3$, and α_i are constant. Thus (4.1) gives

$$(4.3) \qquad 0 = f_1(X)\alpha_1 U_1 - p_3(\Phi_1 X)U_2 + p_2(\Phi_1 X)U_3 - \Phi_1 A \Phi_1 X + p_3(X)U_3 + p_2(X)U_2 - AX$$

for any $X \in TM$. Analogously

$$(4.4) \qquad 0 = f_2(X)\alpha_2 U_2 + p_3(\Phi_2 X)U_1 - p_1(\Phi_2 X)U_3 - \Phi_2 A \Phi_2 X + p_3(X)U_3 + p_1(X)U_1 - AX$$

for any $X \in TM$.

Let $X \in \mathbf{D}$. Suppose that $AX = \lambda X$. From (4.3) we get $-p_3(\Phi_1 X)U_2 + p_2(\Phi_1 X)U_3 - \Phi_1 A \Phi_1 X + p_3(X)U_3 + p_2(X)U_2 - \lambda X = 0$. If in the above equation we take the $\mathbf{D}$ -component, we obtain $-\Phi_1 A \Phi_1 X = \lambda X$. Then $-\Phi_1^2 A \Phi_1 X = \lambda \Phi_1 X$. From (2.1), $A\Phi_1 X = \lambda \Phi_1 X$, from (4.4) and the corresponding equation obtained from $(L_{U_3}\Phi_3)X = 0$, we have that

$$(4.5) \qquad A\Phi_i X = \lambda \Phi_i X, \quad i = 1, 2, 3, \ \textit{for any } X \in \mathbf{D} \textit{ such that } AX = \lambda X$$

Taking such a unit X in (2.6) we get from (4.5), for $Y = U_1$,

$$(4.6) \qquad -\Phi_1 X = p_3(X)(\alpha_1 - \alpha_2)U_2 - p_2(X)(\alpha_1 - \alpha_3)U_3 + \lambda(\alpha_1 \,\mathrm{Id} - A)\Phi_1 X - U_1(\lambda)X + (A - \lambda \,\mathrm{Id})\nabla_{U_1} X.$$

Taking the scalar product of (4.6) and X we have $U_1(\lambda) = 0$. Bearing this in mind and taking the scalar product of (4.6) and $\Phi_1 X$ we obtain $-1 = \lambda(\alpha_1 - \lambda)$. Thus $\lambda \neq 0$. Similarly we get

$$(4.7) \qquad -1 = \lambda(\alpha_1 - \lambda), \quad i = 1, 2, 3.$$

From (4.7) and the fact that $\lambda \neq 0$, we get $\alpha_1 = \alpha_2 = \alpha_3$. Thus the result follows from [8].

Proof of Theorems 3 and 5. Consider $X = U_1, Y = U_2$ and $Z = U_3$. From (2.5) the condition $\sigma(R(X, Y)AZ) = 0$ gives

$$(5.1) \qquad \Phi_1 AU_1 + \Phi_2 AU_2 + \Phi_3 AU_3 = 0.$$

Let now take $Z \in \mathbf{D}$. From (2.5) and our condition we have

$$(5.2) \qquad \begin{aligned} &\{g(\Phi_j Z, AU_j) + g(\Phi_k Z, AU_k)\}U_i + \{g(U_k, AZ) - g(\Phi_i Z, AU_j)\}U_j - \\ &- \{g(U_j, AZ) + g(\Phi_i Z, AU_k)\}U_k - 2\Phi_i AZ + \{g(U_j, AU_j) + \\ &+ g(U_k, AU_k)\}\Phi_i Z - g(U_i, AU_j)\Phi_j Z - g(U_i, AU_k)\Phi_k Z = 0, \end{aligned}$$

where (i, j, k) is a cyclic permutation of (1,2,3). Taking the scalar product of (5.2) and U_i we have $g(\Phi_j Z, AU_j) + g(\Phi_k Z, AU_k) = 0$, where $j \neq k$, $j, k = 1, 2, 3$. Thus

$$(5.3) \qquad g(\Phi_i Z, AU_i) = 0, \quad i = 1, 2, 3.$$

Take the scalar product of (5.2) and Z. We shall obtain

$$g(AZ, \Phi_i Z) = 0, \quad i = 1,2,3. \tag{5.4}$$

If we now take the scalar product of (5.2) and $\Phi_t Z$, $t = j, k$, we get from (5.4)

$$g(U_i, AU_j) = 0, \quad i \neq j, \quad i,j = 1,2,3. \tag{5.5}$$

The same reasoning with $\Phi_i Z$ gives $-2g(AZ,Z)+g(U_j,AU_j)+g(U_k,AU_k) = 0$, for any $j,k \in \{1,2,3\}$. Therefore

$$g(U_i, AU_i) = g(U_j, AU_j) = g(Z, AZ), \quad i,j \in \{1,2,3\}. \tag{5.6}$$

for any $Z \in \mathbf{D}$. Thus from equations (5.h), $h = 1,...,6$, we get $\Phi AU_i = 0$, $i = 1,2,3$. That means that $g(A\mathbf{D}, \mathbf{D}^\perp) = \{0\}$. Then, [8], $U_i, i = 1,2,3$, are principal.

But (5.6) implies that they must have the same principal curvature as any other tangent vector field Z, that is, M must be totally umbilical. The proof of Theorem 3 now concludes from [2].

Let us now prove Theorem 5. We call $H_i = a(A\Phi_i + \Phi_i A) - \Phi_i$, $i = 1,2,3$. Then from (1.1) and (2.6) we obtain

$$\sum_{i=1}^{3}\{f_i(X)H_iY - f_i(Y)H_iX - 2g(H_iX,Y)U_i\} = 0. \tag{5.7}$$

for any X, Y tangent to M. If in (5.7) we take $X, Y \in \mathbf{D}$ we get $-2\sum_{i=1}^3 g(H_iX,Y)U_i = 0$, and as U_i, $i = 1,2,3$ are mutually orthogonal,

$$g(H_iX, Y) = 0 \quad i = 1,2,3 \tag{5.8}$$

for any $X, Y \in \mathbf{D}$. Take now $Y = U_1$ and $X \in \mathbf{D}$. Then (5.7) gives $0 = -H_1X - 2\sum_{i=1}^3 g(H_iX,U_1)U_i$ and if we take its scalar product with U_j, $j = 1,2,3$, we obtain

$$0 = -g(H_1X, U_j) - 2g(H_jX, U_1) \quad j = 1,2,3 \tag{5.9}$$

for any $X \in \mathbf{D}$. Taking $j = 1$ we have $g(H_1X, U_1) = 0$. In general

$$g(H_iX, U_i) = 0 \quad i = 1,2,3 \tag{5.10}$$

for any $X \in \mathbf{D}$. Thus from (5.8) and (5.10) we can write $H_iX = -2\sum_{j\neq i} g(H_jX,U_i)U_j$, for any $X \in \mathbf{D}$. Therefore we have $g(H_1X, U_2) = -2g(H_2X, U_1)$ and $g(H_2X, U_1) = -2g(H_1X, U_2)$ and this implies that $g(H_1X, U_j) = 0$, $j = 1,2,3$. Similarly and from (5.8) we obtain

$$H_jX = 0 \quad j = 1,2,3. \tag{5.11}$$

for any $X \in \mathbf{D}$.

Take $X = Y = U_1$ in (5.7). Then we obtain $0 = -2g(HU_1,U_1)U_1 - 2g(H_2U_1,U_1)U_2 - 2g(H_3U_1,U_1)U_3$. This implies

$$g(H_jU_i, U_i) = 0 \quad i,j = 1,2,3 \tag{5.12}$$

Take $X = U_1, Y = U_2$ in (5.7). Therefore $0 = H_1U_2 - H_2U_1 - 2g(H_1U_1,U_2)U_1 - 2g(H_2U_1,U_2)U_2 - 2g(H_3U_1,U_2)U_3$. Take the scalar product of this expression and U_1 bearing in mind (5.12). Thus we have $0 = g(H_1U_2,U_1) - 2g(H_1U_1,U_2) = -3g(H_1U_1,U_2)$ because H_i is skewsymmetric, $i = 1,2,3$. This implies that $g(H_iU_i, U_j) = 0, i \neq j, i,j = 1,2,3$ and then

$$H_iU_i = 0 \quad i = 1,2,3 \tag{5.13}$$

Now we have obtained that $a\Phi_i AU_i = 0$, $i = 1,2,3$. Thus $a\Phi_i^2 AU_i = 0$ and as $a \neq 0$, from (2.1), $AU_i = f_i(AU_i)U_i$, $i = 1,2,3$. Thus $U_i, i = 1,2,3$, are eigenvectors and from [1] M must be an open subset of either a tube of radius r, $0 < r < \pi/2$, over QP^k, $k \in \{0,...,m-1\}$ or of a tube of radius r, $o < r < \pi/4$, over CP^m.

For the second kind of tubes the principal curvatures are $\cot r$, $-\tan r$, $2\cot(2r)$ and $-2\tan(2r)$ with respective multiplicities $2(m-1), 2(m-1), 1$ and 2. Take X such that $AX = (\cot r)X$. Then $H_1X = 0 = (2a\cot r)\Phi_1X - \Phi_1X$. Thus $\cot r = 1/(2a)$. But $H_2X = 0 = a(\cot r - \tan r)\Phi_2X - \Phi_2X$. Thus $a(\cot r - \tan r) = 1$. This implies $a^2 = -1/4$ and this case cannot occur.

The principal curvatures for tubes over $QP^k, k \in \{0,...,m-1\}$ are $\cot r$, $-\tan r$, and $2\cot(2r)$ with respective multiplicities $4l, 4k$ and 3, $l = m-k-1$. Let us take X such that $AX = (\cot r)X$, $H_iX = 0 = (2a\cot(r) - 1)\Phi_iX$ gives $\cot r = 1/(2a)$. Take Y such that $AY = (-\tan r)Y$. Then $H_iY = 0 = (-2a\tan(r) - 1)\Phi_iY$ gives $\tan r = -1/(2a)$. Thus $4a^2 = -1$ and there cannot exist such a Y. That is $k = 0$, concluding the proof of Theorem 5.

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ON THE GEOMETRY OF LEAVES ON A SEMI-INVARIANT $\xi^\perp$ -SUBMANIFOLD IN A KENMOTSU MANIFOLD

BY

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The semi-invariant $\xi^\perp$-submanifold (res. semi-invariant ξ -submanifolds) of a Kenmotsu manifold have been introduced and studied by the author of the present paper in [6] (see also [4] and [5]). The main purpose of this paper is to obtain new results on the geometry of semi-invariant $\xi^\perp$ - submanifolds in a Kenmotsu manifold. We remark that similar results for semi-invariant submanifolds of a Sasakian manifold have been obtained by A.B e j a n c u and the present author in [2] (see also [1]).

First, in section 1, I recall some fundamental results from [6] which are used in the next sections.

In section 2, I study the geometry of leaves of distributions which are involved in the definition of a semi-invariant $\xi^\perp$ - submanifold of a Kenmotsu manifold.

Finally, in the third section I study the natural f -structure on a semi-invariant $\xi^\perp$ - submanifold M and obtain some decomposition theorems for M.

1.Some fundamental results on semi-invariant $\xi^\perp$ -submanifolds of a Kenmotsu manifold. Let $\tilde{M}$ be a $(2n+1)$-dimensional almost contact metric manifold and denote by (φ,ξ,η,g) the almost contact metric structure on $\tilde{M}$, where φ is a tensor field of type (1.1), ξ is a vector field, η is a 1-form and g is a Riemmanian metric. As is well known, these tensor fields are related by

$$\varphi^2X = -X + \eta(X)\xi, \varphi\xi = 0, \eta_0\varphi = 0, \eta(\xi) = 1 \tag{1.1}$$

and

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{1.2}$$

for any vector fields X, Y tangent to $\tilde{M}$.

It is also known (see [4], [6]) that an almost contact metric manifold $\tilde{M}$ is a Kenmotsu manifold if and only if

$$(\tilde{\nabla}_X\varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X \tag{1.3}$$