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ON THE GEOMETRY OF LEAVES ON A SEMI-INVARIANT $\xi^\perp$ -SUBMANIFOLD IN A KENMOTSU MANIFOLD

BY

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The semi-invariant $\xi^\perp$-submanifold (res. semi-invariant ξ -submanifolds) of a Kenmotsu manifold have been introduced and studied by the author of the present paper in [6] (see also [4] and [5]). The main purpose of this paper is to obtain new results on the geometry of semi-invariant $\xi^\perp$ - submanifolds in a Kenmotsu manifold. We remark that similar results for semi-invariant submanifolds of a Sasakian manifold have been obtained by A.B e j a n c u and the present author in [2] (see also [1]).

First, in section 1, I recall some fundamental results from [6] which are used in the next sections.

In section 2, I study the geometry of leaves of distributions which are involved in the definition of a semi-invariant $\xi^\perp$ - submanifold of a Kenmotsu manifold.

Finally, in the third section I study the natural f -structure on a semi-invariant $\xi^\perp$ - submanifold M and obtain some decomposition theorems for M.

1.Some fundamental results on semi-invariant $\xi^\perp$ -submanifolds of a Kenmotsu manifold. Let $\tilde{M}$ be a $(2n+1)$-dimensional almost contact metric manifold and denote by (φ,ξ,η,g) the almost contact metric structure on $\tilde{M}$, where φ is a tensor field of type (1.1), ξ is a vector field, η is a 1-form and g is a Riemmanian metric. As is well known, these tensor fields are related by

$$\varphi^2X = -X + \eta(X)\xi, \varphi\xi = 0, \eta_0\varphi = 0, \eta(\xi) = 1 \tag{1.1}$$

and

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{1.2}$$

for any vector fields X,Y tangent to $\tilde{M}$.

It is also known (see [4], [6]) that an almost contact metric manifold $\tilde{M}$ is a Kenmotsu manifold if and only if

$$(\tilde{\nabla}_X\varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X \tag{1.3}$$

for any vectors fields X, Y tangent to $\tilde{M}$, where $\tilde{\nabla}$ denotes the Riemannian connection on $\tilde{M}$. From (1.1), (1.2) and (1.3) we also get

$$\tilde{\nabla}_X \xi = X - \eta(X)\xi \tag{1.4}$$

for any vector field X tangent to $\tilde{M}$. Denote by Ω the contact distribution on $\tilde{M}$, i.e.

$$\Omega = \{X \in \Gamma(T\tilde{M}), \eta(X) = 0\}$$

Then (1.4) implies that Ω is always integrable, therefore we can consider two classes of submanifolds in a Kenmotsu manifold, according to the position of the structure vector field ξ with respect to the tangent bundle of the submanifolds.

Let M be an m-dimensional Riemmanian manifold isometrically immersed in a Kenmotsu manifold $\tilde{M}$. We say that the submanifold M is a $\xi^\perp$ -submanifold (resp. a ξ -submanifold) if the structure vector field ξ of $\tilde{M}$ is normal (resp. tangent) to the submanifold M.

Denote by $\{\xi\}$ the 1-dimensional distribution defined by the structure vector field ξ. For a submanifold M of $\tilde{M}$, denote by TM and $TM^\perp$ the tangent bundle of M and the normal bundle to M, respectively. Also for any vector bundle $\mathcal{H}$ on M, denote by $\Gamma(\mathcal{H})$ the module of all differentiable sections of $\mathcal{H}$. Throughout the paper, all manifolds and morphisms are supposed to be differentiable of class C^∞.

Let M be a $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$.

Definition, [6]. *The $\xi^\perp$ -submanifold M of $\tilde{M}$ is called a* **semi-invariant $\xi^\perp$ -submanifold** *if there exist on M two differentiable orthogonal distributions D and $D^\perp$ such that the following conditions are fulfilled*

(i) $TM = D \oplus D^\perp$;

(ii) the distribution D is inavariant by φ, i.e. $\varphi(D_x) = D_x$ for each $x \in M$;

(iii) the distribution $D^\perp$ is anti-invariant by φ, i.e. $\varphi(D_x) \subset T_xM^\perp$ for each $x \in M$.

The distributions D and $D^\perp$ are called respectively **the inavariant distribution** and **the anti-invariant distribution** of M.

A semi-invariant $\xi^\perp$ -submanifold is said to be an **invariant** (resp. an **anti-inavariant**) $\xi^\perp$ -submanifold if we have $D_x^\perp = \{0\}$)resp. $D_x = \{0\}$) for each $x \in M$. A semi-invariant $\xi^\perp$ -submanifold is said to be **proper** if it is neither an invariant nor an anti-invariant $\xi^\perp - submanifold$.

Some examples of semi-invariant $\xi^\perp$ -submanifolds are given in [6].

Now, let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. We shall denote by the same symbol g both metrics on M and $\tilde{M}$. The projection morphisms of TM to D and $D^\perp$ are denoted respectively by P and Q. Then, for any $X \in \Gamma(TM)$ we have

$$X = PX + QX \tag{1.5}$$

For any vector field N in the normal bundle $TM^\perp$, we put

$$\varphi N = tN + fN \tag{1.6}$$

where tN and fN are the tangential and normal components of φN, respectively. We denote by ∇ the Riemmanian connection on M with respect to the metric g and by $\nabla^\perp$ the linear connection induced by $\tilde{\nabla}$ on the normal bundle $TM^\perp$. Then the Gauss and Weingarten formulas for M are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{1.7}$$

and

$$\tilde{\nabla}_X N = -A_N X + \nabla_X^\perp N \tag{1.8}$$

for all $X, Y \in \Gamma(TM)$ and $N \in \Gamma(TM^\perp)$, where h is the second fundamental form of M and A_N is the Weingarten map with respect to the normal section N. We also have

$$g(h(X,Y), N) = g(A_N X, Y) \tag{1.9}$$

for any $X, Y \in \Gamma(TM)$ and $N \in \Gamma(TM^\perp)$.

Concerning the semi-invariant $\xi^\perp$ -submanifolds of a Kenmotsu manifold we recall the following fundamental results from [6].

Lemma 1.1 *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then we have*

$$P\nabla_X \varphi PY - PA_{\varphi QY} X = \varphi P \nabla_X Y, \tag{1.10}$$

$$Q\nabla_X \varphi PY - QA_{\varphi QY} X = th(X,Y); \tag{1.11}$$

$$h(X, \varphi PY) + \nabla_X^\perp X \varphi QY = \varphi Q \nabla_X Y + fh(X,Y) + g(\varphi PX, Y)\xi \tag{1.12}$$

for any vector fields $X, Y \in \Gamma(TM)$.

Lemma 1.2 *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then we have*

$$A_{\varphi X} Y = A_{\varphi Y} X \tag{1.13}$$

for any $X, Y \in \Gamma(D^\perp)$.

Lemma 1.3 *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then we have*

(a) the anti-invariant distribution $D^\perp$ is always integrable ;

(b) the invariant distribution D is integrable if and only if we have

$$h(X, \varphi Y) - h(\varphi X, Y) = 2g(\varphi X, Y)\xi \tag{1.14}$$

for any $X, Y \in \Gamma(D)$.

Other special results concerning the classes of submanifolds of a Kenmotsu manifold are given in [4], [5] and [6].

2.Geometry of leaves on a semi-invariant $\xi^\perp$ -submanifold in a Kenmotsu manifold . Let M be a proper semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Denote by ν the orthogonal complementary vector bundle to $\varphi D^\perp \oplus \{\xi\}$ in $TM^\perp$. Thus, $TM^\perp$ has the decomposition

$$TM^\perp = \varphi D^\perp \oplus \nu \oplus \{\xi\} \tag{2.1}$$

where $g(\varphi D^\perp, \nu) = 0, g(\nu, \{\xi\}) = 0$.

By using the Weingarten formula and (1.4) we get

$$\eta(h(X,Y)) = -g(X,Y) \tag{2.2}$$

for any $X, Y \in \Gamma(TM)$.

Now we introduce some weaker conditions on the second fundamental form h for the submanifold M of $\tilde{M}$.

We say that the semi-invariant $\xi^\perp$ -submanifold M of $\tilde{M}$ is *D -totally umbilical* (resp. *$D^\perp$ - tatally umbilical*) if

$$h(X,Y) = -g(X,Y)\xi \tag{2.3}$$

for any $X, Y \in \Gamma(D)$ (resp. $X, Y \in \Gamma(D^\perp)$).

The semi-invariant $\xi^\perp$ -submanifold M of $\tilde{M}$ is called *$(D, D^\perp)$ -geodesic* if

$$h(X,Y) = 0 \tag{2.4}$$

for any $X \in \Gamma(D)$ and $Y \in \Gamma(D^\perp)$.

Remark. If the proper semi-invariant $\xi^\perp$ -submanifold M of a Kenmotsu manifold $\tilde{M}$ is D -totally umbilical, then the invariant distribution D is integrable.

Theorem 2.1. *Let M be a D-totally umbilical semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then each leaf of the invariant distribution D is a totally umbilical invariant submanifold immersed in $\tilde{M}$.*

Proof: First, using (1.3) and (1.7) - (1.9) we obtain

$$\begin{aligned} g(\nabla_X Z, \varphi Y) &= g(\tilde{\nabla}_X Z - h(X,Z), \varphi Y) = \\ &= -g(Z, \tilde{\nabla}_X \varphi Y) = g(\varphi Z, h(X,Y)) \end{aligned} \tag{2.5}$$

for any $X, Y \in \Gamma(D), Z \in \Gamma(D^\perp)$.

Now, let M^* be a leaf of the invariant distribution D. Then, from (2.5) we get

$$g(A^*_Z X, Y) = g(-\nabla_X Z + \nabla^{*\perp}_X Z, Y) = -g(\nabla_X Z, Y) = g(h(X,\varphi Y), \varphi Z) = 0 \tag{2.6}$$

for all $X \in \Gamma(TM^*), Y \in \Gamma(D)$ and for Z normal to M^* but tangent to M, where A^*_Z is the Weingarten map of M^* with respect to Z. From (2.6) , by using (1.10) we get

$$h^*(X,Y) = 0 \tag{2.7}$$

for any $X, Y \in \Gamma(TM^*)$, where h^* is the second fundamental form of M^* in M. If we denote by $\tilde{h}$ the second fundamental form of M^* in $\tilde{M}$ we have

$$\tilde{h}(X,Y) = h^*(X,Y) + h(X,Y) \tag{2.8}$$

for any $X, Y \in \Gamma(TM^*)$.

Thus the assertion of Theorem 2.1 follows from (2.8), (2.7) and (2.3).

As we have seen in Lemma 1.3, the distribution $D^\perp$ is always integrable. Denote by $M^\perp$ a leaf of the distribution $D^\perp$.

Theorem 2.2. *The leaf $M^\perp$ is totally geodesic immersed in the semi-invariant $\xi^\perp$ -submanifold M if and only if*

$$h(X,Y) \in \Gamma(\nu) \tag{2.9}$$

for all $X \in \Gamma(D^\perp)$ and $Y \in \Gamma(D)$.

Proof: First, from (2.2) we have

$$\eta(h(X,Y)) = 0 \tag{2.10}$$

for any $X \in \Gamma(D^\perp)$ and $Y \in \Gamma(D)$.

Next, using (1.9) and (1.10) we obtain

$$g(\varphi P \nabla_X Z, Y) = -g(A_{\varphi Z} X, Y) = -g(h(X,Y), \varphi Z) \tag{2.11}$$

for all $X, Z \in \Gamma(D^\perp)$ and $Y \in \Gamma(D)$.

We denote by ∇' the linear connection induced by ∇ on $M^\perp$ and by h' the second fundamental form of the immersion of $M^\perp$ in M. Hence, the Gauss equation is of the form

$$\nabla_X Z = \nabla'_X Z + h'(X,Z) \tag{2.12}$$

for all X, Z tangent to $M^\perp$. Then from (2.10) and (2.11) we obtain

$$g(h'(X,Z), \varphi Y) = g(\nabla_X Z, \varphi Y) = -g(\varphi \nabla_X Z, Y) = g(h(X,Y), \ \varphi Z) \tag{2.13}$$

for X, Z tangent to $M^\perp$ and $Y \in \Gamma(D)$. Finally, the assertion of Theorem 2.2 follows from (2.9), (2.13) and (2.10).

Now, using Theorem 2.2 we state

Theorem 2.3. *Let M be a $(D, D^\perp)$- geodesic semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then we have*

(i) each leaf of the anti-invariant distribution is totally geodesic immersed in M;

(ii) each leaf of the anti-invariant distribution is totally umbilical immersed in the Kenmotsu manifold $\tilde{M}$ if and only if M is $D^\perp$ -totally umbilical.

Proof: Let $M^\perp$ be an arbitrary leaf of the anti-invariant distribution $D^\perp$. Denote by h' and $\tilde{h}$ the second fundamental forms of $M^\perp$ in M and $\tilde{M}$, respectively. Then we have

$$\tilde{h}(X,Y) = h'(X,Y) + h(X,Y) \tag{2.14}$$

for all $X, Y \in \Gamma(D^\perp)$. Then, the assertion (i) of Theorem 2.3 follows from Theorem 2.2 and the assertion (ii) follows by using the assertion (i) and (2.14).

A semi-invariant $\xi^\perp$ -submanifold is said to be *generic* if the vector bundle ν satisfies $\nu = \{0\}$. Then from Theorems 2.2 and 2.3 we obtain

Corollary 2.4. *A generic semi-invariant $\xi^\perp$ -submanifold M of a Kenmotsu manifold $\tilde{M}$ is $(D, D^\perp)$ -geodesic if and only if each leaf of the anti-invariant distribution is totally geodesic immersed in M.*

Moreover, concerning the immersion of each leaf or $D^\perp$ in $\tilde{M}$ we have

Theorem 2.5. *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then each leaf of the anti-invariant distribution is totally umbilical immersed in $\tilde{M}$ if and only if the following conditions are fulfilled:*

(a) $\nabla^\perp_X \varphi Y \in \Gamma(\varphi D^\perp)$ for all $X, Y \in \Gamma(D^\perp)$;

(b) $h(X, Z) \in \Gamma(\nu \oplus \{\xi\})$ for all $X \in \Gamma(D^\perp)$ and $Z \in \Gamma(TM)$.

Proof: For all $X, Y \in \Gamma(D^\perp)$, adding (1.11) and (1.12) we obtain

$$\varphi h(X,Y) = -QA_{\varphi Y}X + \nabla^\perp_X \varphi Y - \varphi Q \nabla_X Y. \tag{2.15}$$

From (2.15) taking into account of (2.2) we get

$$h(X,Y) = \varphi Q A_{\varphi Y} X - \varphi \nabla^\perp_X \varphi Y - Q\nabla_X Y - g(X,Y)\xi \tag{2.16}$$

for all $X, Y \in \Gamma(D^\perp)$.
Next , (1.6) together with (2.16) imply

$$h(X,Y) = \varphi Q A_{\varphi Y} X - f \nabla^\perp_X \varphi Y - g(X,Y)\xi \tag{2.17}$$

and

$$t\nabla^\perp_X \varphi Y + Q\nabla_X Y = 0 \tag{2.18}$$

for all $X, Y \in \Gamma(D^\perp)$.
Soppose each leaf of $D^\perp$ is totally umbilical immersed in the Kenmotsu manifold $\tilde{M}$. Then, by using (2.14), Theorems 2.2 and (2.3) we obtain the condition (b).

Moreover, (2.17) implies $f\nabla^\perp_X \varphi Y = 0$ for all $X, Y \in \Gamma(D^\perp)$ which is equivalent to condition (a); Conversely, suppose conditions (a) and (b) are satisfied. Then, from (2.17) we get $h(X,Y) = -g(X,Y)\xi$ for all $X, Y \in \Gamma(D^\perp)$. On the other hand, from the condition (b), by takimg into account of (2.10) and Theorem 2.2 we infer that any leaf of $D^\perp$ is totally geodesic immersed in M. Finally, by (2.14) it follows that any leaf of $D^\perp$ is totally umbilical immersed in $\tilde{M}$.

Corollary 2.6. *Let M be a generic semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then each leaf of the anti- invariant distribution is totally umbilical immersed in $\tilde{M}$ if and only if M is $(D, D^\perp)$ -geodesic and $D^\perp$ -totally umbilical immersed in $\tilde{M}$.*

3.The indeced f-structure on a semi-invariant $\xi^\perp$ -submanifold. Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. We put $F = \varphi_0 P$ and using (1.1) we obtain $F^3 + F = 0$ that is F is an f -structure on M. In fact, we have $F^2 X = -X$ for each $X \in \Gamma(D)$ and $FY = 0$ for all $Y \in \Gamma(D^\perp)$. The purpose of this section is to study the fundamental properties of the f -structure F. First, by direct computation, using (1.5), (1.10) and (1.11) we get

Lemma 3.1. *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then we have*

$$th(X,Y) = \nabla_X \varphi PY - \varphi P \nabla_X Y - A_{\varphi QY} X = (\nabla_X F)Y - A_{\varphi QY} X \tag{3.1}$$

for all $X, Y \in \Gamma(TM)$.

The f -structure F is said to be *parallel* if $\nabla_X F = 0$ for all $X \in \Gamma(TM)$. Then Lemma 3.1 implies

Lemma 3.2. *Let M be a semi-invariant $\xi^\perp$ -submanifold of a Kenmotsu manifold $\tilde{M}$. Then the f-structure F is parallel if and only if*

$$th(X,Y) = -QA_{\varphi QY}X \tag{3.2}$$

for any $X, Y \in \Gamma(TM)$.

Now we can state

Theorem 3.3. *Let M be an m-dimensional generic semi-invariant $\xi^\perp$ -submanifold of a (2n+1) -dimensional Kenmotsu manifold $\tilde{M}$. If the f-structure F is parallel, then M is locally the Riemmanian direct product $M_1 \times M_2$, where M_1 is a $(2m-2n)$ -dimensional totally umbilical invariant $\xi^\perp$ -submanifold of $\tilde{M}$ and M_2 is a $(2n-m)$ -dimensional anti -invariant $\xi^\perp$ -submanifold of $\tilde{M}$.*

Proof: Since M is a generic semi-invariant $\xi^\perp$ -submanifold we have $\varphi = t$ on $TM^\perp$. Then from (3.2), by using (2.2) we get

$$h(X,Y) = -g(X,Y)\xi \tag{3.3}$$

for any $X \in \Gamma(TM), Y \in \Gamma(D)$.
By using (3.3) and (1.12) we obtain

$$\nabla^\perp_X \varphi QY - \varphi Q \nabla_X Y = 0 \tag{3.4}$$

for all $X, Y \in \Gamma(TM)$.
Now, we take $Y \in \Gamma(D)$. Then (3.4) implies $\nabla_X Y \in \Gamma(D)$ for each $X \in \Gamma(TM)$. Consequently, the distribution D is parallel.
Since $\nabla_X F = 0$, we have

$$\nabla_X \varphi PY - \varphi P \nabla_X Y = 0 \tag{3.5}$$

for any $X, Y \in \Gamma(TM)$. Then (3.5) implies $\nabla_X Y \in \Gamma(D^\perp)$ for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D^\perp)$, which means that the distributions $D^\perp$ is also parallel. Therefore, M is locally the Riemmanian direct product $M_1 \times M_2$, where M_1 is a leaf of the invariant distribution D and M_2 is a leaf of $D^\perp$. Of course, M_1 is an invariant $\xi^\perp$ -submanifold of $\tilde{M}$ and M_2 is an anti-invariant $\xi^\perp$ -submanifold of $\tilde{M}$. Moreover, by (3.3) the conditions of Theorem 2.1 are satisfied, hence M_1 is a totally umbilical $\xi^\perp$ -submanifold of $\tilde{M}$. The dimensions of M_1 and M_2 are obtained by direct computation using the definition of a generic semi-invariant $\xi^\perp$ -submanifold. The proof is complete.

By using Lemma 3.2 we have

Lemma 3.4. *Let M be a semi-invariant $\xi^\perp$ -submanifolds of a Kenmotsu manifold $\tilde{M}$. Then the f-structure F is parallel if and only if*

$$h(X, Y) \in \Gamma(\nu \oplus \{\xi\}). \tag{3.6}$$

for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D)$.

Finally we state

Theorem 3.5. *Let M be a $(2p + q)$ -dimensional semi-invarint $\xi^\perp$ -submanifold of a ($2p + 2q + 2s + 1$) -dimensional Kenmotsu manifold $\tilde{M}$, where* dim. $D_x = 2p$,dim. $D_x^\perp = q$, dim. $\nu_x = 2s$, *for any $x \in M$. If the f-structure F is parallel, then M is locally the Riemmanian direct product $M_1 \times M_2$, where M_1 is a $2p$ - dimensional invariant $\xi^\perp$ -submanifold of $\tilde{M}$ totally geodesic immersed in M and M_2 is a q-dimensional anti-invariant $\xi^\perp$ -submanifold of $\tilde{M}$.*

Proof: From (1.12) we obtain

$$h(X, \varphi Y) = \varphi Q \nabla_X Y + fh(X, Y) + g(\varphi PX, Y)\xi \tag{3.7}$$

for all $X \in \Gamma(TM), Y \in \Gamma(D)$.
Since the f -structure F is parallel, from (3.6) and (3.7) we get

$$\varphi Q \nabla_X Y = 0 \tag{3.8}$$

and

$$h(X, \varphi Y) = fh(X, Y) + g(\varphi X, Y)\xi \tag{3.9}$$

for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D)$. Then (3.8) implies $\nabla_X Y \in \Gamma(D)$ for any $X \in \Gamma(TM), Y \in \Gamma(D)$, i.e. the distribution D is parallel. By taking $Y \in \Gamma(D^\perp)$

in (3.5) we get $\nabla_X Y \in \Gamma(D^\perp)$ for all $X \in \Gamma(TM)$ and $Y \in \Gamma(D^\perp)$, i.e. the distribution $D^\perp$ is parallel too. Moreover, by using (3.9) in a similar way as in the proof of Theorem 2.1 we obtain that any leaf of the invariant distribution D is totally geodesic immersed in the semi-invariant $\xi^\perp$ -submanifold M. Finally, the assertion of Theorem 3.5 follows in a similar way as the last part of the proof of Theorem 3.3.

Remark. Examples of semi-invariant $\xi^\perp$ -submanifolds in a Kenmotsu manifold which are locally the Riemannian direct products $M_1 \times M_2$ may be obtained if we consider CR-products in a Kähler manifold $\tilde{N}$ (see [1], [3]) and we take into account that any CR -submanifold of a Kähler manifold $\tilde{N}$ is a semu-invariant $\xi^\perp$ -submanifold of the warped product $\tilde{M} = R \times_f \tilde{N}$, where $f(t) = ce^t, c > 0$, [6].

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