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THE REFLEXIVE PARADOXES IN SET THEORY: THE LOGICAL RELATION OF THE ABIAN PARADOX TO THE CURRY PARADOX

BY

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The Abian paradox, formulated in 1985, is a generalization of the Russel paradox because it applies to both sets and classes.

We formulate the Abian paradox as

$$(*) \qquad (Ax)(A \in x \longleftrightarrow x \notin x)$$

To prove that the Abian paradox applies to both sets and classes, we show that A in $(*)$ cannot be a class, regardless of whether quantification in $(*)$ is over sets or classes. Indeed replacing x in $(*)$ by Φ (which is both a set and a class), we arrive at the contradiction $A \in \Phi \longleftrightarrow \Phi \notin \Phi$ since $A \in \Phi$ is always false and $\Phi \notin \Phi$ is always true. It is not difficult to show that the Abian paradox does not totally depend upon the existence of Φ.

The Russel paradox is weaker than the Abian paradox, since the Russel paradox fails in cases where the Abian paradox holds. The reflexive paradoxes have been analyzed by G o d d a r d, J o h n s t o n [1], G o d d a r d [2] as arising in quantified formulas of the form $\neg(\exists)(\forall y)[f(y,x) \longleftrightarrow \neg f(y,y)]$, where a property f presented in the formula entails the negation of an quantifier instantiation for the formula containing presuppose any such condition(s). They argue that resolution of such reflexive paradoxes is achieved only if the value chosen to instantiate the existential quantifier is some how removed from the range of the universal quantifier. The Abian paradox exhibits this structure of reflexive paradoxes presented by G o d d a r d and J o h n s t o n and requires the same conditions for its resolution.

The present status of research described above justifies in raising the open question in the context of Russel's paradox: How to define a paradoxical class (or to point to some already defined one), the paradoxality of which cannot be generated by the uniform principle which is, in fact, the application of Cantor's theorem to the universe of all classes. This question has not been answered completely but this deserves answer, and giving this answer is the subject of this project.

Definition. *A class A for which there is an infinite progression of classes $A_1, A_2, \ldots$ (not necessarily all distinct) such that $\ldots \in A_2 \in A_1 \in A$ is said to be groundless.*

Definition. *A class which is not groundless is said to be grounded.*

Lemma 1. *Let K be the class of all grounded classes. Then K is groundless and grounded.*

Proof. Let us assume that K is a groundless class. Then there is an infinite progression of classes $A_1, A_2, \ldots$ such that $\ldots \in A_2 \in A_1 \in K$.

Since $A_1 \in K$, A_1 is a grounded class; since $\ldots \in A_3 \in A_2 \in A_1$ A is also a groundless class. But this is impossible.

Therefore K is a grounded class. Hence $K \in K$, and we have $\ldots \in K \in K \in K$. Therefore K is also a groundedless class.

Lemma 2. *If $f : V \to V$ is an onto map, then we can make two disjoint paradoxical classes using f.*

Proof. First $\{x : x \notin f'x\}$ is one. And define $g : V \to V$ by $g'x = (f'x)^c$ for all x. Then g is onto since for given y, there is x such that $f'x = y^c$ and so $g'x = (f'x)^c = y$. Hence $\{x : x \notin g'x\} = \{x : x \notin (f'x)^c\}$ is another paradoxical class and since $\{x : x \notin g'x\} = \{x : x \notin f'x\}^c$, two classes are disjoint.

Theorem. *There are infinitely many paradoxes classes which are not generated by the uniform method.*

Proof. Since $P(V) \subseteq V$, V is a paradoxical class. Suppose V is generated by a map f. Then by Lemma 2, $V^c = \Phi$ would be a paradoxical class, which is absurd.

Now for any nonparadoxical singleton $\{a\}$, consider a class $V \setminus \{a\}$. Let $g_a : P(V \setminus \{a\}) \to V \setminus \{a\}$ be a map defined by

if $a \notin P(V \setminus \{a\})$, $g_a(x) = x$ for all $x \in P(V \setminus \{a\})$,

if $a \in P(V \setminus \{a\})$, $g_a(x) = \begin{cases} x & x = a \\ V & x \neq a. \end{cases}$

Then clearly g_a is $1-1$, and so $V \setminus \{a\}$ is a paradoxical class.

$V \setminus \{a\}$ is not generated by the uniform method, since its complement $\{a\}$ is not paradoxical.

Remark. It remains still open whether there exists a map α such that $\alpha : V \to V$, $\alpha''V = V$ and $C(\alpha, V) = K = \{x \mid \text{is grounded}\}$ as in Lemma 1.

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THE METHOD OF COMPLEX POTENTIALS IN SOLVING TWO-DIMENSIONAL PROBLEMS FOR MEDIA WITH VOIDS

BY

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0.Introduction. The infinitesimal theory of elastic materials possessing a structure of distributed small voids and some questions concerning the behaviour of these materials have been approached by S.C. C o w i n and J.W. N u n z i a t o in [1]. Starting from this theory, S.C. C o w i n defines in [2] the quasi-static problems of plane strain and plane stress applying the appropriate field equations to the study of the problem of stress concentration around a circular hole in an infinite plate subjected to uni and bi-axial tension.

In this paper we give a general method of solving Cowin's equations for arbitrary domains and boundary data. The method which we propose is that of complex potentials. Accordingly, in the first stage one establishes a representation formulae of the general solution of equilibrium equations in terms of three potentials among two are analytic complex valued and, one - real valued - of Helmholtz's type. Then, the obtained representation is used to reduce the boundary value problems of plane strain and plane stress to some problems of complex function theory.

The content of the paper is the the following: the field equations and some of their consequences are presented in Section 1. After the general solution of equilibrium equations is found - Section 2 - we formulate the fundamental boundary value problem in terms of complex functions in Section 3. In Section 4 we deduce the expression of complex potentials in the case of multiply-connected finite and infinite regions. To illustrate the method we have included solutions of some concrete problems in the final part of Sections 3 and 4.