hence $(G,\cdot)$ is associative. Similarly, $g\cdot xy = (gx)R(g)^{-1}\cdot gy$ implies that $xy\cdot g = x\cdot yg$ for all $x,y,g\in G$.

REFERENCES

1. B a s a r a b, A.S. – *On a class of G-loops (Russian)*, Mat. Issled., 3, vyp. 2,3-24 (1968).
2. B e l o u s o v, V.D. – *Foundations of the theory of quasigroups and loops (Russian)*, (Izdat. Nauka, Moscow 1967).
3. B e u c k, R.H. – *A Survey of binary systems*, (Springer - Verlag, Berlin and New York, 1958).
4. C h e i n, O., P f l u g f e l d e r, H. – *On maps $x\to x^n$ and the isotopy - isomorphy property of Moufang loop*, Acquationes Math. 6(1971) 157 - 161.
5. C o x e t e r, H.S.M. – *Integral Cayley numbers* , Duke Math., J., 13 (1946), 561 - 577.
6. F e n y v e s, F. – *Extra loops I & II*, Publ. Math. Debrecen, 15 (1968), 235 - 238; 16(1969), 187 - 192.
7. G o o d a i r e, E.G., R o b i n s o n, D.A. – *A class of loops which are isomorphic to all loop isotopes*, Can. J. Math. 34 (3), 1982, 662 - 672.
8. N o r t o n, D.A. – *Hamiltonian loops*, Proc. Amer. Math. Soc., 3, 56 - 65 (1952).
9. P f l u g f e l d e r, H.O. – *A special class of Moufang loops*, Proc. Amer. Math. Soc. 21, 583 - 586 (1970).
10. S h a r m a, B.L. – *Laws for characterizing M-loops* , Ann. Soc. Sci. Bruxelles, T. 90, I pp 88 - 94 (1976).
11. S h a r m a, B.L., S o l a r i n, A.R.T. – *On the classification of Bol loops of order $3p(p>3)$*, Comm. Alg. 16, (1988) 37 - 55.
12. S o l a r i n, A.R.T., C h i b o k a, V.O. – *On some classes of G-loops*, (communicated for publication)
13. W i l s o n, R.L.Jr. – *Loop isotopism and isomorphism and extensions of universal algebras*, Ph.D. Thesis, University of Wisconsin, Madison (1969).

Received:6.V.1991
Revised:21.I.1993

Department of Mathematics
Obafemi Awolowo University
Ile-Ife
NIGERIA

ANALELE STIINTIFICE ALE UNIVERSITATII "AL.I.CUZA" IASI
Tomul XXXIX, s.I. a, Matematica, 1993, f.1.

ON CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS

BY

B.A. URALEGADDI and C. SOMANATHA

Introduction. Let A_n denote the class of all functions of the form $f(z) = z + a_{n+1}z^{n+1} + a_{n+2}z^{n+2} + \ldots$ that are regular in the unit disk $E = \{z : |z| < 1\}$. A function $f(z)$ in A_n is said to be starlike if

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0 \ for\ z\in E,$$

and is said to be convex if

$$\mathrm{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\} > 0 \ for\ z\in E.$$

Let

$$D^\alpha f(z) = \frac{z}{(1-z)^{\alpha+1}} * f(z),\ \alpha > -1 =$$
$$= z + \sum_{m=n}^{\infty}\frac{(\alpha+1)(\alpha+2)\ldots(\alpha+m)}{m!}a_{m+1}z^{m+1}$$

and the operation $(*)$ denotes the Hadamard product of the power series. R u s c h e w e y h introduced the symbol D^α in [3].

In this paper we obtain certain properties of functions $f(z)$ in A_n that satisfy $\mathrm{Re}(D^\alpha f(z)/z) > 0$ for $z\in E$. Some of the results of M a c G r e g o r [1,2] are obtained as particular cases.

Theorem 1. *Suppose that $f(z) = z + a_{n+1}z^{n+1} + \ldots$ is regular and satisfies $\mathrm{Re}(D^\alpha f(z)/z) > 0$ for $z\in E$. Then*

$$0 < \mathrm{Re}\frac{D^{\alpha+1}f(z)}{D^\alpha f(z)} > \frac{\alpha}{\alpha+1}\ in\ |z| < \left((n^2+1)^{1/2}-n\right)^{1/n}$$

and this result is sharp.

Proof. Since $\mathrm{Re}(D^\alpha f(z)/z) > 0$, we have

$$\frac{D^\alpha f(z)}{z} = p(z),\ where\ p(z) = 1 + c_n z^n + c_{n+1} z^{n+1} + \dots \tag{1}$$

is regular in $E, p(0) = 1, \mathrm{Re}\, p(z) > 0$ in E. The logarithmic differentiation of (1) yields

$$\frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} = 1 + \frac{1}{(\alpha+1)}\, z\, \frac{p'(z)}{p(z)}. \tag{2}$$

It is well known (see [2]) that

$$\left| z\frac{p'(z)}{p(z)} \right| < \frac{2n|z|^n}{1-|z|^{2n}}.$$

It is follows from (2) that

$$\mathrm{Re}\frac{D^{\alpha+1} f(z)}{D^\alpha f(z)} \geq 1 - \frac{2n|z|^n}{(\alpha+1)(1-|z|^{2n})} >$$
$$> \frac{\alpha}{\alpha+1}\ if\ |z| < ((n^2+1)^{1/2} - n))^{1/n}.$$

The result is sharp for the function $f(z)$ for which $\frac{D^\alpha f(z)}{z} = \frac{1+z^n}{1-z^n}$.

As special cases we obtain several results of M a c G r e g o r (see [1], [2]). Thus, putting $\alpha = 0$, we get

Corollary 1. *If $f(z)$ is regular and satisfies $\mathrm{Re}(f(z)/z) > 0$ in E, then $\mathrm{Re}(zf'(z)/f(z)) > 0$ ($f(z)$ is starlike and univalent) in $|z| < ((n^2+1)^{1/2} - n)^{1/n}$.*

For $\alpha = 1$, Theorem1 yields the following earlier result, of [2].

Corollary 2. *If $f(z)$ is regular and satisfies $\mathrm{Re} f'(z) > 0$ in E then $\mathrm{Re}(1 + zf''(z)/f'(z)) > 0$ in $|z| < ((n^2+1)^{1/2} - n)^{1/n}$.*

Also, the both results are established for $n = 1$ in [1].

Theorem 2. *Let $f(z) = z + a_{n+1}z^{n+1} + \dots$ be regular in E and*

$$F(z) = \frac{\alpha+1}{z^\alpha}\int_0^z t^{\alpha-1} f(t)dt,\ \alpha > -1.$$

Then $\mathrm{Re}(D^{\alpha+1}F(z)/z) > 0$ in E if and only if $\mathrm{Re}(D^\alpha f(z)/z) > 0$ in E.

Proof. From the definition of F we have

$$D^\alpha(zF'(z)) + \alpha D^\alpha F(z) = (\alpha+1)D^\alpha f(z).$$

That is

$$z(D^\alpha F(z))' + \alpha D^\alpha F(z) = (\alpha+1)D^\alpha f(z). \tag{3}$$

By using the identity

$$z(D^\alpha F(z))' = (\alpha+1)D^{\alpha+1}F(z) - \alpha D^\alpha F(z),$$

(3) reduces to

$$D^\alpha f(z) = D^{\alpha+1}F(z)$$

and the result follows.

For $\alpha = 0$, Theorem 2 yields the following result.

Corollary 3. Let $f(z) = z + a_{n+1}z^{n+1} + \dots$ be regular in E and

$$F(z) = \int_0^z (f(t)/t)dt$$

then $\mathrm{Re}F'(z) > 0$ in E if and only if $\mathrm{Re}(f(z)/z) > 0$ in E.

REFERENCES

1. M a c G r e g o r, T.H. – *Functions whose derivative has a positive real part*, Trans. Amer. Math. Soc. 104 (1962), 532 - 537.

2. M a c G r e g o r, T.H. – *The radius of univalence of certain analytic functions*, Proc. Amer. Math. Soc. 14 (1963), 514 - 520.

3. R u s c h e w e y h, St. – *New criteria for univalent functions*, Proc. Amer. Math. Soc. 49 (1975), 109 - 115.

Received:28.VI.1991
Revised:12.IV.1993

Department of Mathematics
Karnatak University
Dharwad - 580 003
INDIA